*Work performed under the auspices of the U. S. Atomic Energy Commission.

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Evidence for Duality Constraints in $\Delta\to\pi+\Delta(1236)$ Decays*

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(Received 22 August 1972)

Partial-wave analysis of $\pi^+ + p \to \pi^0 + \Delta^{++}$ at 1820–2090 MeV c.m. energy shows that this reaction is dominated by the $F_{37}(1950)$ resonance decaying to $\Delta(1236)$ with s-channel helicity $\frac{3}{2}$. The analysis also gives evidence for $F_{35}(1890) \to \pi + \Delta$ via F wave. The coupling of F_{37} to helicity-$\frac{3}{2}$ states, and the unexpected dominance of F- over P-wave decay for $F_{35}(1890)$, can both be interpreted as arising from the constraints of s-t channel duality.

We have made a partial-wave analysis of the reaction $\pi^+ + p \to \pi^0 + \Delta^{++}$ in the c.m. energy interval 1820–2090 MeV.

Phase-shift analysis in the elastic channel shows that this energy region is dominated by the resonance $F_{37}(1950)$.[1] Other isospin-$\frac{3}{2}$ resonances believed present are $F_{35}(1890)$ and $P_{31}(1910)$; there is also some indication for the existence of $D_{35}(1960)$.[1] Our analysis gives evidence for the coupling of $F_{37}(1950)$ and $F_{35}(1890)$ to the $\pi\Delta$ channel with $(\chi_{\pi N}\chi_{\pi\Delta})^{1/2}$ of 0.43 ± 0.06 and 0.20 ± 0.03, respectively. In addition we find strong evidence for duality constraints in $\Delta \to \pi + \Delta(1236)$ resonance decays.

The data comes from a large bubble-chamber exposure at the Bevatron which gave 35 400 events $\pi^+ + p \to \pi^+ + p + \pi^0$ at six incident π^+ momenta: 1.28, 1.34, 1.42, 1.55, 1.67, and 1.84 GeV/c. Details of data processing and of the determination of the $\pi^+ p\pi^0$ cross section have been given in a previous publication on elastic scattering in this experiment.[2]

The $\pi^+ p \pi^0$ channel is dominated by the final states $\pi^0\Delta^{++}$, $\pi^+\Delta^+$, and $\rho^+ p$. The channel cross sections for $\pi\Delta$ and $\rho^+ p$ were determined at each momentum by a maximum-likelihood fit of the $\pi^+ p\pi^0$ events, assuming the following set of amplitudes in the $\pi^+ + p \to \pi^+ + p + \pi^0$ channel: $\pi^0\Delta^{++}$, $\pi^+\Delta^+$, $\rho^+ p$, $\pi^+ N^+(1500)$, and $\pi^+ N^+(1680)$.[3]

To obtain $\pi^0\Delta^{++}$ angular distributions free from $\rho^+ p$ background, we utilized the linear relationship between $M_{\pi^+\pi^0}{}^2$ and $\cos\delta$ at fixed $M_{\pi^+ p}$; δ is the decay angle of the $(\pi^+ p)$ system in the helicity frame. If the ρ band intersects the Δ^{++} band in the interval $1 \geq \cos\delta > 0$ (or $-1 \leq \cos\delta < 0$), we can obtain unbiased $\pi^0\Delta^{++}$ distributions by taking only Δ^{++} events with $-1 \leq \cos\delta < 0$ (or $1 \geq \cos\delta > 0$). This technique takes advantage of the symmetry of the Δ^{++} distributions about $\cos\delta = 0$, and was used at 1.28, 1.34, 1.42, 1.55, and 1.84 GeV/c. At 1.67 GeV/c, where the ρ^+ band intersects the $\cos\delta = 0$ line, the mass conjugation technique of

Eberhard and Pripstein was used to eliminate ρ^+p background.[4]

Residual ρ^+p contamination is most serious at 1.55, 1.67, and 1.84 GeV/c, where the ρ^+ band is at or near the center of the Dalitz plot. From the known ρ^+ production and decay distributions,[3] we calculate that the ρ^+p background contaminates most strongly the angular range $0.9 < \cos\theta_\pi < 1$ for $\pi^0\Delta^{++}$ events. We have, therefore, omitted this angular range at momenta 1.55, 1.67, and 1.84 GeV/c. We estimate that any remaining ρ^+p background is $\lesssim 5\%$.[5]

The input data to the partial-wave analysis were the cross section, and the distributions in $\cos\theta_\pi$, $\rho_{33}{}^s(\cos\theta_\pi)$, $\mathrm{Re}\rho_{31}{}^s(\cos\theta_\pi)$, and $\mathrm{Re}\rho_{3-1}{}^s(\cos\theta_\pi)$ at the six c.m. energies from 1820–2090 MeV. Figure 1 shows these distributions at c.m. energies 1850, 1950, and 2090 MeV. The χ^2 minimizing routine LSQ MIN was used to find the set of partial-wave amplitudes which best fitted the experimental data.[6]

From πN phase-shift analysis, the $I=\frac{3}{2}$ partial-wave amplitudes with significant inelastic cross sections at 1820–2090 MeV are S_{31}, P_{31}, P_{33}, D_{33}, D_{35}, F_{35}, and F_{37}. The corresponding partial waves in $\pi+N\to\pi+\Delta$ are $SD1$, $PP1$, $PP3$, $PF3$, $DS3$, $DD3$, $DD5$, $DG5$, $FP5$, $FF5$, $FF7$, and $FH7$, with notation $LL'2J$, where L (L') is the orbital angular momentum in the πN ($\pi\Delta$) channel. For each spin-parity state in the πN channel, two orbital angular momenta are accessible in the $\pi\Delta$ channel (except for $J=\frac{1}{2}$).

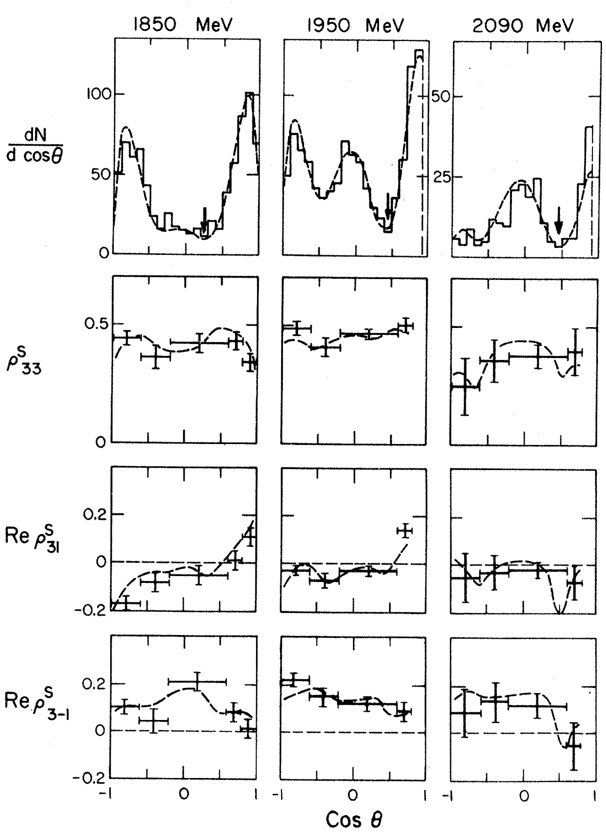

FIG. 1. Angular distribution and helicity-frame density-matrix elements for $\pi^+ + p \to \pi^0 + \Delta^{++}$ at c.m. energies 1850, 1950, and 2090 MeV. Arrows indicate $-t = 0.5\ \mathrm{GeV}^2$. Broken lines show best fit to the data.

The predominantly resonant F_{37} amplitude dominates the π^+p elastic and inelastic channels in our energy range.[1] At 1950 MeV, the total inelastic π^+p cross section for this amplitude is $\simeq 8$ mb, of which $\simeq 7$ mb is resonant.[1]

We made an "energy-dependent" fit, i.e., we fitted the data at all energies simultaneously. The dependence of the partial-wave amplitudes on c.m. momentum p was assumed to have the form $A_{\mathrm{BG}} = x_1 \exp[i(x_2 + x_3 p)] + (x_4 + ix_5)$, with five parameters x_i. In other words the partial waves can traverse an arc of a circle of radius of curvature x_1, centered anywhere (x_4, x_5) in the complex energy plane.

When testing for the presence of a resonance in a partial wave, we used a six-parameter form combining a Breit-Wigner amplitude $A_{\mathrm{BW}}(M, \Gamma, \chi_L\chi_{L'})$ with a constant background amplitude: $\exp(ix_1)A_{\mathrm{BW}}(x_2, x_3, x_4) + (x_5 + ix_6)$.[7]

The F_{37} amplitude was described by a nine-parameter form consisting of a resonant amplitude coupling to both F- and H-wave final states, plus a five-parameter background in $FF7$: $A_{\mathrm{BG}}(FF7) + A_{\mathrm{BW}}(M, \Gamma, \chi_F\chi_F, \chi_F\chi_H)$. The unknown overall phase for the partial-wave amplitudes in $\pi+N\to\pi+\Delta$ was defined by fixing the phase of the F_{37} resonance at zero (this phase is zero or π if the F_{37} amplitude is purely resonant).

We made an extensive series of fits with all amplitudes except F_{37} having the five-parameter "background" form. In these fits, and in all our subsequent fits, the F_{37} amplitude was found to be dominant. The other amplitudes demanded by the data were $SD1$, $PP1$, $PP3$, $DS3$, $DD3$, $DD5$, and $FF5$. We found no evidence for $DG5$ or $FP5$, and can set limits $|DD5/DG5|$ and $|FF5/FP5| > 2$. A satisfactory fit (fit 3), with a χ^2 per degree of freedom of 214/170, was obtained for the hypothesis of resonant F_{37} and background waves $SD1$, $PP1$, $PP3$, $DS3$, $DD3$, $DD5$, and $FF5$.

In some solutions, the amplitudes $PP1$, $DD5$, and $FF5$ appeared resonantlike, i.e., they executed a counterclockwise semicircle in the complex energy plane. These amplitudes couple to

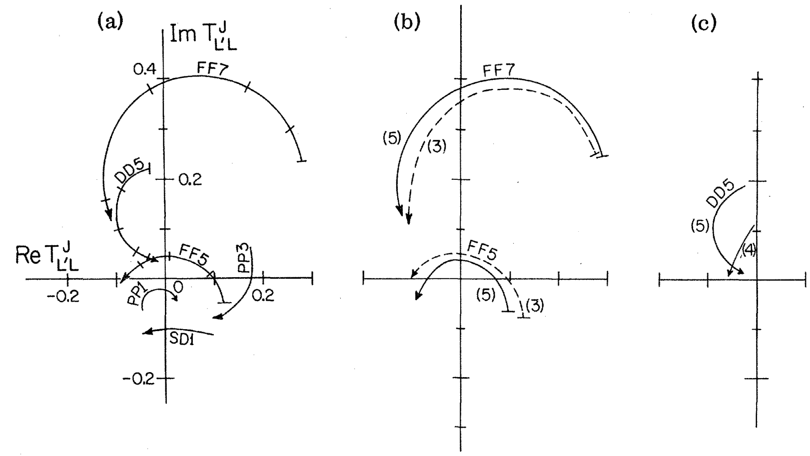

FIG. 2. (a) Argand diagram of prominant partial waves for best fit (fit 1 in Table I). (b), (c) Range of solutions for partial-wave amplitudes $FF7$, $FF5$, and $DD5$. Numbers refer to fits in Table I.

P_{31}, D_{35}, and F_{35}, respectively, for which there is evidence of resonant properties in the πN channel.[1] We therefore made a series of fits in which one or more of these amplitudes were given the six-parameter resonant form described above.

The best fit (fit 1) with a χ^2 per degree of freedom of 199/168 was obtained for the assumption of F_{37}, $FF5$, and $DD5$ resonant. This partial-wave solution is shown in Fig. 2(a); the corresponding fitted distributions are compared with the experimental data in Fig. 1. The F_{37} and F_{35} resonance parameters for the best fit are in good agreement with those deduced from phase-shift analysis in the elastic channel.

The range of resonance parameters obtained in our fits is shown in Table I. Figure 2 indicates the extent of the variation of the $FF7$, $FF5$, and $DD5$ amplitudes in these solutions.

We may summarize the results on resonance coupling to $\pi\Delta$ as follows.

$F_{37}(1950)$.—The $\pi^+p \to \pi^0\Delta^{++}$ channel at 1830–2090 MeV is dominated by the $F_{37}(1950)$ reso-

TABLE I. Parameters of F_{37}, F_{35}, and D_{35} resonances from partial-wave analysis of $\pi^+ + p \to \pi^0 + \Delta^{++}$ at 1820–2090 MeV.

Fit	χ^2/Degrees of Freedom	Resonant Amplitudes	Mass (MeV)	Γ (MeV)	$\sqrt{x_{\pi N} x_{\pi\Delta}}$	$\sqrt{x_{L'+2}/x_{L'}}$
1	199/168	F_{37}	1920	269	0.48	0.06
		FF5	1890	300	0.23	> 2
		DD5	1824	138	0.19	> 2
2	202/168	F_{37}	1926	266	0.46	0.06
		FF5	1911	294	0.22	> 2
		DD5	1824	158	0.19	> 2
3	214/170	F_{37}	1951	254	0.37	0.08
4	215/168	F_{37}	1926	267	0.46	.06
		FF5	1913	322	0.18	> 2
5	220/168	F_{37}	1923	234	0.40	0.07
		FF5	1986	273	0.19	> 2
		DD5	1822	174	0.18	> 2

nance with best-fit parameters $M = 1920$ MeV, $\Gamma = 269$ MeV, $(\chi_{\pi N}\chi_{\pi\Delta})^{1/2} = 0.48$, and $(\Gamma_H/\Gamma_F)^{1/2} = 0.06$. The error on these estimates is indicated by the range of values listed in Table I. Although the parameters of the F_{37} resonance vary from fit to fit, the combined resonant plus background wave [Figs. 2(a) and 2(b)] is essentially the same in all solutions.

$F_{35}(1890)$.—The F_{35} resonance parameters from the best fit are $M = 1890$ MeV, $\Gamma = 300$ MeV, and $(\chi_{\pi N}\chi_{\pi\Delta})^{1/2} = 0.23$. In fits 1, 2, and 4, the parameters of $F_{35}(1890)$ are in good agreement with those measured in the elastic channel. One fit (fit 5) gave a mass of 1986 for the F_{35} resonance. The analysis gave no evidence for the $FP5$ partial wave and we can set a limit $(\Gamma_F/\Gamma_P)^{1/2} > 2$ for $F_{35}(1890)$.

$D_{35}(1960)$.—There is no evidence for $D_{35}(1960)$ in the $\pi\Delta$ channel. When the $DD5$ wave was parametrized as resonant, we obtained acceptable solutions with a resonance mass of 1820 MeV.

$P_{31}(1910)$.—The analysis gave no evidence for this resonance in the $\pi\Delta$ channel.

These results have considerable implications for the duality concept. In a recent paper, Gell, Horn, Jacob, and Weyers[8] have shown that channels with helicity flip $\Delta\lambda > 1$ amplitudes, such as $\pi N \to \pi\Delta$, allow a much more stringent test of the duality hypothesis than channels with $\Delta\lambda \leq 1$ such as $\pi^- + p \to \pi^0 + n$. For $\pi + N \to \pi + \Delta$, Gell *et al.* show that the requirement of duality between peripheral resonances and the ρ Regge amplitude is met if the resonance decays predominantly to a state of helicity $\frac{3}{2}$ in the s channel. At 1950 MeV, $\rho_{33}{}^s$ in Fig. 1 is close to the maximum value of 0.5, corresponding to 100% helicity $\frac{3}{2}$ in the s channel. At the same energy, the partial-wave analysis establishes that the resonant F_{37} amplitude accounts for $\simeq 85\%$ of the $\pi + N \to \pi + \Delta$ cross section. The idea that peripheral resonances are constrained by duality to couple to helicity $\frac{3}{2}$ in the $\pi\Delta$ channel is thus strongly supported by this experiment. We note that the angular distributions in Fig. 1 show a dip at $-t \simeq 0.5$ GeV^2 as required by duality.

The requirement that $\rho_{33}{}^s = 0.5$ for each individual resonance can also be expressed in terms of the ratio Γ_{L+2}/Γ_L for the $\pi\Delta$ decay of the resonance. For $F_{35}(1890)$ the duality requirement is $(\Gamma_F/\Gamma_P)^{1/2} = 1.2$, contrary to the P-wave dominance expected from centrifugal barrier arguments. This experiment gave $(\Gamma_F/\Gamma_P)^{1/2} > 2$. For $(\Gamma_F/\Gamma_P)^{1/2}$ in the range 2–4, the corresponding $\rho_{33}{}^s$ values are 0.48–0.41. The experimental observations are thus consistent with $F_{35}(1890)$ decaying to $\pi\Delta$ with predominant s-channel helicity $\frac{3}{2}$. The dominance of F- over P-wave decay for $F_{35}(1890)$ must be attributed to dynamical factors and is a striking confirmation of the prediction of Gell *et al.*[8]

To our knowledge, this is the first example of a resonance decay in which the higher orbital angular momentum state dominates.

We thank D. Horn for helpful discussions.

*Work supported in part by the U. S. Atomic Energy Commission under Contract No. AT (04-3)34 P107B.

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