Cusps, Dips, and Resonances

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Striking regularities in plots of π^+p elastic differential cross-section data at fixed c.m. angles are exhibited, a simple interpretation in terms of strongly correlated resonances is proposed, and further experimental tests of this idea are suggested. A novel and simple method for the determination of resonance energies and total widths based on the proposed model is also pointed out.

Several years ago we pointed out the usefulness of fixed-angle plots of the experimental data in pion-nucleon scattering as a powerful tool for extracting information on the resonance spectrum in a sinple fashion.[1] Not until recently, however, have we started a systematic study of all the available data. In this note we report striking features observed in the plots of $k^2 d\sigma/d\Omega$ (k is the c.m. momentum) for π^+p elastic scattering at fixed c.m. angles, point out a simple phenomenological interpretation in terms of strongly correlated resonances, propose further tests of our scheme, and suggest a novel and simple method for the determination of resonance energies and total widths directly from the data (i.e., without performing phase-shift analyses).

We have studied the behavior of π^+p elastic differential cross sections at 22 different angles as a function of the incident laboratory momentum p from threshold-up to 5 GeV/c in the backward region and up to 2.8 GeV/c at all the other angles. As most of the data was taken at different values of θ, a large amount of interpolation was necessary.[2] The results may be summarized as follows: (a) In the region $-1 \leq \cos\theta \leq -0.725$, the plots (Fig. 1) show dips with almost vanishing cross sections in the neighborhood of $p = 0.7$, 2.15, and 3.5 GeV/c. (b) In the region $0.325 \leq \cos\theta \leq 0.625$, the plots [Figs. 2(a)–2(d)] show a striking cusp at $p = 0.82$ GeV/c. Another cusp in the vicinity of $p = 2.08$ GeV/c seems to be present, at least at some of the angles. (c) The region $-0.725 < \cos\theta < 0.325$ is a transition region from the regime observed in (b) to the regime in (a). (In these three regions a full wavelength of a more or less distorted sinusoidal curve is observed between 0.82 and 2.08 GeV/c.) (d) For the forward peak, the plots (Fig. 3) in the region between 0.82 and 2.08 GeV/c look like severely distorted sinusoidal curves with initial phase equal to $-\pi/2$. A dip is observed in the neighborhood of 0.82 GeV/c.

All of these features may be correlated by means of a simple model whose basic assumptions are the following:

(1) There exists a spectrum of baryonic resonances with levels approximately degenerate in mass and width.[3,4]

(2) The resonance contribution from states with the same isospin to the spin-nonflip (a) and spin-flip (b) amplitudes in pseudoscalar meson–spin-$\frac{1}{2}$ baryon scattering may be written

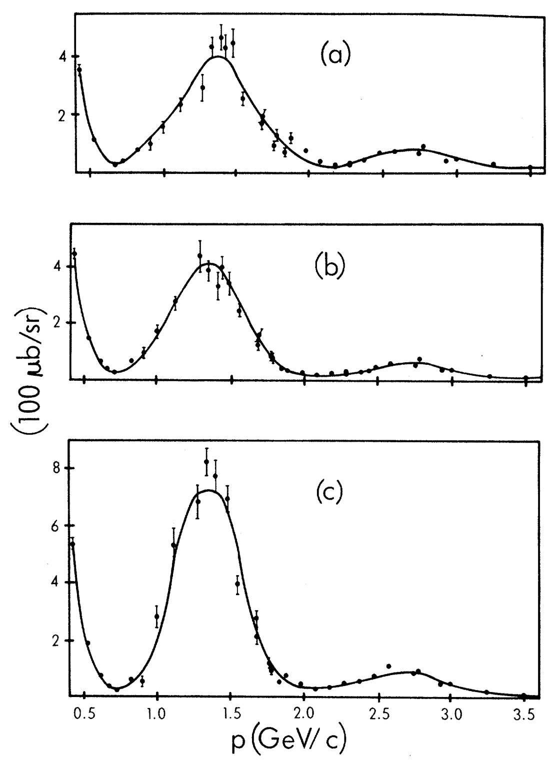

FIG. 1. Plot of $(k^2/k_s^2)\, d\sigma/d\Omega$ in π^+p elastic scattering as a function of the incident momentum p for (a) $\cos\theta = -0.725$, (b) -0.825, and (c) -0.925 (k is the c.m. momentum and k_s its value for $p = 2.07$ GeV/c). The experimental points were obtained from Ref. 2 by linear interpolation when required. The continuous lines are freehand curves drawn to guide the eye.

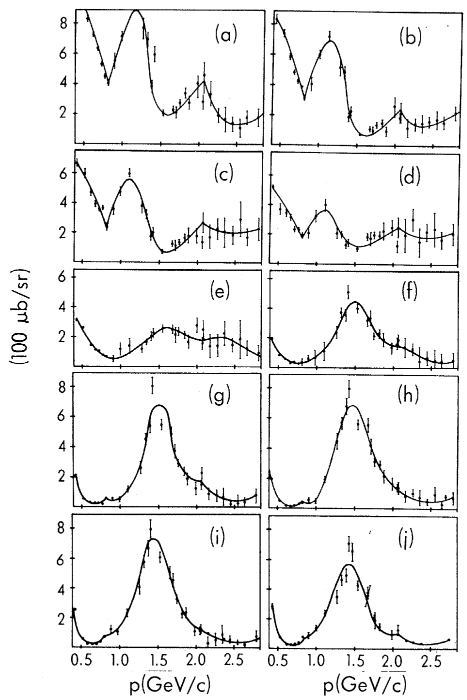

FIG. 2. Plot of $(k^2/k_s^2)d\sigma/d\Omega$ in π^+p elastic scattering a a function of the incident momentum for (a) $\cos\theta = 0.625$, (b) 0.525, (c) 0.425, (d) 0.325, (e) 0.125, (f) -0.125, (g) -0.225, (h) -0.325, (i) -0.525, and (j) -0.625. The experimental points were obtained from Ref. 2 by linear interpolation when required. The continuous lines are freehand curves drawn to guide the eye.

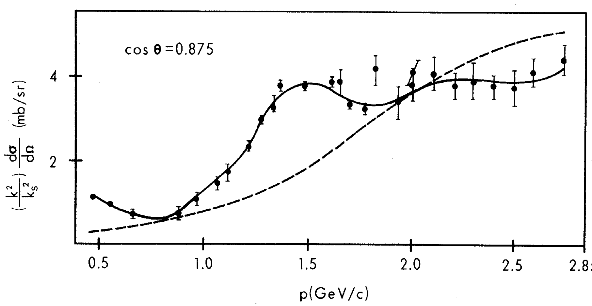

FIG. 3. Plot of $(k^2/k_s^2)d\sigma/d\Omega$ for $\cos\theta = 0.875$ in π^+p elastic scattering as a function of p. The continuous line was drawn by hand to guide the eye. The dashed line represents a possible shape of the background contribution consistent with our scheme. (Data from Ref. 2.)

at all energies

$$a_r = \sum f_l^J (J+\tfrac{1}{2}) P_l(\cos\theta),$$
$$b_r = \sum f_l^J (-1)^{J-l+1/2} P_l'(\theta), \qquad (1)$$

where

$$f_l^J = \tfrac{1}{2} i k^{-1} [(x_l^J)_1 (x_l^J)_2]^{1/2} (1 - e^{2i\delta});$$

$\delta(s)$ is a (common) rising phase,[5] and $[x_l^J(s)]_1$ and $[x_l^J(s)]_2$ (the initial and final "elasticities") are functions which change at most slowly within the (squared c.m.) energy intervals from $\delta(s) = (n-1)\pi$ to $\delta(s) = n\pi$ but may undergo a sudden change (and some of them do it in fact) at the energies for which $\delta(s) = n\pi$, referred to from now on as junction energies. The summation extends over all the possible values of l and J, but the elasticities are set equal to zero for noncontributing partial waves. The exact energy dependence of $\delta(s)$, which may be approximated by either a nonrelativistic[3] or a relativistic Breit-Wigner form in the neighborhood of a level, is not needed for the predictions. It should be noticed that the amplitudes a_r and b_r vanish exactly at the junction energies.

A standard calculation gives for the differential cross section of pure-isospin processes the result

$$d\sigma/d\Omega = (|a_b|^2 + |b_b|^2) + |V| k^{-1}\{\sin[2\delta(s) - \gamma] + \sin\gamma\}$$
$$+ \tfrac{1}{2}k^{-2}\{[\sum (x_l^J)_{1,2}(J+\tfrac{1}{2})P_l(\cos\theta)]^2 + [\sum (x_l^J)_{1,2}(-1)^{J-l+1/2}p_l'(\theta)]^2\}\{\sin[2\delta(s) - \tfrac{1}{2}\pi] + \sin\tfrac{1}{2}\pi\}, \qquad (2)$$

where $(x_l^J)_{1,2} = [(x_l^J)_1 (x_l^J)_2]^{1/2}$,

$$V = |V|e^{i\gamma} = \sum (x_l^J)_{1,2}[(J+\tfrac{1}{2})P_l(\cos\theta)a_b + (-1)^{J-l+1/2}p_l'(\theta)b_b],$$

and $a_b(s, \cos\theta)$ and $b_b(s, \cos\theta)$ are the background contributions to a and b.

In order to be able to make a large number of qualitative predictions, some (extra) assumptions regarding the behavior of these amplitudes at fixed angle are needed. On general grounds the magnitudes of these amplitudes are not expected to depart very much from a k^{-1} dependence at fixed angle (except near threshold), and their phases (α, β) are expected to remain approximately constant as s increases.[6] For simplicity we will assume exact k^{-1} dependence of $|a_b|$ and $|b_b|$ and exact constancy of the phases as functions of s. It should be understood that those predictions which depend specifically on these assumptions are only approximate.

This expression for $d\sigma/d\Omega$ becomes identical for elastic scattering $[(x_l^J)_1 = (x_l^J)_2]$ to one given by the author in the past.[3] It has been written however in a slightly different fashion in order to make certain features more transparent. It gives the following behavior for $k^2 d\sigma/d\Omega$ at fixed angle:

(I) Angle region where the background is negligible (possibly a certain region in the backward hemisphere). In this case the first two terms in the expression for $d\sigma/d\Omega$ (background and background-resonance interference contributions) may be neglected in comparison with the last one (resonance contribution). The plot of $k^2 d\sigma/d\Omega$ at fixed angle as either a function of s or a function of p in the region for which $(n-1)\pi \leq \delta(s) \leq n\pi$ is a full sinusoidal curve with an initial phase equal to $-\pi/2$ and displaced in the upward direction by an amount equal to its amplitude; $k^2 d\sigma/d\Omega$ vanishes at the junction energies. The amplitude of the sinusoidal curve changes abruptly at these energies, where pronounced dips are observed.

(II) Angle region where the background is large (possibly a certain region in the forward hemisphere). In this situation we may neglect the third term (resonance contribution) in the expression for $d\sigma/d\Omega$. The plot of $k^2 d\sigma/d\Omega$ at fixed angle in the region for which $(n-1)\pi \leq \delta(s) \leq n\pi$ is a full sinosoidal curve with an initial phase equal to $-\gamma$ and displaced vertically in general. At the junction energies the complex function V changes abruptly; the change in $|V|$ (γ) gives rise to a change in the amplitude (horizontal displacement) of the sinusoidal curve, i.e., except in some special situations[7] a cusp appears at the junction. This cusp should be particularly striking when the complex function V changes sign (γ increases by π).[8] When the background is very large (neighborhood of $\theta = 0°$ in elastic scattering) the resonance-background interference contribution to $d\sigma/d\Omega$ might be so small in comparison with the background term that its effect on the plot of $k^2 d\sigma/d\Omega$ becomes very difficult to detect experimentally.

(III) Any angle region. It is not hard to see that a full sinosoidal curve between two consecutive junction energies should be observed even if neither the background nor the resonance contribution to the amplitudes dominates,[9] and a cusp should appear at the junctions so long as there is a background (except in some special situations). These cusps are expected to be pronounced (and therefore noticeable) when the background and resonance amplitudes are of the same order of magnitude, but they are expected to be rather faint (and therefore hard to detect experimentally) when the background amplitudes are relatively small.

It should be noticed that only the prediction of (isolated) cusps ocurring at fixed energy independently of the angle is exact.

It is immediately seen that the behavior given in prediction (I) is identical to the one observed in Fig. 1, the one given in the general predictions (III) is observed in Fig. 2, and that one given in (II) for the case of $\gamma = -\frac{1}{2}\pi$ is present in Fig. 3.[10]

Several comments are now in order:

(1) It should be pointed out that the junction energies determined by the dip method have a constant separation $\Delta s = 2.6$ GeV^2. Only two junction energies could be determined by the cusp method and their separation is 2.4 GeV^2. The cusp method is more reliable, however, because it does not rest on an approximation. It gives as the general expression for the junction energies $s = 2.4n$ GeV^2 (assuming constant spacing), which has the interesting feature of giving $s = 0$ for $n = 0$.

(2) Once the junction energies are found experimentally the resonance energies are easy to locate since they are displaced by half of a wavelength (i.e., the phase δ changes by $\frac{1}{2}\pi$ from a cusp energy to the adjacent resonance energy). Also the total widths are easily determined since the energies for which $\delta = (n \pm \frac{1}{4})\pi$ are a quarter of a wavelength away from the corresponding resonance energy. Regions for which the initial phase is either 0 or π [such as in Figs. 2(a)–2(d)] provide the most sensitive determination, and regions for which the sinusoidal curves are distorted the least provide the most reliable determination. We obtain for the squared c.m. energy of the level located between 0.82 and 2.08 GeV/c the value $s = 3.52$ GeV^2 and for its total width (using the nonrelativistic Breit-Wigner expression) $\Gamma = 245$ MeV. This gives for the energy of the resonance pole $1866 - 122.5i$ MeV.[11]

(3) It should be noticed that, according to Eq. (2), the differential cross section at the junction energies is made up entirely of the background contribution. This means that, if the background is structureless (as we observed a few years ago[3]), no structure in the plot of $d\sigma/d\Omega$ at the junction energies as a function of $\cos\theta$ should appear. It is encouraging to observe that in the plots of $d\sigma/d\Omega$ at 0.82 GeV/c and in the neighborhood of 2.08 GeV/c the structure has almost

washed out completely.

Clearly, more experimental tests of these ideas are needed. In particular, we suggest *accurate* measurements of π^+p elastic differential cross sections at fixed angles in the region $0.4 \leq \cos\theta \leq 0.8$, $1.8 \leq p \leq 5.5$ GeV/c. Ideally these experiments should be carried at small incident momentum steps.[12] We also suggest similar testing of all pure-isospin $B=1$ two-body reactions, for which our model gives essentially the same predictions.

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[1]G. T. Hoff, Phys. Rev. 154, 1331 (1967), and unpublished.

[2]We have used the fixed-angle data of R. A. Sidwell *et al.*, Phys. Rev. D 3, 1523 (1971), and of G. E. Kalmus *et al.*, Phys. Rev. D 4, 676 (1971), the data of W. Busza *et al.*, Phys. Rev. 180, 1339 (1969), and older data taken from G. Giacomelli, P. Pini, and S. Stagni, CERN Report No. CERN-HERA 69-1, 1969 (unpublished).

[3]We were led to this assumption by considerations of a practical nature a few years ago. See G. T. Hoff, Phys. Rev. 18, 816 (1967).

[4]Another way of stating this assumption is by saying that there is a single pole in the complex energy plane associated with each level.

[5]Rising phases have been used by other authors in $\pi\pi$ scattering: A. M. Gleeson, W. J. Meggs, and M. Parkinson, Phys. Rev. Lett. 25, 74 (1970). Their approach is, however, somewhat different from ours.

[6]These properties are easily obtained from expressions given by Dalitz for the background contribution to each partial wave. See R. N. Dalitz, Annu. Rev. Nucl. Sci. 13, 339 (1963).

[7]When γ is equal to either $\frac{1}{2}\pi$ or $-\frac{1}{2}\pi$. This situation arises in particular when the background amplitudes are purely imaginary.

[8]When γ is equal to either $\frac{1}{2}\pi$ or $-\frac{1}{2}\pi$ a change of phase by the amount π gives rise to an inflection point, not to a cusp.

[9]An exception to this statement is the case when the background-resonance interference and resonance contributions cancel each other at a given angle.

[10]The extreme distortion of the sinusoidal curve in Fig. 3 could be interpreted as due to a strong departure of the background from a k^{-1} dependence (the total cross section is still increasing in this energy region).

[11]It is currently believed that there are five established resonances between 0.82 and 2.08 GeV/c: $\Delta(1650)$, $\Delta(1670)$, $\Delta(1890)$, $\Delta(1910)$, and $\Delta(1950)$, with full widths ranging from 165 to 325 MeV. The average spread for each resonance energy (total width) is 87 MeV (126 MeV). See P. Söding *et al.*, Phys. Lett. 39B, 1 (1972).

[12]Efforts should be made to find a solution to all the available π^+p elastic data constrained to satisfy our basic assumptions.

Why Does the Pomeranchuk Singularity Have a Unit Intercept?

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We suggest, contrary to the spirit of the Amati-Fubini-Stanghellini multiperipheral model and a φ^3 ladder theory, that the relation $\alpha_P(0)=1$ is independent of the strength of the strong interaction. We propose that the "zero strength" ($g\to 0$) limit of a future correct strong-interaction theory will collapse into a vector-type field theory, guaranteeing $\alpha_P(0)=1$ even as $g\to 0$. The intercept of the leading non-Pomeranchukon exchange then becomes lower when g increases. Specific models which are consistent with this approach lead to a new relation between this intercept and the moments of the multiplicity distribution.

The nature of the Pomeranchuk singularity has always been a mystery. Empirically, we know that all total hadronic cross sections seem to approach a constant at high energies (modulo factors of $\ln s$). Recent data[1] from Serpukhov, the National Accelerator Laboratory, and the Intersecting Storage Rings seem to confirm this trend, which has earlier been observed at lower energies. It is often said that the constancy of σ_{tot} is due to the unit intercept of the Pomeranchuk singularity, which dominates the imaginary part of the forward elastic scattering amplitude. Needless to say, this statement does not clarify the nature of this singularity, nor does it explain why $\alpha_P(0)=1$.

The solution to both of these puzzles is almost certainly buried in the unitarity relation which simply states that the sum of all the cross sec-