faster with pressure, and it appears to win in the competition in all the Eu monochalcogenides with the exception of EuO. EuO is thus the first Eu chalcogenide to exhibit a valence transformation from the 2^+ towards 3^+ state due to $4f$-to-$5d$ electron delocalization, prior to the CsCl transformation. Qualitative optical observation shows that the CsCl form of EuO is also metallic in nature. *Per contra,* the CsCl phases of EuTe, EuSe, and EuS do not exhibit any metallic reflectivity to about 350 kbar pressure,[13] which suggests that there is no significant change in the valence state of Eu in these cases up to the above pressure.

We wish to thank W. M. Walsh, Jr., for critically reading the manuscript and for some suggestions, and R. G. Maines for assistance.

[1]A. Chatterjee, A. K. Singh, and A. Jayaraman, Phys. Rev. B 6, 2285 (1972).

[2]A. Jayaraman, unpublished.

[3]W. A. Bassett, T. Takahashi, and P. W. Stook, Rev. Sci. Instrum. 38, 37 (1967).

[4]We believe that 400 kbar pressure attained in the present study is a new record of highest pressure for the diamond-anvil camera.

[5]L. Liu and W. A. Bassett, to be published.

[6]D. B. McWhan, P. C. Souers, and G. Jura, Phys. Rev. 143, A385 (1966).

[7]D. B. McWhan [J. Chem. Phys. 44, 3528 (1966)] has suggested 1030 ± 130 kbar as his estimate for B_0 for EuO.

[8]A. Jayaraman, V. Narayanamurti, and E. Bucher, Phys. Rev. Lett. 25, 4130 (1970).

[9]The metallic phase of SmS exhibits a golden yellow luster: J. L. Kirk, K. Vedam, V. Narayanamurti, A. Jayaraman, and E. Bucher, Phys. Rev. B 6, 3023 (1972).

[10]C. J. M. Rooymans, Ber. Bunsenges. Phys. Chem. 70, 1036 (1966), and Solid State Commun. 3, 421 (1965).

[11]J. O. Dimmock, IBM J. Res. Develop. 14, 301 (1970), and references cited therein. Also S. J. Cho, Phys. Rev. B 1, 4589 (1970).

[12]P. Wachter, Solid State Commun. 7, 693 (1969).

[13]A. Jayaraman, unpublished.

Experimental Evidence of the Anomalous Hall Effect in a Nonmagnetic Semiconductor

J. N. Chazalviel and I. Solomon

Laboratoire de Physique de la Matière Condensée, Ecole Polytechnique, Paris, France*

(Received 30 October 1972)

The anomalous Hall effect is measured in InSb at low temperature. In this nonmagnetic semiconductor the polarization of the conduction electrons is obtained by application of an external magnetic field B_0. The spin-dependent Hall effect is separated from the much larger ordinary Hall effect by magnetic resonance of the conduction electrons. Two physical mechanisms responsible for the anomalous Hall effect are separated and the experimental results agree within a factor of 2 with the theoretical predictions without any adjustable parameter.

The discovery of a strong anomalous Hall effect in ferromagnetic materials initiated an abundant literature on the theory of the spin-dependent Hall effect,[1] but this theory was obscured for a long time by the extreme complexity of the ferromagnetic systems at finite temperatures. The theories conflicted in the interpretation of the physical mechanisms responsible for the effect, although most of them agreed on the importance of the spin-orbit interaction. Recently,[2] the consideration of much simpler physical systems like III-V semiconductors has somewhat clarified the situation, acknowledging for the most part the validity of the pioneering work of Luttinger.[3] However, an experimental study of the anomalous Hall effect in one of those simple nonmagnetic systems, permitting a direct comparison with the theory, was still lacking. In this work we present the first anomalous Hall effect obtained in a semiconductor, thus providing experimental evidence of the validity of the theoretical model. In this material there is no spontaneous magnetization, and the polarization of the conduction electrons is obtained by application of an external magnetic field B_0. The experimental difficulty is to separate unambiguously the small spin-dependent Hall effect, corresponding to the equilibrium polarization, from the much larger ordinary Hall effect. This is achieved by observing the effect on the Hall voltage of a change of the electronic spin polarization. This change is obtained by a magnetic resonance method.[4] The experiment is performed in indium antimonide at low temperature (1.3°K). The applied magnetic field is $B_0 = 130$ G which corresponds, with a g factor of $g^* = -51$, to a spin resonant frequency $\omega/2\pi = 9.2$ GHz.[5]

We present first a short review of the current theoretical situation relevant to a comparison with our experimental results.[2,6,7] Two different mechanisms contribute to the spin-dependent Hall effect in a semiconductor at low temperature:

(a) The scattering of a polarized spin on an impurity, when calculated to the second-order Born approximation,[8,9] contains an asymmetric part. The resulting *skew scattering* on an individual impurity leads to an average scattering angle θ_s. If one neglects multiple scattering, this angle is precisely the angle of the spin-dependent Hall effect due to this mechanism. Its calculated value is[10]

$$\theta_s = -\frac{5\pi}{12}(2-g^*)\,\frac{2E_g+\Delta}{E_g+\Delta}\,\frac{m^*}{m}\,\frac{V}{E_g}\,(n_\uparrow - n_\downarrow), \qquad (1)$$

where E_g is the forbidden gap and Δ the spin-orbit splitting of the valence band. This is a simplified expression obtained for a short-range scattering potential, so that $V(k,k') = V$ is independent of k, k'. To obtain the order of magnitude of the effect, we consider the scattering by an ionized donor, i.e., by a screened Coulomb potential. If the inverse Debye screening radius k_s is much larger than the value of k at the Fermi level, the assumption that V is independent of k, k' is justified and one has simply $V = -2E_F/3n$. For degenerate electrons, the equilibrium polarization in a magnetic field corresponding to the Larmor frequency ω is $p = 3\hbar\omega/4E_F$. Inserting these values in Eq. (1), we obtain

$$\theta_s = \frac{5\pi}{24}(2-g^*)\frac{m^*}{m}\,\frac{2E_g+\Delta}{E_g+\Delta}\,\frac{\hbar\omega}{E_g}$$
$$\approx 0.86\times 10^{-4}\ \text{rad}. \qquad (2)$$

The effect changes sign from an attractive to a repulsive potential and will therefore be proportional to the degree of compensation.[8]

(b) The other mechanism contributing to the anomalous Hall effect is the *transverse displacement* undergone by an electron subject to collisions or to a longitudinal force.[6,11,12] It can be shown that for a wave packet of longitudinal velocity $\langle v_x\rangle$ the transverse position (along the y axis) is not the same for a spin-up and a spin-down electron,[13] and that this transverse displacement is proportional to the longitudinal velocity. The origin of this displacement lies in the fact that the quantum state of an electron in the conduction band is not a pure spin state, but is admixed with the orbital variables through the spin-orbit interaction Δ in the valence band[14]; the effect is particularly important in InSb where the spin-orbit interaction Δ is large (0.9 eV) and the gap E_g is small (0.235 eV). On application of an electric field E_x, we obtain the usual longitudinal current J_x; but also, because of this transverse displacement, there appears a transverse current J_y proportional to the polarization p of the conduction electrons along the z axis. The relevant parameter for this effect is therefore the *transverse mobility* $\mu_{yx} = J_y/neE_x$. Its theoretical value is[15]

$$\mu_{yx} = -(2-g^*)\,\frac{2E_g+\Delta}{E_g+\Delta}\,\frac{e\hbar}{2m}\,\frac{1}{E_g}\,p. \qquad (3)$$

Note that μ_{yx} depends only on the intrinsic properties of the material. For the type of samples used in the present experiment (electronic density of the order of 10^{14} cm^{-3}) and the same polarization as that considered before, Eq. (3) gives a value $\mu_{yx} \simeq -7.0$ cm^2 V^{-1} sec^{-1}. This mechanism will therefore give an anomalous Hall effect of the same order of magnitude as the effect of the skew scattering for the highest mobility samples ($\approx 8\times 10^4$ cm^2 V^{-1} sec^{-1}) and a larger effect for lower mobilities.

The principle of the experimental method is to change the spin polarization leaving the applied field B_0 constant. The ordinary Hall effect remains constant and the recorded change of the Hall voltage measures the variation of the spin-dependent Hall effect, thus separated from the much larger ordinary Hall effect. The straightforward method of decreasing the spin polarization by passing through the spin resonance line with high-power microwave field[16] (the so-called "saturation" of the line) cannot be used here because of the microwave heating of the conduction electrons. This heating is due to the microwave electric field E_1, whereas the saturation of the line increases with the microwave magnetic field B_1. In order to maximize the ratio B_1/E_1, we use a very thin sample (50 μm) placed on the bottom of a TE_{102} cavity. With this arrangement, B_1 is still limited to some 10^{-2} G for a heating of the order of 1°K. This imposes a severe limitation on the saturation of the spin resonance line. For a linewidth $2\Delta B$ of 2 G, the decrease of the polarization at resonance is of the order of $-\delta p/p$

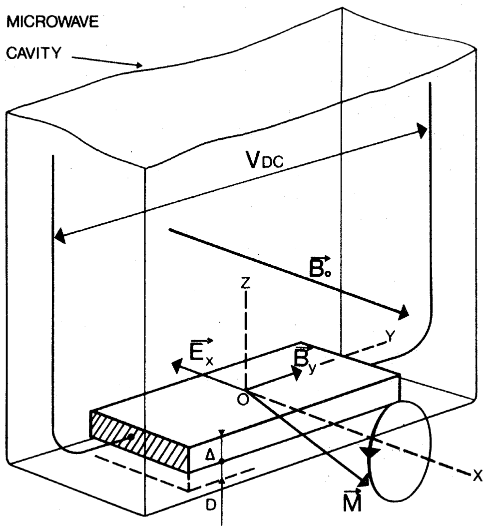

FIG. 1. The arrangement of Juretschke (Ref. 4) to measure the spin-dependent Hall effect. Near resonance, the magnetization $\vec{M}$ precesses around the applied field $\vec{B}_0$. The product of the microwave electric field E_x with the component M_z of $\vec{M}$ gives a dc Hall voltage along Oy.

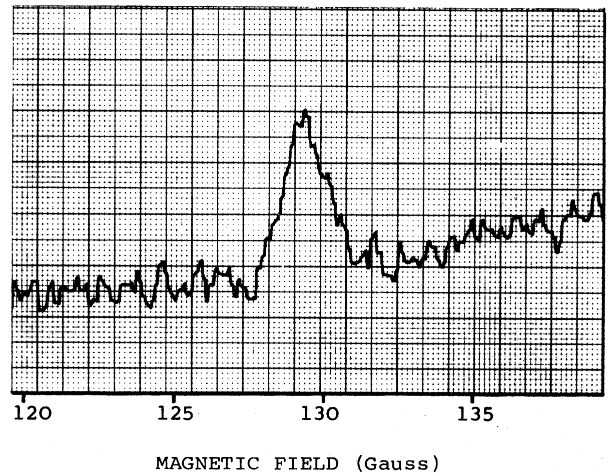

FIG. 2. Spin-dependent Hall voltage obtained along the y axis (Fig. 1) when the magnetization of the conduction electrons is affected by passage through spin resonance at 9.2 GHz. (InSb, n type 10^{14} cm^{-3}, 1.3°K). The recorded signal, after lock-in detection at 300 Hz, has the shape of the derivative of a dispersion curve.

$\simeq (B_1/\Delta B)^2 = 10^{-4}$. With this method the decrease of the Hall voltage is thus only a fraction times 10^{-4} of the full spin-dependent Hall effect, which is beyond our detecting capability. We have therefore used the ingenious arrangement of Juretschke,[4] which does not use the decrease of the polarization along B_0 but the appearance, near resonance, of a transverse rotating magnetization (the Larmor precession). The magnitude of this transverse magnetization is a fraction $B_1/\Delta B = 10^{-2}$ of the equilibrium magnetization. We thus gain a factor $\Delta B/B_1 = 100$ over the saturation method, which is enough to make the detection of the effect compatible with our sensitivity. Figure 1 shows the experimental arrangement. The applied current J_x is due to the microwave electric field E_1 along the x axis, and the rotating magnetization varies with time at the same microwave frequency. Since the spin-dependent Hall voltage along the y axis is proportional to the cross term $J_x M_z$, it has a dc component V_y that gives a measure of the effect. The relevant magnetization is the component along the z axis which is in phase with the microwave electric field; passing through resonance, it is easily seen to be proportional to the "dispersion" curve.[17] As usual, the magnetic field is modulated (300 Hz) and the recorded signal, after lock-in detection, has the shape of the derivative of a dispersion curve (Fig. 2).

In an attempt to separate the two mechanisms responsible for the Hall effect, we considered two samples with approximately the same carrier content[18] but with different compensations:

"Compensated" InSb: The number of carriers is $N_D - N_A = 1.1\times10^{14}$ cm^{-3} and the mobility is $\mu \simeq 2.2\times10^4$ cm^2 V^{-1} sec^{-1}, from which we estimate a compensation $N_A/N_D \simeq 60-70\%$. In such a sample the Hall angle associated with the displacement effect is enhanced because of the rather low longitudinal mobility; in contrast, the skew scattering is strongly decreased because of the existence of impurity potentials of both signs. Therefore, the Hall angle should be dominated in this sample by the displacement effect which has a calculated value, from Eq. (3), of $\theta = \mu_{yx}/\mu = -3.1\times10^{-4}$. The measured value, extrapolated for the equilibrium polarization, is $-(2.6\pm0.5)\times10^{-4}$.

"Noncompensated" InSb: Our less-compensated sample has a number of carriers $N_D - N_A = 1.3\times10^{14}$ cm^{-3} and a mobility $\mu \simeq 4\times10^4$ $cm^2 V^{-1}$ sec^{-1} which indicates a compensation N_A/N_D hopefully less than 30%. With a mobility nearly doubled, we expect the contribution of the displacement effect to be reduced by a factor of $\frac{1}{2}$, with a value getting thus very close to the contribution of the skew scattering. Since the two mechanisms have opposite signs we should obtain almost complete cancelation and the measured spin-dependent Hall effect in this sample should be very small.

The measured value is $\theta = +0.3\times10^{-4}$, nearly 1 order of magnitude smaller than for the compensated sample. The effect has changed sign, indicating a skew scattering $\theta_s = (1.6\pm0.5)\times10^{-4}$ which is nearly twice the calculated value $\theta_s = 0.86\times10^{-4}$. In view of the experimental uncertainties, particularly because of the macroscopic inhomogeneities of the sample, the result can be considered as very satisfactory.

In conclusion, the measurement of the anomalous Hall effect in indium antimonide has provided an unambiguous and quantitative verification of the theory. The existence of the two physical mechanisms responsible for the effect, the skew scattering and the transverse displacement, is illustrated by the variation of the magnitude and sign of the Hall effect from a compensated to a noncompensated sample.

We wish to thank Professor P. Nozières and Mrs. C. Lewiner for extremely valuable discussions concerning many aspects of the anomalous Hall effect.

*Equipe de Recherche du Centre National de la Recherche Scientifique.

[1]For a short but critical review of both experiment and theory, see F. E. Maranzana, Phys. Rev. 160, 421 (1967).

[2]C. Lewiner, O. Betbeder-Matibet, and P. Nozières, to be published.

[3]J. M. Luttinger, Phys. Rev. 112, 739 (1958).

[4]H. J. Juretschke, J. Appl. Phys. 31, 1401 (1960), and J. Appl. Phys. 34, 1223 (1963).

[5]G. Bemski, Phys. Rev. Lett. 4, 62 (1960).

[6]R. C. Fivaz, Phys. Rev. 183, 586 (1969).

[7]A more complete description of the experiment and a detailed discussion of the theoretical results, with special emphasis on the physical mechanisms responsible for the anomalous Hall effect, will be published elsewhere.

[8]P. Leroux-Hugon and A. Ghazali, J. Phys. C: Proc. Phys. Soc., London 5, 1072 (1972).

[9]V. N. Abakumov and I. N. Yassievich, Zh. Eksp. Teor. Fiz. 61, 2571 (1972) [Sov. Phys. JETP 34, 1375 (1972)].

[10]The relevant physical quantity for this mechanism is the Hall angle which results only from the individual collisions. In Ref. 2 the calculated quantity is the transverse conductivity [Eqs. (53)–(56)], $\sigma_{an}^{(2)}$, which contains the collision rate $\Gamma = \hbar/\tau$. By dividing by the conductivity $\sigma = ne^2\tau/m^*$, we obtain the Hall angle θ_s which is indeed independent of the relaxation time τ.

[11]J. Smit, Physica (Utrecht) 21, 877 (1955).

[12]L. Berger, Phys. Rev. B 2, 4559 (1970).

[13]A. M. de Graaf and A. W. Overhauser, Phys. Rev. 180, 701 (1969).

[14]E. O. Kane, J. Phys. Chem. Solids 1, 249 (1957).

[15]$ne\mu_{yx}$ is the quantity $\sigma_{an}^{(1)} = -\sigma_{an}^{0}$ of Ref. 2. We have explicitly displayed the parameter $\xi = (2-g^*)(2E_g+\Delta)(2E_g+2\Delta)^{-1}$ to emphasize the influence of the large g^* value of InSb to obtain an anomalous Hall effect.

[16]J. Schmidt and I. Solomon, C. R. Acad. Sci. 263, 169 (1966).

[17]A. Abragam, *The Principles of Nuclear Magnetism* (Oxford Univ. Press, Oxford, England, 1961), Chap. III.

[18]Because of the microwave power absorbed at resonance, there is a gradient of heating of the conduction electrons along the z axis, resulting in a Nernst voltage along Oy. This spin-dependent Nernst voltage is very troublesome for high-concentration samples but is negligible for a spin concentration in the range of 10^{14} cm^{-3}.

Translational Mobility of Hard Ferromagnetic Bubbles

J. C. Slonczewski

IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598

(Received 19 October 1972)

We show that steady translation of a magnetic bubble possessing Bloch lines requires the application of both longitudinal and transverse field gradients. The predicted longitudinal gradient obeys the usual relation valid for a normal bubble. The transverse gradient is proportional to the product of velocity and number of Bloch lines. Experiments of Tabor *et al.* support the theory.

Instances of magnetic-bubble motion in a direction different from that of the applied field gradient have been reported recently.[1,2] The experiment of Tabor *et al.*[1] shows that such skewed motion is a general property of "hard" magnetic bubbles, which are known to contain vertical Bloch lines.[1,3,4] Vella-Coleiro, Rosencwaig, and Tabor[5] proposed a nonlinear theory of both longitudinal and transverse components of mobility to explain this effect. Although it is consistent