The measured value is $\theta = +0.3\times10^{-4}$, nearly 1 order of magnitude smaller than for the compensated sample. The effect has changed sign, indicating a skew scattering $\theta_s = (1.6\pm0.5)\times10^{-4}$ which is nearly twice the calculated value $\theta_s = 0.86\times10^{-4}$. In view of the experimental uncertainties, particularly because of the macroscopic inhomogeneities of the sample, the result can be considered as very satisfactory.

In conclusion, the measurement of the anomalous Hall effect in indium antimonide has provided an unambiguous and quantitative verification of the theory. The existence of the two physical mechanisms responsible for the effect, the skew scattering and the transverse displacement, is illustrated by the variation of the magnitude and sign of the Hall effect from a compensated to a noncompensated sample.

We wish to thank Professor P. Nozières and Mrs. C. Lewiner for extremely valuable discussions concerning many aspects of the anomalous Hall effect.

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# Translational Mobility of Hard Ferromagnetic Bubbles

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We show that steady translation of a magnetic bubble possessing Bloch lines requires the application of both longitudinal and transverse field gradients. The predicted longitudinal gradient obeys the usual relation valid for a normal bubble. The transverse gradient is proportional to the product of velocity and number of Bloch lines. Experiments of Tabor *et al.* support the theory.

Instances of magnetic-bubble motion in a direction different from that of the applied field gradient have been reported recently.[1,2] The experiment of Tabor *et al.*[1] shows that such skewed motion is a general property of "hard" magnetic bubbles, which are known to contain vertical Bloch lines.[1,3,4] Vella-Coleiro, Rosencwaig, and Tabor[5] proposed a nonlinear theory of both longitudinal and transverse components of mobility to explain this effect. Although it is consistent with experiment, their theory cannot be considered complete because it employs a drive-field–dependent function $f$ determined only from the dynamic data to be explained.

In the present Letter, we propose a linear model of skewed bubble motion, and suggest that the apparent nonlinearity of the experimental data[1] is primarily a consequence of coercivity, which tends to obscure the basic linearity of the underlying phenomenon. Our results enjoy some support from the data.

Let us consider first a static cylindrical domain with radius $r$ in a uniaxial magnetic film, as illustrated in Fig. 1. The cylinder axis, which is parallel to the magnetically easy $z$ axis, lies normal to the $xy$ plane of the film. The equilibrium distribution of the magnetization vector $\vec{M}(x,y)$ is determined by a balance of torques due to exchange, anisotropy, demagnetizing field, and a uniform applied bias field $H_{zb}$. Within the cylinder, $\vec{M}$ lies nearly parallel to $z$, while outside, nearly antiparallel to $z$. Only within the interior of the domain-wall region of thickness $\Delta$, assumed small compared to $r$, does it tilt much away from the $z$ axis. In any wall region small compared to $r$, the *amount* of tilt of $\vec{M}$ measured by the spherical polar angle $\theta$ has the Bloch-wall form[6]

$$\tan(\theta/2)=\exp[(\xi-q)/\Delta], \tag{1}$$

$$\Delta=(A/K)^{1/2}, \tag{2}$$

where $\xi$ is a local Cartesian coordinate normal to the wall, and $q$ marks the center of the wall. The *direction* of tilt constitutes the Bloch-line structure, described further on.

Turning to the dynamical problem, we assume that in first-order approximation the static pattern $\vec{M}(x,y)$ translates without distortion whatsoever, and we calculate the applied-field distribution $H_z(x,y)+H_{zb}$ required to produce the assumed motion. Let $\psi(t)$ be the azimuthal angle, measured from the plane tangent to the wall, of $\vec{M}$ at any point $P$ (fixed in the laboratory) which happens to lie on the moving wall at the time $t$. The motion of $\psi$ in the laboratory frame is governed by one component of the Landau-Lifshitz equation

$$d\psi/dt=\gamma H_z+(\alpha/\sin\theta)\partial\theta/\partial t, \tag{3}$$

where $\gamma$ is the gyromagnetic ratio and $\gamma H_z$ represents the Larmor precession due to $H_z$. The last term in this equation represents the damping in Gilbert's form with coefficient $\alpha$.[7] The omitted terms due to exchange, anisotropy, demagnetization, and $H_{zb}$ cancel, according to our assumption of motion without distortion. Moreover, the function (1) is unchanged, though now $q$ depends on $t$ because we consider $\xi$ to be defined in the laboratory frame. Thus, with the help of Eq. (1) we have

$$\partial\theta/\partial t=(\partial\theta/\partial q)(\partial q/\partial t)=-\Delta^{-1}(\partial q/\partial t)\sin\theta. \tag{4}$$

Substituting Eq. (4) in (3) we have

$$\partial\psi/\partial t=\gamma H_z-\alpha\Delta^{-1}\partial q/\partial t, \tag{5}$$

a special case of one of the general equations of wall motion.[8]

Let us now assume a uniform domain velocity $V$ ($>0$) in the $x$ direction. If the plane polar angle $\beta$ describes the position of $P$ on the cylinder wall (see Fig. 1), then the local wall velocity normal to the wall is

$$\partial q/\partial t=V\cos\beta. \tag{6}$$

Given a static Bloch-line structure $\psi(\beta)$ in the reference frame of the moving domain, the motion of $\psi$ in the laboratory frame is

$$\partial\psi/\partial t=(d\psi/d\beta)Vr^{-1}\sin\beta. \tag{7}$$

Let us consider the limit of high Bloch-line density. Then for small $V$, $\psi(\beta)$ tends to the static constant-twist expression $(n\beta/2)+\text{const}$,[3,5] where $n$ is the even number (positive or negative) of Bloch lines. In this limit, substitution of Eqs. (6) and (7) in (5) gives

$$\gamma H_z=(nV/2r)\sin\beta+\alpha V\Delta^{-1}\cos\beta. \tag{8}$$

Comparing this equation with the field distribution $H_z(x,y)$ in the presence of a uniform gradi-

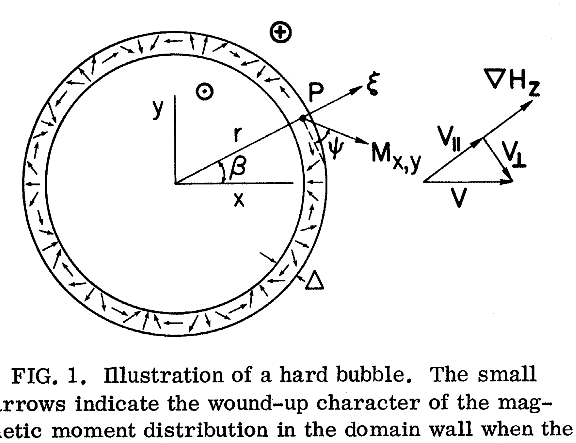


FIG. 1. Illustration of a hard bubble. The small arrows indicate the wound-up character of the magnetic moment distribution in the domain wall when the number of Bloch lines is large.

ent, one finds easily

$$\partial H_z/\partial x = (\alpha V/\gamma\Delta r) + C, \quad (9)$$

$$\partial H_z/\partial y = nV/2\gamma r^2. \quad (10)$$

Here the *ad hoc* constant $C$ ($>0$) has been added without proof to represent dynamic coercivity, an effect present in all specimens but not well understood. We mention in passing that Ref. 1 assumed the coercive force to be parallel to the *applied gradient,* in the course of data reduction. It seems to us more natural to take it parallel to the *velocity,* as above.

As stated by Rosencwaig, Tabor, and Nelson,[4] the presence of Bloch lines tends to decrease the Bloch-wall thickness. For sufficiently large Bloch-line density, the exchange energy density term $A(\nabla\psi)^2\sin^2\theta$ provides the dominant influence. Since $|\nabla\psi|$ has the constant value $|n|/2r$, this term is equivalent to a uniaxial anisotropy contribution $A(n/2r)^2\sin^2\theta$. Replacing $K$ by $K+A(n/2r)^2$ in Eq. (2), we find the more general relation

$$\Delta = [(K/A)+(n/2r)^2]^{-1/2}, \quad (11)$$

which is a special case of Eq. (9) of Ref. 4, except for a numerical factor. (Our $n$ is defined as twice theirs.)

Experimental results for a garnet of composition $YGdTmGa_{0.8}Fe_{4.2}O_{12}$ were expressed in terms of the velocity components parallel ($V_\parallel$) and perpendicular ($V_\perp$) to the applied field gradient.[1] However, the corresponding expressions $V_\parallel(|\nabla H_z|)$, $V_\perp(|\nabla H_z|)$ derived by inverting Eqs. (9) and (10) are cumbersome. It is simpler to compare two other relations. The first relation is (10), which may be expressed in the form

$$2V^2/\gamma d V_\perp = H_a/n, \quad (12)$$

where $H_a$ $(=d|\nabla H_z|)$ is the applied field difference across the bubble diameter $d$. It differs from $\Delta H$ of Refs. 1 and 5 by a threshold value of $H_t = 0.87, 0.72, 0.72$ Oe for the three bubbles with $d = 6.2, 3.9, 3.3$ $\mu$m, respectively.[9] The left-hand quantity in Eq. (12) is plotted versus $H_a$ in Fig. 2, using the data of Ref. 1 and the value $\gamma = 1.76\times10^7$ sec$^{-1}$ Oe$^{-1}$. The two larger bubbles obey Eq. (12) quite well, assuming they have different values of $n$. From the slopes of the fitted lines we find $n=250$ and 190 for the 6.2- and 3.9-$\mu$m bubbles, respectively,[10] which compares favorably with an estimate $n=100$ based on bubble statics.[4,5]

The second relation measures the dependence of direction on $V$. Assuming the limit of *very* high Bloch-line density, we neglect $K$ in the thickness formula (11). Then we substitute the result in Eq. (9) and divide the latter by Eq. (10) to find

$$\frac{V_\parallel}{V_\perp} = \frac{\delta H_z/\delta x}{\delta H_z/\delta y} = \pm\left[\alpha + \frac{H_c}{|n|}\frac{\gamma d}{2V}\right], \quad (13)$$

where $H_c = Cd$ is the dynamic–coercive-field difference across the diameter, and the sign is that of $n$. Experimental values of $V_\parallel/V_\perp$ versus $\gamma d/2V$ are shown in Fig. 3. The plotted points partially support the linear relation predicted by Eq. (13). Also the difference between $n$ for the larger bubbles is qualitatively reflected in the fact that points for $d=3.9$ $\mu$m lie above the line fitted to the points for $d=6.2$ $\mu$m. Considering all of the data in Fig. 3, the value of $\alpha$ given by the vertical intercept may lie in the range 0 to 0.15. The independent estimate $\alpha=0.2$ for this garnet[5]

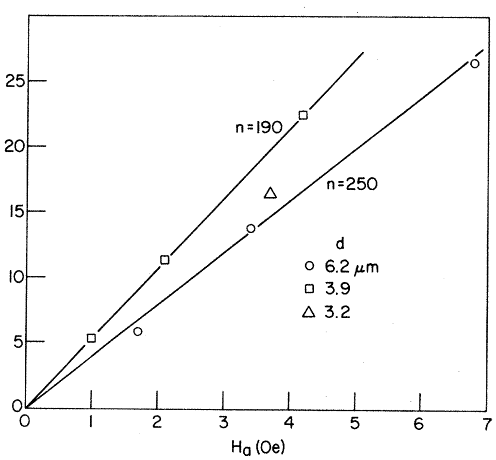


FIG. 2. Test of the drive dependence of velocity predicted by Eq. (12), using data of Ref. 1 for bubbles of three diameters.

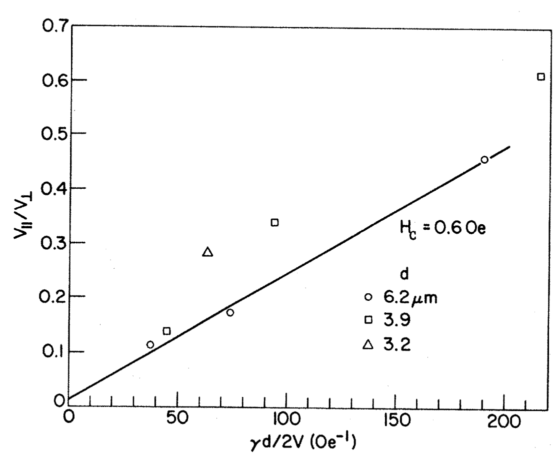


FIG. 3. Test of the velocity dependence of direction predicted by Eq. (13), using data of Ref. 1.

falls outside of our range. However, a more satisfactory estimate $\alpha = 0.07$ follows by linear interpolation of the quantity $\lambda/\gamma^2 = \alpha M/\gamma$, where $\lambda$ is Landau-Lifshitz damping, from a table of ferromagnetic resonance values for pure garnets.[11] Taking $n = 250$, as determined above, the slope of the line for $d = 6.2$ $\mu$m provides the value $H_c = 0.6$ Oe comparable to the static $H_t = 0.87$ Oe. The poorer agreement in Fig. 3 compared with Fig. 2 may reflect inaccuracy in our representation of coercivity, which plays no role in Fig. 2.

Some qualitative differences between the present theory and that of Ref. 5 are apparent. Their expression for $V_\perp/V_\parallel$ [see their Eq. (15)] depends on drive only through the function $f$, which they determine from the experimental velocity data by inverting this expression. The function $f$ measures the drive-dependent redistribution of the Bloch lines, according to their Eq. (10), which in our notation is equivalent to

$$\psi = n\beta/2 + \text{const} + a\cos(\beta - \beta_0). \tag{14}$$

where the $V$-dependent parameter $a$ is proportional to $f$. Thus, they clearly attribute the nonlinearity represented by the drive dependence of $V_\perp/V_\parallel$ (Fig. 3) to this redistribution. Although we acknowledge the general existence of the redistribution, we consider its invocation unnecessary because the coercivity $H_c$ leads to $V$ (or drive) dependence of $V_\perp/V_\parallel$, according to Eq. (13). Moreover, one can easily show that the last term in Eq. (14), when substituted into our relations above, contributes nothing to $\nabla H_z$ but only to higher terms in the Taylor-series expansion of $H_z(x,y)$ which would tend to distort the domain. In short, we assert the adequacy of the static Bloch-line distribution to explain the data, and also refute the idea that redistribution greatly affects mobility in general. Particularly, our linear model accounts naturally for the linearity displayed in Fig. 2, which cannot be simply explained with the inherently nonlinear theory of Ref. 5.

The expression of Ref. 5 for $V_\parallel/V_\perp$ is made equivalent to our Eq. (13), for the case $H_c = 0$ and $\Delta = d/|n|$, by assigning to $f$ the constant value $(1 - \sqrt{2}\alpha^2)/(1 + \sqrt{2})$. In this case, the two theories are approximately equivalent in the limit of small $\alpha$, Ref. 5 differing from Eq. (10) by the factor $2^{3/2}$. In more general circumstances the predictions of the theories disagree.

It is interesting to notice that the longitudinal mobility of the hard bubble, described by Eq. (9), has the standard form valid for a normal domain without Bloch lines, granted the correction (11) for Bloch-wall thickness.[10] Thus it does not suffer the enormous reduction (factor $\alpha^2$) effected by Bloch lines in bubble-collapse experiments.[12] (This statement presupposes the application of an appropriate transverse "bias" gradient, which does no work, to maintain $\vec{V}$ parallel to the "driving" gradient.) The difference arises from the fact that a pure *gradient* drive pushes the Bloch lines in both senses around the perimeter so that the net force vanishes. Since the Bloch lines thereby assume static positions, the elementary concepts of domain wall dissipation are still valid. However, a *uniform* drive field pushes the Bloch lines in one sense only, causing a continual circulation which diminishes the radial mobility.[12]

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[10]In the present heuristic theory only one component of the Landau-Lifshitz equation was employed. In an unpublished, more rigorous, calculation based on both components, we find that the first term in Eq. (9) requires the correction factor $1 + (n\Delta/2r)^2$, which ranges between 1 and 2; and that Eq. (10) requires a correction factor of 2. Therefore, our estimates of $n$ derived here from dynamical data should be halved and thus put in excellent agreement with the static value quoted. Our values of $\alpha$ and $H_c$ remain unchanged by these corrections.

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