In the transition region of diffusion where collisional and convective diffusions are comparable, the total diffusion will be the sum of two different types of diffusions. Thus, we have

$$D_{\mathrm{tot}} = (\alpha r_L^2/\tau_n) + \tfrac{1}{8}(\omega_p/\omega_c)(\omega_p/n\lambda_D), \tag{7}$$

which agrees well with the simulation results as shown in Fig. 1(a). The break points from classical to convective diffusion are also predicted correctly. They are given by

$$\omega_p/\omega_c = \omega_p\tau_n/8n\lambda_D^3\alpha.$$

Finally, we should like to point out that the convective diffusion may also explain measurements of Moore and Kessler[8] on the magnetic moment of a germanium plasma diffusing across a magnetic field. They found that for $\omega_c\tau_n \lesssim 3$ the measured magnetic moment agrees well with the theory based on the ambipolar diffusion which gives

$$M = -2en\omega_c\tau_n D_a, \tag{8}$$

where

$$D_a = v_t^2\tau_n/(1+\omega_c^2\tau_n^2). \tag{9}$$

However, for $\omega_c\tau > 3$, the observation deviates from Eq. (8), which predicts $M \sim 1/B$, and is more or less independent of B.

As we have already pointed out, the convective plasma transport is more important than the collisional transport for $\omega_c/\tau_n \gtrsim 3$ and D should vary as $1/B$ there, which would explain the observed fact that the magnetic moment is independent of B.

In closing, it would be interesting to observe the three different regions of plasma diffusion[2-4] for a finite-length solid-state plasma. This may be possible if $n\lambda_D^3$ is larger than, say, 100 and the electron lattice collision rate is small enough.

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$$m_i\frac{d\vec{v}_i}{dt} = \sum_{j\neq i}\frac{q_i q_j(\vec{r}_i - \vec{r}_j)}{\epsilon|\vec{r}_i - \vec{r}_j|^3} + \frac{q_i}{c}\vec{v}_i\times\vec{B},$$

where ϵ is the dielectric constant of the medium. Therefore, the plasma frequency and the Debye length in a dielectric will be $\omega_p \equiv (4\pi nq^2/\epsilon m)^{1/2}$ and $\lambda_D^{-2} = \sum_j 4\pi nq_j^2/\epsilon kT_j$ which are used throughout the paper.

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Dynamical Behavior of He^3-He^4 Mixtures near the Tricritical Point

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The dynamical behavior of the normal fluid phase of liquid He^3-He^4 mixtures near the tricritical point is studied. The superfluid order parameter and the entropy fluctuation are shown to have a common characteristic frequency which is the same as that predicted by dynamical scaling for the characteristic frequency of second sound in the ordered phase. The concentration fluctuation is shown to have a smaller characteristic frequency. The critical anomalies of the transport coefficients also are determined.

The unusual thermodynamic properties manifest near tricritical points have recently been the subject of several experimental and theoretical investigations.[1] Here we report on the first theoretical study of the dynamical behavior near tricritical points, concerning ourselves in particular with the liquid He^3-He^4 mixtures in the normal fluid phase. We adopt the notation recently proposed by Griffiths[1] for the critical exponents and the scaling fields g and λ. Here it is useful to introduce the straight line through the tricritical point which lies tangent to the line of critical points and the first-order line in the Δ-T plane (where Δ denotes the chemical potential difference $\mu_3-\mu_4$). Then the λ axis may be chosen to lie parallel to this critical line (say $\lambda=T-T_t$) and the g axis may be chosen so as to cut this critical line [say $g=\Delta-\Delta_1(T)$, where $\Delta_1(T)$ defines the straight line].[2,3] We choose the set of gross dynamical variables $\{a_j\}$ to be Fourier components of those local fluctuations which exhibit diffusive or relaxational behavior in the normal fluid phase, namely the superfluid order parameter ψ, $\psi^\dagger$, the entropy density fluctuation δS, the local concentration fluctuation δx, and the local transverse velocity $\vec{u}$.[4] The variable δS is chosen in such a way that its average square fluctuation is proportional to the specific heat at constant pressure and concentration, $C_{p,x}$, which remains finite at the tricritical point.[1,5b,6] For simplicity we ignore memory effects and write the macroscopic law for the gross variables in the following form (dots stand for time derivatives):

$$\dot{a}_j=-\sum_l(\chi_j/\chi_l)^{1/2}\Gamma_{jl}a_l, \tag{1}$$

where $\chi_j=\langle|\hat{a}_j|^2\rangle$, $\hat{a}_j$ is the quantum mechanical operator corresponding to a_j, and where $\Gamma_{jl}=\Gamma_{lj}{}^*$ contains the transport coefficients [see Eq. (9)]. If one approximates the mode coupling theory to the lowest order in which only two modes are allowed in intermediate states then Γ_{jl} is given as[5]

$$\Gamma_{jl}=\frac{L_{jl}{}^0}{(\chi_j\chi_l)^{1/2}}+\frac{1}{2}\sum_{m,n}\frac{V(jmn)V^*(lmn)}{\Gamma_m+\Gamma_n}, \tag{2}$$

where $L_j{}^0$ are the so-called bare Onsager kinetic coefficients which are assumed to be finite at the critical point[5] and where only the diagonal elements of the Γ's, $\Gamma_m=\Gamma_{mm}$, are retained in the denominator of the second term.[7] The quantity $V(jmn)$ is a mode-coupling coefficient given by[5]

$$V(jmn)=-(k_BT/\hbar)\langle[\hat{a}_j/\sqrt{\chi_j},\varphi_{mn}(\{\hat{a}\})]\rangle, \tag{3}$$

where

$$\varphi_{mn}(\{\hat{a}\})=(\chi_m\chi_n)^{-1/2}\{\hat{a}_m\hat{a}_n-\sum_p\langle\hat{a}_m\hat{a}_n\hat{a}_p\rangle\hat{a}_p/\chi_p\}, \tag{4}$$

and the brackets denote a commutator. To discuss the tricritical point, we use the parameter scaling assumption of Riedel and Wegner[2,3] according to which the order-parameter correlation function $G(k,\lambda,g)$ takes the following scaling form:

$$G(k,\lambda,g)=k^{-(2-\eta_t)}Y(g/k^{1/\nu_t},\lambda/k^{1/\varphi\nu_t}), \tag{5}$$

and the correlation function $G_0(k,\lambda,g)$ of the nonordering density x is

$$G_0(k,\lambda,g)=k^{-(2-\eta_0)}Y_0(g/k^{1/\nu_t},\lambda/k^{1/\varphi\nu_t}). \tag{6}$$

Since the χ's and V's involve only long-wavelength Fourier components, it is not difficult to obtain the commutators among the a's and their scaling behavior.[8] For example, $V(S\psi\psi^\dagger)$ can be shown to scale as $g^{3\nu_t/2}(\lambda^\varphi/g)^{3(\nu_t-\nu)/2}$. In this way one can determine the scaling behavior of Γ_{jl}, where in the cases in which the second term of Eq. (2) dominates the first term one must solve the equations in a self-consistent manner. The results can be expressed in the following scaling forms: In the tricritical region,

$$\Gamma_{jl}(k)=g^{\sigma_3}f_3(k/g^{\nu_t},\lambda^\varphi/g)+g^{\zeta_3}F_3(k/g^{\nu_t},\lambda^\varphi/g). \tag{7}$$

Near the λ line,

$$\Gamma_{jl}(k)=g^{\sigma_2}\lambda^{\varphi(\sigma_3-\sigma_2)}f_2(k/g^\nu\lambda^{\varphi(\nu_t-\nu)})+g^{\zeta_2}\lambda^{\varphi(\zeta_3-\zeta_2)}F_2(k/g^\nu\lambda^{\varphi(\nu_t-\nu)}), \tag{8}$$

where k is the wave number of the macroscopic mode and where the first and second terms of (7) and (8) correspond, respectively, to those of Eq. (2). The scaling function f_3 reduces to a function of k/g^{ν_t} for $\lambda^\varphi/g\to0$, and tends to $(\lambda^\varphi/g)^{\sigma_3-\sigma_2}f_2(k/g^\nu\lambda^{\varphi(\nu_t-\nu)})$ in the limit as λ^φ/g approaches infinity. The same remarks apply to F_2 and F_3. The critical exponents σ_i and ζ_i (where $i=2,3$) for the five elements of Γ which appear in the present problem are tabulated in Table I. The critical exponents γ_3, ν_3, ... are the tricritical exponents γ_t, ν_t, ..., and the critical exponents γ_2, ν_2, ... are the λ-line critical exponents γ, ν, ... The mode-coupling contribution to Γ_u was found to be negligible and thus was omitted. Hence the shear viscosity always remains finite. On the other hand, the mode-coupling contributions in which two order-parameter modes (ψ and $\psi^\dagger$) are excited in intermediate states dominate in all the other Γ's if one assumes the accepted values $\alpha \approx 0$, $\nu \approx \frac{2}{3}$, $\gamma \approx \frac{4}{3}$ and that $\gamma_t \approx 1$, $\nu_t \approx \alpha_t \approx \frac{1}{2}$, $\varphi \approx 2$, as has been suggested recently.[1-3]

TABLE I. The critical exponents σ_i and ζ_i, $i=2,3$, for the matrix elements Γ_{mn}, where $\Gamma_{mm} = \Gamma_m$. The subscripts $i=2,3$ denote the λ line and tricritical exponents, respectively.

	Γ_ψ	Γ_s	Γ_x	Γ_{xs}	Γ_u
σ_i	γ_i	$2\nu_i$	$2\nu_i+\alpha_i$	$2\nu_i+\alpha_i/2$	$2\nu_i$
ζ_i	$3\nu_i/2$	$3\nu_i/2$	$3\nu_i/2+\alpha_i$	$3\nu_i/2+\alpha_i/2$	$\cdots$

In the superfluid phase the second sound velocity u_2 is proportional to $\rho_s^{1/2}$ (where ρ_s is the superfluid density), since $C_{p,x}$ remains finite at the tricritical point.[11] Since ρ_s scales as the inverse of the correlation (coherence) length ξ,[12] the characteristic frequency associated with the second sound propagation, $\omega_2 = ku_2$, scales as $\omega_2 \sim \xi^{-3/2} \sim g^{3\nu_t/2}(\lambda^{\varphi}/g)^{3(\nu-\nu_t)/2}$ provided the critical exponents are the same as those in the normal phase. Hence dynamical scaling[13] holds for the second sound propagation, the order-parameter relaxation, and the entropy diffusion.

The results obtained above also allow us to predict the critical anomalies of all the transport coefficients of the normal fluid phase by noting that in the hydrodynamic regime the Γ's are simply related to the transport coefficients:

$$\Gamma_s(k) = \kappa_s k^2/(\rho C_{p,x}), \quad \Gamma_x(k) = Dk^2, \quad \Gamma_{xs}(k) = \Gamma_{sx}(k) = [(\partial\Delta/\partial x)_{p,T}/TC_{p,x}]^{1/2} L_T k^2, \tag{9}$$

where D, L_T, and κ_s are the diffusion constant, the thermal diffusion coefficient and the thermal conductivity (measured in the absence of a concentration gradient), respectively, and where ρ and Δ are the mass density and the chemical potential difference, respectively. Comparison of Eq. (9) with Eqs. (7) and (8) and Table I immediately yields for the transport coefficients in the tricritical region

$$\kappa_s \sim L_T \sim g^{-\nu_t/2} h(\lambda^{\varphi}/g), \quad D \sim g^{\alpha_t - \nu_t/2} h(\lambda^{\varphi}/g), \tag{10}$$

and near the λ line

$$\kappa_s \sim L_T \sim g^{-\nu/2}, \quad D \sim g^{\alpha - \nu/2}. \tag{11}$$

The function h in general differs for different transport coefficients but behaves like $h(x) \sim x^{(\nu-\nu_t)/2}$ for $x \gg 1$. We note that the concentration conductivity $\alpha \equiv \rho D (\partial\Delta/\partial x)_{p,T}^{-1}$ behaves in the same way as κ_s and L_T. We also note that a thermal conductivity is normally measured with a heat current but no diffusion current, such that the resulting thermal conductivity κ is given by[14]

$$\kappa = \kappa_s - (\rho L_T^2/TD)(\partial\Delta/\partial x)_{p,T}. \tag{12}$$

Now an explicit calculation shows that the two mode-coupling coefficients $V(S\psi\psi^\dagger)$ and $V(x\psi\psi^\dagger)$ which enter in the dominant contributions to κ_s, L_T, and D are simply related for $k\xi \ll 1$ by

$$\frac{V(S_{\vec{k}}\psi_{-\vec{k}'}\psi_{\vec{k}-\vec{k}'}^\dagger)}{V(x_{\vec{k}}\psi_{-\vec{k}'}\psi_{\vec{k}-\vec{k}'}^\dagger)} = -\frac{Ts_4}{xC_{p,x}(\partial\Delta/\partial x)_{p,T}}, \tag{13}$$

where s_4 is the partial molar entropy of He^4 per unit mass. Since this ratio is independent of $\vec{k}'$ one can show that these dominant mode-coupling contributions cancel in Eq. (12). As there is no other mode-coupling process which leads to a divergence in κ_s in the present approximation, we are led to predict that whereas κ_s will diverge, κ will remain finite near the tricritical point and the λ line, a prediction which is in agreement with Ahlers's experimental measurements of κ near the λ line.[15] In particular the approach described here can be readily extended to the superfluid phase as well as to other tricritical systems like magnets. We plan to investigate these problems including applications to critical scattering experiments.

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Onset of Striations in Barium Clouds*

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A numerical technique is used to solve equations which describe growth of a barium cloud in the ionosphere, including arbitrary end-shorting ratio λ. For λ corresponding to actual releases, one finds growth of a limited range of wavelengths k grouped around a maximum growing k. The growth rate and k range are sensitive functions of electric field E and λ. Stability occurs below a critical E and for strong shorting. The growing modes are localized on the rear of the cloud. All modes are unstable for no shorting.

One of the most striking phenomena exhibited by artificial plasma clouds released in the ionosphere is the onset of well-defined magnetic-field-aligned striations. These appear at some time after the initial release, the delay varying from experiment to experiment, but in the neighborhood of tens of minutes. There seems to be fairly general agreement that the $E\times B$ instability[1] (sometimes called the gradient drift instability) is responsible for the onset of these ionospheric striations. The problem has been to demonstrate that this is actually so by calculating delay times and scale sizes and comparing with experiment. Previous attempts to calculate this have failed for reasons which are now quite clear. One approach has been to ignore the end shorting entirely and treat the instability in a slab model.[2] (End shorting here means electron and ion currents in the surrounding ionosphere across magnetic lines passing through the cloud.) The result is a prediction of instability at all times and at all wavelengths. This is just what one would expect if the entire stabilizing effect of end shorting is removed. [Indeed the result is already there in Ref. 1 if one takes the same limit as in Ref. 2, namely $(\Omega\tau)_- \to \infty$, where Ω is the gyro-