washed out completely.

Clearly, more experimental tests of these ideas are needed. In particular, we suggest *accurate* measurements of π^+p elastic differential cross sections at fixed angles in the region $0.4 \leq \cos\theta \leq 0.8$, $1.8 \leq p \leq 5.5$ GeV/c. Ideally these experiments should be carried at small incident momentum steps.[12] We also suggest similar testing of all pure-isospin $B = 1$ two-body reactions, for which our model gives essentially the same predictions.

I am indebted to R. Stanek and N. Thalassinos for helping with some of the interpolations and to Erich J. Hoff for plotting many graphs.

[1]G. T. Hoff, Phys. Rev. 154, 1331 (1967), and unpublished.

[2]We have used the fixed-angle data of R. A. Sidwell *et al.*, Phys. Rev. D 3, 1523 (1971), and of G. E. Kalmus *et al.*, Phys. Rev. D 4, 676 (1971), the data of W. Busza *et al.*, Phys. Rev. 180, 1339 (1969), and older data taken from G. Giacomelli, P. Pini, and S. Stagni, CERN Report No. CERN-HERA 69-1, 1969 (unpublished).

[3]We were led to this assumption by considerations of a practical nature a few years ago. See G. T. Hoff, Phys. Rev. 18, 816 (1967).

[4]Another way of stating this assumption is by saying that there is a single pole in the complex energy plane associated with each level.

[5]Rising phases have been used by other authors in $\pi\pi$ scattering: A. M. Gleeson, W. J. Meggs, and M. Parkinson, Phys. Rev. Lett. 25, 74 (1970). Their approach is, however, somewhat different from ours.

[6]These properties are easily obtained from expressions given by Dalitz for the background contribution to each partial wave. See R. N. Dalitz, Annu. Rev. Nucl. Sci. 13, 339 (1963).

[7]When γ is equal to either $\frac{1}{2}\pi$ or $-\frac{1}{2}\pi$. This situation arises in particular when the background amplitudes are purely imaginary.

[8]When γ is equal to either $\frac{1}{2}\pi$ or $-\frac{1}{2}\pi$ a change of phase by the amount π gives rise to an inflection point, not to a cusp.

[9]An exception to this statement is the case when the background-resonance interference and resonance contributions cancel each other at a given angle.

[10]The extreme distortion of the sinusoidal curve in Fig. 3 could be interpreted as due to a strong departure of the background from a k^{-1} dependence (the total cross section is still increasing in this energy region).

[11]It is currently believed that there are five established resonances between 0.82 and 2.08 GeV/c: $\Delta(1650)$, $\Delta(1670)$, $\Delta(1890)$, $\Delta(1910)$, and $\Delta(1950)$, with full widths ranging from 165 to 325 MeV. The average spread for each resonance energy (total width) is 87 MeV (126 MeV). See P. Söding *et al.*, Phys. Lett. 39B, 1 (1972).

[12]Efforts should be made to find a solution to all the available π^+p elastic data constrained to satisfy our basic assumptions.

Why Does the Pomeranchuk Singularity Have a Unit Intercept?

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We suggest, contrary to the spirit of the Amati-Fubini-Stanghellini multiperipheral model and a φ^3 ladder theory, that the relation $\alpha_P(0) = 1$ is independent of the strength of the strong interaction. We propose that the "zero strength" ($g \to 0$) limit of a future correct strong-interaction theory will collapse into a vector-type field theory, guaranteeing $\alpha_P(0) = 1$ even as $g \to 0$. The intercept of the leading non-Pomeranchukon exchange then becomes lower when g increases. Specific models which are consistent with this approach lead to a new relation between this intercept and the moments of the multiplicity distribution.

The nature of the Pomeranchuk singularity has always been a mystery. Empirically, we know that all total hadronic cross sections seem to approach a constant at high energies (modulo factors of $\ln s$). Recent data[1] from Serpukhov, the National Accelerator Laboratory, and the Intersecting Storage Rings seem to confirm this trend, which has earlier been observed at lower energies. It is often said that the constancy of σ_{tot} is due to the unit intercept of the Pomeranchuk singularity, which dominates the imaginary part of the forward elastic scattering amplitude. Needless to say, this statement does not clarify the nature of this singularity, nor does it explain why $\alpha_P(0) = 1$.

The solution to both of these puzzles is almost certainly buried in the unitarity relation which simply states that the sum of all the cross sections for specific final states adds up to the total cross section and must therefore produce the Pomeranchuk singularity. One way of expressing σ_{tot} is in terms of the trivial relation

$$\sigma_{tot}=\sum_n \sigma_n, \tag{1}$$

where σ_n is the cross section for producing n hadrons in the final state. Experimentally, we know that over the energy range in which σ_{tot} stays roughly constant, the various σ_n's vary dramatically [e.g., between $P_{lab}=13$[2] and 300 GeV/c[3] $\sigma_{tot}(pp)$ changes by less than 5%, while $\sigma(pp\to 4$ prongs) drops by a factor of 2.7, and $\sigma(pp\to 8$ prongs) increases by a factor of 10]. We therefore have to understand how a sum of *energy-dependent partial* cross sections is able to produce an *energy-independent total* cross section, thereby creating the Pomeranchuk singularity at $\alpha_P(0)=1$.

In this note we address ourselves to the question: Why does the Pomeranchuk singularity have a unit intercept? We propose a general answer to this question and discuss various consequences of our proposal within the framework of simple models.

Before we begin our discussion, we must, however, mention an important technical point. We have quoted the experimental observation that the σ_n's vary significantly with energy. The strong energy dependence of the two-, four-, and six-prong pp cross sections indicates that *most* of the contributions to a given σ_n come from nondiffractive mechanisms, involving no Pomeranchukon exchange in the production of a given n-particle final state. The sum of these *nondiffractive* contributions for any given σ_n drops like a power of s as $s\to\infty$. Now, in reality, there probably *is* a small diffractive contribution to any given σ_n (especially for small n values). For the sake of simplicity we ignore these diffractive contributions, assuming that the sum of all diffractive terms in the various σ_n's add to a (small) constant fraction of σ_{tot}, while the sum of all nondiffractive, energy-dependent pieces of the σ_n's adds to another (large) constant fraction of σ_{tot}.[4] In the following, we focus our attention on the latter term, which has the property of a constant total cross section produced by a sum of terms, each of which falls like a power of s. The inclusion of the small diffractive terms would make our discussion more cumbersome, while none of the essential features of our conclusions would change.

We are now ready for our basic question: How do the various σ_n's add up to a constant σ_{tot}, producing a singularity at $\alpha_P(0)=1$?

There are two basic approaches to this question. The first approach relates the value of $\alpha_P(0)$ to the strength of the strong interactions. More precisely, within the framework of specific models which advocate this approach $\alpha_P(0)$ is determined by an equation involving a strong-interaction coupling constant. In such models, we face the following proposition: *Had nature selected a somewhat weaker (or stronger) strong interaction, $\alpha_P(0)$ would be smaller (or larger) than 1.* The most famous (and earliest) model of this variety is the Amati-Fubini-Stanghellini (AFS) multiperipheral model[5] involving the exchange of spin-0 mesons. Later models involve field-theoretical exercises with φ^3 interactions.[6] In all of these cases, every σ_n obeys

$$\sigma_n \xrightarrow[s\to\infty]{} (g\ln s)^n s^{-2}, \tag{2}$$

and the constancy of σ_{tot} can be achieved only if the coupling constant g has a certain specific value. In particular, the same models, in the limit $g\to 0$, would predict $\sigma_{tot}\to s^{-2}$, yielding $\alpha_P(0)=-1$. Many versions of this approach exist in the literature and in all of them

$$\alpha_P(0)=-1+f(g), \tag{3}$$

where $f(g)$ is a function of the coupling constant, obeying $f(g)\to 0$ as $g\to 0$. On the other hand, the *energy* dependence of any given σ_n in such models is independent of g [Eq. (2)]. The value $\alpha_P(0)=1$ seems to be an "accident" in this type of approach. *We feel that such accidents are very unlikely and that the observed constancy of σ_{tot} is much more fundamental.*

We therefore prefer a second approach which can be formulated in the following way: We assume that $\alpha_P(0)=1$ is a fundamental result of strong-interaction dynamics, a result which would remain stable under an (imaginary) variation of the strength of the strong interactions. In particular, we believe that if we consider the $g\to 0$ limit of a future correct theory of the strong interactions, we would still have $\alpha_P(0)=1$.[7] In principle, there is a very simple way of achieving this. Any field-theory model involving vector exchanges (such as a massless Yang-Mills theory) would yield a total cross section which is constant in energy in the limit $g\to 0$. If we then start with such a theory for $g\to 0$ and gradually increase g, we would be starting with $\alpha_P(0)=1$, and would be prevented from increasing $\alpha_P(0)$ by the Froissart bound. A complete unitary theory of the strong interactions may be a dream for the future, but it is clear that any such theory that would collapse into a vector field theory for $g\to 0$ would have $\alpha_P(0)=1$ for all possible strengths of the strong interactions. It is also reasonable to assume that the $g\to 0$ limit of a future strong-interaction theory would collapse into *some* kind of a field theory, and a vector theory is the only one with $\alpha_P(0)=1$ for $g\to 0$.

What will be the energy dependence of the various σ_n's in such a theory? For $g\to 0$, every σ_n will involve simple vector exchange diagrams, and will therefore remain constant in energy (modulo $\ln s$ terms). Furthermore, as $g\to 0$, $\sigma_n/\sigma_{n-1}\to 0$ and $\sigma_2\to\sigma_{\text{tot}}$. If we now gradually increase g, the high-multiplicity cross sections will become more and more important. However, since the total cross section must remain energy independent, each individual σ_n will probably fall faster and faster with s, and the simple vector exchange will be replaced by the exchange of a more complicated (unitarized, renormalized) object. We may assume the parametrization

$$\sigma_n \xrightarrow[s\to\infty]{} s^{2\alpha_R(g)-2}, \tag{4}$$

where $\alpha_R(g)$ is the $t=0$ intercept of the leading non-Pomeranchukon exchange, for a coupling constant g (ignoring $\ln s$ terms). We then find that

$$\alpha_R(g) \xrightarrow[g\to 0]{} 1, \tag{5}$$

while as g increases $\alpha_R(g)$ decreases below 1! In other words, in the $g\to 0$ limit, the leading non-Pomeranchukon exchange is our vector field. When g increases, each σ_n begins to fall as $s\to\infty$ and α_R moves away from a unit intercept (and probably acquires a t dependence as well). *The stronger the interaction is, the lower the intercept of the leading non-Pomeranchukon term* !

The contrast between our proposal and the first (AFS or φ^3) approach[5,6] is now clear. In our case $\alpha_P=1$ for all g, while $\alpha_R=1-F(g)$, where $F(g)\geq 0$, $F(0)=0$. In the first approach, $\alpha_R=0$ for all g while $\alpha_P=-1+f(g)$, where $f(g)\geq 0$, $f(0)=0$.

Until this point we have concentrated on the somewhat imaginary possibility of varying the strength of the strong interaction. Such a discussion might be interesting, but it cannot lead to experimental tests of our theoretical conjectures. The only way to test such considerations is to consider the g dependence of several measurable quantities and to eliminate g, thereby deriving new relations between previously unrelated quantities. We will now study this possibility within the framework of several simple models.

The simplest and most naive realization of our approach is the Reggeistic multiperipheral model of Chew and Pignotti.[8] These authors assume that every σ_n is accounted for by a simple ladder diagram involving the exchange of one type of meson trajectory α_R. They further assume that the produced hadrons are independently emitted. They find (g is the squared coupling constant)

$$\sigma_n \propto \frac{(g\ln s)^n}{n!}\, s^{2\alpha_R-2}. \tag{6}$$

If we further assume that $\sigma_{\text{tot}}\to s^{\alpha_P(0)-1}=\text{const}$, we obtain

$$\alpha_R = 1-\tfrac{1}{2}g. \tag{7}$$

However, in the same model the average multiplicity $\langle n\rangle$ is given by

$$\langle n\rangle \xrightarrow[s\to\infty]{} c_1 \ln s, \tag{8}$$

where $c_1=g$. We therefore find

$$\alpha_R = 1-\tfrac{1}{2}c_1, \tag{9}$$

where both α_R and c_1 are measurable quantities (they can be deduced, respectively, from the energy dependence of σ_n and $\langle n\rangle$). We see immediately from Eq. (9) that *larger multiplicities (stronger interaction !) are necessarily associated with a lower intercept for the meson trajectory.*

A more realistic model would abandon the assumption of an independent emission of the final hadrons. Any model in which σ_n is dominated by *tree diagrams* (in which the internal lines may be Reggeons, sums of direct channel resonances, etc.) would lead to a relation of the form

$$\sigma_n = g^n f(n,s) s^{2\alpha_R(g)-2}, \tag{10}$$

where the s dependence in $f(n,s)$ is at most logarithmic as $s\to\infty$, and $\alpha_R(g)$ is the $t=0$ intercept of the leading non-Pomeranchukon trajectory. All reasonable multiperipheral and multi-Regge models as well as most versions of the dual-resonance model (with no loops) must obey Eq. (10).

We now assume

$$\sigma_{\text{tot}} = \textstyle\sum_n \sigma_n = \sum_n g^n f(n,s) s^{2\alpha_R(g)-2} \xrightarrow[s\to\infty]{} K(g). \tag{11}$$

Hence

$$\textstyle\sum_n g^n f(n,s) \to K(g) s^{2-2\alpha_R(g)}. \tag{12}$$

We now define the generating function[9]

$$Q(z) = \textstyle\sum z^n \sigma_n, \tag{13}$$

such that the multiplicity moments f_m are given by

$$f_m = d^m \ln Q(z)/dz^m|_{z=1}, \quad (14)$$

and $f_1 = \langle n\rangle$, $f_2 = \langle n(n-1)\rangle - \langle n\rangle^2$, etc. Using Eq. (10), we then find

$$Q(z) = \sum_n (zg)^n f(n,s) s^{2\alpha_R(g)-2} = K(zg) s^{2\alpha_R(g) - 2\alpha_R(zg)}, \quad (15)$$

$$f_m = d^m \ln Q(z)/dz^m|_{z=1} = g^m(d^m/dg^m)[-2\alpha_R(g)\ln s + \ln K(g)]. \quad (16)$$

Our first conclusion is the interesting result that all the moments f_m (including $\langle n\rangle$) behave like[9]

$$f_m = c_m \ln s + d_m. \quad (17)$$

We further learn, however, that

$$c_m = -2g^m d^m \alpha_R(g)/dg^m. \quad (18)$$

Expanding $\alpha_R(g)$ around a point g, we find

$$\alpha_R(g') = \alpha_R(g) + \sum_{m=1}^{\infty} \frac{(g'-g)^m}{m!} \frac{d^m\alpha_R(g')}{dg'^m}\bigg|_{g'=g}. \quad (19)$$

If we now choose g as the "g value of the real world" and apply the expansion to the point $g'=0$, we obtain [using Eqs. (5) and (18)]

$$1 = \alpha_R(g) - \frac{1}{2}\sum_{m=1}^{\infty}(-1)^m \frac{c_m}{m!} \quad (20)$$

or

$$\alpha_R = 1 - \tfrac{1}{2}c_1 + \tfrac{1}{4}c_2 \tfrac{1}{12}c_3 + \cdots. \quad (21)$$

The independent emission case of Chew and Pignotti[8] corresponds to $f_m = 0$, $c_m = 0$ for $m \geq 2$, and it is clearly a special case of our new relation [compare Eq. (9)].

A detailed comparison of Eq. (21) with experiment requires a careful separation of the diffractive part[4] and we shall return to it in a future publication. Here we only remark that the data indicate that $\alpha_R > 1 - \frac{1}{2}c_1$, predicting a positive value for c_2 (if the following terms are neglected). Since f_2 increases with energy, c_2 is probably indeed positive, in agreement with our prediction.

Another model which is consistent with our approach is the version of the parton model proposed by Kogut and Susskind.[11] Their version is similar to the Chew-Pignotti scheme, and it leads to Eq. (9).

It is, of course, possible to invent more complicated and more sophisticated models within the framework of our approach. A particularly interesting challenge would be to construct an explicit dual-resonance model in which the relative strength of the n-point and $(n-1)$-point amplitudes is related to the intercept of the leading input trajectory. In this connection it is interesting to note that the Virasoro model collapses in the $g\to 0$ limit into a massless Yang-Mills theory.[12] In that model, however, $\alpha_R = 1$ for all g, while we are looking for a model in which $\alpha_R \to 1$ for $g \to 0$, but $\alpha_R < 1$ for $g \neq 0$.

None of the points discussed above shed any light on the nature (rather than the location) of the Pomeranchuk singularity. We must add, however, that nowhere had we assumed that the Pomeranchukon is a pole. Any complicated singularity would suit us, as long as its leading term corresponds to $\alpha_P(0) = 1$.

We finally summarize our main points again: We suggest that $\alpha_P(0) = 1$ is a fundamental relation in hadron dynamics, rather than an "accident," as it appears to be in scalar ladder theories. We propose that the "zero-strength limit" of a future correct theory of the strong interactions is a vector field theory. In such a case, it is very plausible that, for all g, $\alpha_P(0) \geq 1$. Since the Froissart bound tells us that for all g, $\alpha_P(0) \leq 1$, we necessarily conclude that $\alpha_P(0) = 1$, regardless of the value of g and regardless of the specific nature of the singularity. Specific models based on this philosophy may lead to relations among measurable quantities. One such model yields the interesting relation of Eq. (21), which is, at present, in qualitative agreement with experiment. A conclusive test of our ideas will emerge, however, only when a correct theory of the strong interactions is finally achieved.

The author would like to thank Adam Schwimmer for many helpful discussions.

[1]See, e.g., G. Giacomelli, Rapporteur's talk, in Proceedings of the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Illinois, 1972 (to be published).

[2]D. B. Smith *et al.*, Phys. Rev. Lett. 23, 1064 (1969).

[3]F. T. Dao *et al.*, to be published.

[4]An approximate description of the various σ_n values can be given in terms of a two-component description involving a diffractive component and a multiperipheral-type, nondiffractive component. Such a description has been discussed by, among others, K. G. Wilson, Cornell University Report No. CLNS-131 (to be published); J. Ellis, J. Finkelstein, and R. D. Peccei,

SLAC Report No. SLAC-PUB-1020 (unpublished), and to be published; A. Białas, K. Fialkowski, and K. Zalewski, Krakow Report No. TPJU-5/72 (to be published).

[5] D. Amati, S. Fubini, and A. Stanghellini, Nuovo Cimento 26, 896 (1962).

[6] Recent discussions of multiparticle production via φ^3 interactions can be found in D. K. Campbell and S. J. Chang, Phys. Rev. D 4, 1151 (1971); T. D. Lee, to be published; and many other papers.

[7] In this sense we take the opposing view to the notion that $\alpha_P(0)=1$ is related to a "maximal strength" of the strong interaction. In fact, quantum electrodynamics is an obvious example of a "weak" theory which, nevertheless, yields a constant total cross section. We suspect that the constancy of σ_{tot} in hadron physics has nothing to do with a "maximal strength."

[8] G. F. Chew and A. Pignotti, Phys. Rev. 176, 2112 (1968).

[9] See, e.g., A. H. Mueller, Phys. Rev. D 4, 150 (1971).

[10] L. Caneschi [Nucl. Phys. B35, 406 (1971)] has proposed, in a different context, an expansion around the "g value of the real world."

[11] J. B. Kogut and L. Susskind, to be published.

[12] A. Neveu and J. Scherk, Nucl. Phys. B36, 155 (1972).

O(4) Treatment of the Electromagnetic-Weak Synthesis*

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A gauge theory is outlined in which the existence of the Cabibbo angle θ, leptonic CP-invariance violation, and μ-e nonuniversality are linked. To $O(G)$, the sole source of nonleptonic decays is a neutral current. There is no conflict with present neutrino data. This CP-invariance violation induces only superweak effects in K decays. A strategy for finding θ is indicated.

Imagine a semileptonic interaction of the form[1] $ig_1(\bar{\mathcal{P}}\mathfrak{N}+\bar{\nu}_e e+\bar{\nu}_\mu \nu)_L W^1 + ig_2\{\bar{\mathcal{P}}\lambda + a(e^{i\varphi}\bar{\nu}_e e + e^{i\psi}\bar{\nu}_\mu \mu)\}_L W^2$ + H.c. $W^{1,2}$ are charged vector bosons with masses $M_1 \neq M_2$; $\mathcal{P}$, $\mathfrak{N}$, and λ are quarks. The real number a and the phases φ and ψ are observable. To leading order, μe universality is valid in $\mathfrak{N}\to\mathcal{P}$ and in $\lambda\to\mathcal{P}$ β decays. The condition that the absolute values of the amplitudes for $\mu\to e$, $\mathfrak{N}\to\mathcal{P}$, $\lambda\to\mathcal{P}$ are in the ratios[1] $1:\cos\theta:\sin\theta$ implies $(1-a^2)\tan\theta = 2a\cos(\psi-\varphi)$. If $\psi=\varphi$, $a\neq 1$, μe universality is strict. If $\psi\neq\varphi$, it is violated together with CP. To $O(g_1^2; g_2^2)$, CP and T are conserved as is seen by redefining phases; but not, in general, in higher order. In this note, these ideas are examined[2] in the context of a CPT-invariant O(4)-gauge theory, with the choice $a=1$, $\varphi=0$, $\psi=\pi/2$. While then CP-invariance violation is maximal for the muonic terms in the $\Delta Q=\pm 1$ currents, the physical effects thereof will turn out to be minuscule for[1] $q^2/M^2 \ll 1$.

Six gauge fields, A_μ^i, C_μ^i, $i=1, 2, 3$, appear in the gauge-invariant derivative $D_\mu = \partial_\mu - i(g_1\vec{A}_\mu\cdot\vec{t} + g_2\vec{C}_\mu\cdot\vec{\rho})$. Here $[\vec{t},\vec{\rho}]=0$; $\vec{t}\times\vec{t}=i\vec{t}$; $\vec{\rho}\times\vec{\rho}=i\vec{\rho}$. The charge operator is eQ; $Q=t_3+\rho_3$. For now, we bypass the option $g_1/g_2\neq 1$ and put

$$g_1 = g_2 = e\sqrt{2}. \qquad (1)$$

Then there is a further invariance under reflections R:$\vec{t}\leftrightarrow\vec{\rho}$ in O(4). Consider a scalar field quartet H with charges $(+,0,0,-)$. Here the action of $\vec{t},\vec{\rho}$ is $2\vec{t}=\vec{\tau}\otimes 1$; $2\vec{\rho}=1\otimes\vec{\tau}$. 1, $\vec{\tau}$ are Pauli matrices. Let H have vacuum expectation values $\langle 0,a,a',0\rangle$, *a,a′ real*. Denote such an H as $H(a,a')$. Introduce two such H's: $H(a,a')$ and $H(0,b)$; a,a', $b\neq 0$. The H's generate vector masses and D_μ becomes

$$D_\mu = \partial_\mu - ieQA_\mu - ie(t_3-\rho_3)Z_\mu - ie\cdot 2^{-1/2}\{W_\mu^1(t_+ - \rho_+) + \text{H.c.}\}, \qquad (2)$$

$$A_\mu\sqrt{2} = A_\mu^3 + C_\mu^3, \quad Z_\mu\sqrt{2} = A_\mu^3 - C_\mu^3, \qquad (3)$$

$$2W_\mu^{1,2} = [A^1 \mp C^1 - i(A^2 \mp C^2)]_\mu, \quad \xi \equiv M_0^2/(M_1^2+M_2^2) = 1, \qquad (4)$$

where M_0 is the mass of the neutral heavy Z-vector meson. $W_\pm^2$ and A are even under R and are associated with a subalgebra O(3); $W_\pm^1$ and Z are R odd.

All[1] f_L are grouped in quartets $(\frac{1}{2},\frac{1}{2})$. The electron (muon) quartet will be denoted by E_L (M_L). For baryons, two quartets Q_L^1, Q_L^2 of quarks will be introduced. If we assume *no group extension* (see the