the fit to our momentum. His extrapolated prediction yields a value of 77 for a multiperipheral model and 84 for a fragmentation model. Our value of 89 ± 3 is in closer agreement with the latter. Finally, in Fig. 1(c) $\langle n_{ch}\rangle(\langle n_{ch}^2\rangle - \langle n_{ch}\rangle^2)^{-1/2}$ is shown. The apparent constancy of this expression has been noted by a number of authors[12] and in particular Koba, Nielsen, and Olesen point out that it follows from one of their scaling laws.

Significant scientific contributions to this work have been made by many physicists other than the authors. They have built and operated the National Accelerator Laboratory, its extraction system, and the hadron beam line to the bubble chamber. It has been a privilege for us to work with them during the course of this experiment, and we thank them for their enthusiastic participation. We gratefully acknowledge the dedicated support of the operation staffs of the accelerator, the Neutrino Laboratory, the 30-in. Bubble Chamber, and Film Analysis Facility.

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[1]G. Barbiellini *et al.*, Phys. Lett. 39B, 663 (1972).

[2]Definitions of f_n and g_n are as follows: $g_1 = \langle n\rangle$, $g_2 = \langle n(n-1)\rangle$, $g_3 = \langle n(n-1)(n-2)\rangle$, $g_4 = \langle n(n-1)(n-2)(n-3)\rangle$, $f_2 = g_2 - g_1^2$, $f_3 = g_3 - 3g_2g_1 + 2g_1^3$, $f_4 = g_4 - 4g_1g_3 + 12g_1^2g_2 - 3g_2^2 - 6g_1^4$.

[3]We would like to thank E. Berger for pointing out an overestimation of our errors in an earlier version.

[4]J. W. Chapman *et al.*, University of Rochester Report No. UR-395 (to be published).

[5]G. Charlton *et al.*, Phys. Rev. Lett. 29, 515 (1972).

[6]G. Alexander *et al.*, Phys. Rev. 154, 1284 (1967).

[7]D. B. Smith *et al.*, Phys. Rev. Lett. 23, 1064 (1969), and Lawrence Radiation Laboratory Report No. UCRL-20632, March 1971 (unpublished).

[8]H. Boggild *et al.*, Nucl. Phys. 27B, 285 (1971).

[9]Soviet-French Mirabelle Collaboration, submitted to the Proceedings of the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Illinois, September 1972 (to be published).

[10]M. Briedenback *et al.*, Phys. Lett. 39B, 654 (1972).

[11]E. L. Berger, Phys. Rev. Lett. 29, 887 (1972).

[12]O. Czyzewski and K. Rybicki, Institute of Nuclear Physics, Cracow, Report No. 800/PH, 1972 (unpublished); Z. Koba, H. B. Nielsen, and P. Olesen, Nucl. Phys. B40, 317 (1972); S. N. Ganguli and P. K. Malhotra, Tata Institute Report No. TIFR-BC-72-6, July 1972 (to be published).

Measurement of the Ξ^- Magnetic Moment*

R. L. Cool,† G. Giacomelli,‡ E. W. Jenkins,§ T. F. Kycia, B. A. Leontić,‖ K. K. Li, and J. Teiger**
Brookhaven National Laboratory, Upton, New York 11973
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A measurement of the magnetic moment of the Ξ^- hyperon yielded the value $\mu_\Xi = (-2.2 \pm 0.8)\mu_N$. The Ξ^- polarization averaged over the acceptance angle and the four-momenta at which data were taken was $\bar{P} = 0.30 \pm 0.05$.

We report a measurement of the magnetic moment of the Ξ^- hyperon made at the Brookhaven alternating gradient synchrotron using a method similar to that of our earlier measurement of the Λ hyperon.[1] Transversely polarized Ξ^- were produced and subsequently decayed through the reactions

$$\begin{aligned} K^- + p &\to \Xi^- + K^+ \\ &\quad \llcorner \Lambda + \pi^- \\ &\quad\quad \llcorner p + \pi^-. \end{aligned} \quad (1)$$

We selected for analysis those Ξ^- which passed through a strong longitudinal magnetic field and subsequently decayed through the chain (1).

Since parity is not conserved in the Ξ^- and Λ decays, the angle through which the polarization vector has precessed about the magnetic field can be obtained by measuring the angular distribution of the two decay processes with respect to the plane of production. In the Ξ^- rest frame, the equation of motion of the polarization vector $\hat{\sigma}_\Xi$ in a magnetic field $\vec{H}$ is

$$d\hat{\sigma}_\Xi/dt = (\mu_\Xi/s_\Xi\hbar)\hat{\sigma}_\Xi \times \vec{H}, \quad (2)$$

where μ_Ξ and $s_\Xi\hbar$ are, respectively, the magnetic moment and spin angular momentum of the Ξ^-. In our apparatus, a Ξ^- with a magnetic moment of $1\mu_N$ would precess through an angle of about 20°.

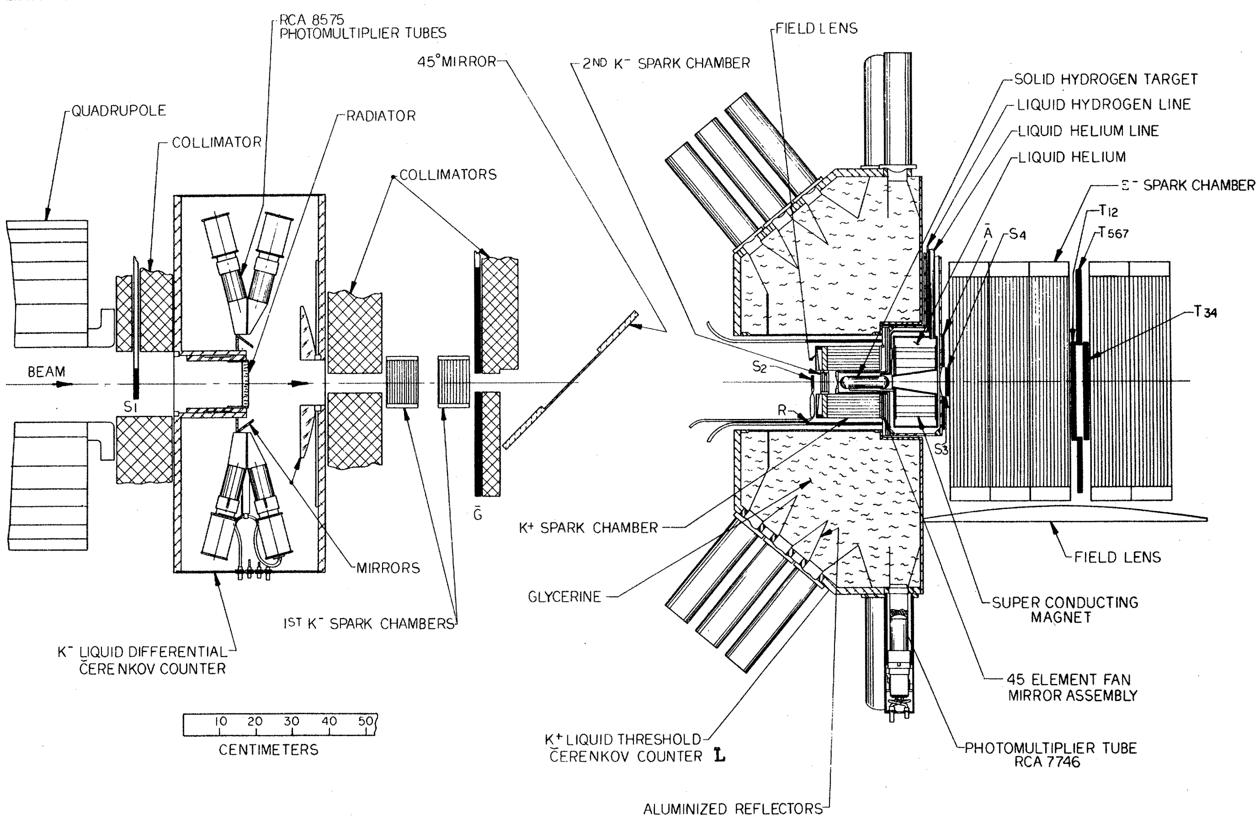

FIG. 1. General experimental layout.

We used a partially separated K^- beam, the separation of which was accomplished by two electrostatic separators. The beam momentum was nominally set at 1.83 GeV/c with $\Delta p/p = \pm 2\%$. A flux of 3×10^4 K^- per 10^{12} circulating protons was achieved within a final image of less than 2.8 cm in diameter. The π^-/K^- ratio after separation was less than 2.

Figure 1 shows the experimental layout. The incoming beam was defined by a two-counter telescope (S_1, S_2), and the K^- mesons were identified by a liquid differential Cherenkov counter.[2] The Ξ^- were produced in a solid H_2 target 2.6 cm in diameter and 14 cm in length. The requirement that a K^+ meson be produced in the collision was the principal electronic trigger used to reject undesired interactions. K^+ mesons from (1) produced between 75° and 120° crossed the cylindrical scintillation counter R and stopped in the glycerine-filled threshold Cherenkov counter L.[3] The index of refraction of the glycerine is such that the K^+ were too slow to produce Cherenkov radiation, but such that most of the fast mesons and pions from the decay would radiate. The signal from counter L was required to be delayed with respect to R between 5.5 and 55 nsec. The remaining trigger requirements were based upon the known topology of the desired Reaction (1). Two thin dE/dx counters (S_3, S_4) required one and only one charged particle in the forward cone of the magnet, while anticoincidence counter $\bar{A}$ rejected events with particles outside this cone. The T-counter assembly required two or more particles. The latter selection demanded that both the Ξ^- and the Λ of (1) decay before reaching T. Great care was exercised to maintain cylindrical symmetry in the geometrical selection behind the magnet to avoid introducing a bias in the observed decay distributions.

The 15-cm-long magnet was a superconducting solenoid wound with niobium-tin ribbon.[4] It was cooled with a He refrigerating system and was operating between 105 and 117 kG. Optical spark chambers were arranged before and after the magnet to observe the production kinematics and both the Ξ^- and Λ decay products and vertices.

All photographs were scanned three times and the geometry of each selected candidate was reconstructed. After preliminary fitting of the production and decay vertices to obtain starting

TABLE I. Lifetimes and decay parameters.

	τ_{K^+} (10^{-8} sec)	τ_{Ξ} (10^{-10} sec)	τ_Λ (10^{-10} sec)	$\alpha_\Xi\alpha_\Lambda$
Accepted value[a]	1.237 ± 0.003	1.660 ± 0.037	2.517 ± 0.024	-0.259 ± 0.021
This experiment	1.29 ± 0.05	1.637 ± 0.050	2.42 ± 0.10	-0.261 ± 0.037

[a]Ref. 4.

values, a final simultaneous three-vertex fit was made. The geometrical and kinematical reconstruction program and its resolution were checked by Monte Carlo simulation. This simulation gave 3° for the full width at half-maximum resolution of the K^+ azimuthal angle and 0.8° for the Ξ^-. The acceptance of the program for Monte Carlo events was about 97%.

Data were taken with zero magnetic field at 1.74, 1.80, and 1.87 GeV/c, and with the field both parallel (+) and antiparallel (−) to the Ξ^- trajectory at 1.83 GeV/c. The field-on data were taken in about 600 h of running time. For the sample of events reported here, we required (1) that the K^- and K^+ tracks, the Ξ^- decay vertex, and the subsequent decay $\Lambda \to p + \pi^-$ be observed; (2) that the Ξ^- and Λ decay vertices be separated by two spark-chamber gaps; and (3) that the three-vertex fit give $\chi^2 \leqslant 20$. After this selection, our sample consists of 1302 events with zero field and 1134 events with magnetic field. For this data sample, we obtain the K^+, Ξ^-, and Λ lifetimes. The results, which are given in Table I, are in good agreement with published values[4] and show that with our criteria a clean sample of Reaction (1) was selected.

From our events, we obtained four decay angular distributions. The first is the distribution of the polar angle of the Λ momentum with respect to the direction of the Ξ^- polarization (i.e., the normal to the plane of production) in the Ξ^- rest frame. The other three are the angular distributions of the proton from the Λ decay with respect to each axis of a suitably defined set in the rest frame of the Λ.[5]

For Ξ^- spin $=\frac{1}{2}$ and zero magnetic field, in the rest system of the decaying hyperon, all angular distributions take the form

$$I(\theta_i) \propto (1 + A_i \cos\theta_i), \qquad (3)$$

with

$$A_1 = \alpha_\Xi \overline{P}, \quad \cos\theta_1 = (\hat{\sigma}_\Xi \cdot \hat{p}_\Lambda), \qquad (3a)$$

$$A_2 = \alpha_\Lambda \alpha_\Xi, \quad \cos\theta_2 = (\hat{p}_p \cdot \hat{p}_\Lambda), \qquad (3b)$$

$$A_3 = \tfrac{1}{4}\pi \beta_\Xi \alpha_\Lambda \overline{P}, \quad \cos\theta_3 = \hat{p}_p \cdot (\hat{\sigma}_\Xi \times \hat{p}_\Lambda), \qquad (3c)$$

$$A_4 = \tfrac{1}{3}(1 + 2\gamma_\Xi)\alpha_\Xi \overline{P}, \quad \cos\theta_4 = \hat{\sigma}_\Xi \cdot \hat{p}_p, \qquad (3d)$$

where $\hat{\sigma}_\Xi \equiv \hat{p}_k \times \hat{p}_\Xi$, the $\hat{p}$ are the unit vectors of the subscripted particle momenta, $\overline{P}$ is the average Ξ^- polarization, and α, β, γ are the usual Ξ^- and Λ decay parameters. Distribution (3b) is independent of $\overline{P}$ and the magnetic field. The value of $\alpha_\Lambda \alpha_\Xi$ for all data given in Table I is in excellent agreement with published results.[4]

In a magnetic field which precesses the polarization vector by an angle ϵ, (3a) and (3d), after projecting onto the plane perpendicular to the Ξ^- momentum, take the form

$$D(\eta_i) \propto 1 + \tfrac{1}{4}\pi A_i \cos(\eta_i - \epsilon), \qquad (4)$$

where η_i is the projected angle. Thus, in the projected distribution (4) the effect of the precession is simply to rotate the distribution by the precession angle ϵ.

In Fig. 2, we plot the left-right asymmetry (R) with respect to an axis rotated in the projected plane. R should pass through zero at an angle equal to ϵ.[1] The asymmetry is plotted for both the Ξ^- and Λ decays under each of the three (+, −, 0) field conditions. Table II gives the best-fit results. Within statistics, the data show the correct behavior, namely, (1) for each distribution ϵ reverses sign with reversal of magnetic field, (2) the zero-field distributions are consistent with $\epsilon = 0$, and (3) the amplitude and relative phase of the Ξ^- and Λ distributions are consistent with the known signs and values of α_Ξ, α_Λ, and γ_Ξ.

For the final values which are given in Table III, we made maximum use of all data by a maximum-likelihood fit with only two parameters, μ_Ξ and $\overline{P}$, to be fitted. We assumed $\alpha_\Lambda = 0.645$, $\alpha_\Xi = -0.40$, and $\gamma_\Xi = 0.91$ without errors. The result is $\mu_\Xi = (-2.2 \pm 0.8)\mu_N$ and $\overline{P} = 0.30 \pm 0.05$. For comparison with theory, we have quoted our re-

TABLE II. Results from the R distribution fits.

Data sample	From Ξ^- decay ϵ (deg)	From Ξ^- decay $\bar{P}$	From Λ decay ϵ (deg)	From Λ decay $\bar{P}$	Weighted average ϵ (deg)	Weighted average $\bar{P}$
Positive field	25 ± 21	0.63 ± 0.21	62 ± 21	0.39 ± 0.18	43 ± 15	0.49 ± 0.14
Negative field	-35 ± 55	0.15 ± 0.17	-58 ± 38	0.20 ± 0.14	-51 ± 31	0.18 ± 0.11
Weighted average					44.5 ± 14	0.30 ± 0.09
Zero field	26 ± 24	0.31 ± 0.13	-25 ± 14	0.38 ± 0.10	-12.1 ± 12	0.35 ± 0.08

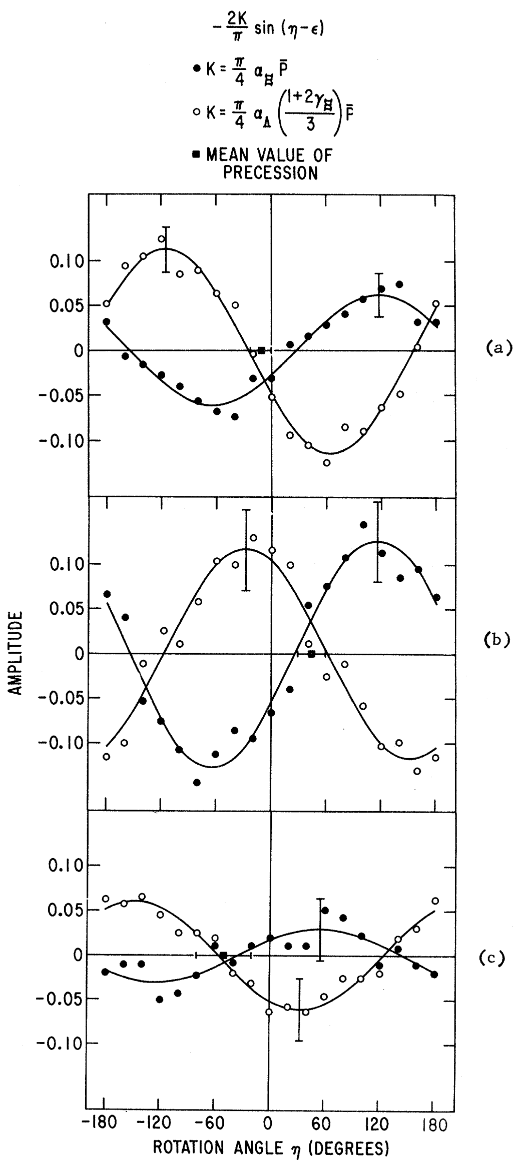

FIG. 2. The left-right asymmetry (R) distributions for (a) no-field data, (b) positive-field data, and (c) negative-field data. The curves are the best fits to the data samples.

sult in nuclear magnetons, assuming the Ξ^- spin $=\frac{1}{2}$. In fact, the experiments measures the gyromagnetic ratio which is $g_\Xi = 4.4 \pm 1.6$.

SU(3) symmetry, without mass breaking, predicts a value of $\mu_\Xi = -(\mu_n + \mu_p) = -0.9\mu_N$.[6] Our value agrees in sign, but gives a most probable value which is somewhat larger. From the experimental point of view, since the measured and predicted values differ by only 1.7 standard deviations, no definitive disagreement is implied by our data. Another measurement of $\mu_\Xi = (-0.1 \pm 2.1)\mu_N$ has been reported.[7] From the theoretical point of view, on the other hand, since m_Ξ is nearly 40% greater than m_p, the presence of an appreciable mass correction term would not be too surprising. At present, no fully acceptable means of calculating such a correction is available.

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TABLE III. Results from maximum-likelihood fits.

Sample	$\bar{P}$	$\mu_\Xi \mu_N$
No field	0.34 ± 0.07	0.4 ± 0.6[a]
Negative field	0.16 ± 0.09	-2.1 ± 1.7
Positive field	0.40 ± 0.12	-2.3 ± 0.9
All data combined	0.30 ± 0.05	-2.2 ± 0.8

[a]Assuming positive field with same average field integral as the field-on data.

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†Present address: The Rockefeller University, New York, N. Y. 10021.

‡On leave of absence from the University of Bologna, Bologna, Italy.

§Present address: University of Arizona, Tucson, Ariz. 85721.

‖Present address: Institute of Physics, University of Zagreb, Zagreb, Yugoslavia.

**Present address: Centre d'Etudes Nucléaires de Saclay, Saclay, France.

[1]R. L. Cool, E. W. Jenkins, T. F. Kycia, D. A. Hill, L. Marshall, and R. A. Schluter, Phys. Rev. 127, 2223 (1962); D. A. Hill, K. K. Li, E. W. Jenkins, T. F. Kycia, and H. Ruderman, Phys. Rev. Lett. 15, 85 (1965), and Phys. Rev. D 4, 1979 (1971).

[2]B. A. Leontic and J. Teiger, BNL Report No. 50031, 1966 (unpublished).

[3]G. Giacomelli, T. F. Kycia, K. K. Li, and J. Teiger, Rev. Sci. Instrum. 38, 1408 (1967).

[4]A. Rittenberg, A. Barbaro-Galtieri, T. Lasinski, A. H. Rosenfeld, T. F. Trippe, M. Roos, G. Bricman, P. Söding, N. Barash-Schmidt, and C. G. Wohl, Rev. Mod. Phys. Suppl. 43, 1 (1971).

[5]T. D. Lee and C. N. Yang, Phys. Rev. 108, 1645 (1957); W. B. Teutsch, S. Okubo, and E. C. G. Sudarshan, Phys. Rev. 114, 1148 (1959); Y. Ueda and S. Okubo, Nucl. Phys. 49, 345 (1963).

[6]S. Coleman and S. L. Glashow, Phys. Rev. Lett. 6, 423 (1961).

[7]G. McD. Bingham, V. Cook, J. W. Humphrey, O. R. Sander, R. W. Williams, G. E. Masek, T. Maung, and H. Ruderman, Phys. Rev. D 1, 3010 (1970).

Evidence for Duality Constraints in $\Delta \to \pi + \Delta(1236)$ Decays*

U. Mehtani,† S. Y. Fung, A. Kernan, T. L. Schalk, and Y. Williamson
University of California, Riverside, California 92502

and

R. W. Birge, G. E. Kalmus,‡ and W. Michael
Lawrence Radiation Laboratory, University of California, Berkeley, California 94720

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Partial-wave analysis of $\pi^+ + p \to \pi^0 + \Delta^{++}$ at 1820–2090 MeV c.m. energy shows that this reaction is dominated by the $F_{37}(1950)$ resonance decaying to $\Delta(1236)$ with s-channel helicity $\frac{3}{2}$. The analysis also gives evidence for $F_{35}(1890) \to \pi + \Delta$ via F wave. The coupling of F_{37} to helicity-$\frac{3}{2}$ states, and the unexpected dominance of F- over P-wave decay for $F_{35}(1890)$, can both be interpreted as arising from the constraints of s-t channel duality.

We have made a partial-wave analysis of the reaction $\pi^+ + p \to \pi^0 + \Delta^{++}$ in the c.m. energy interval 1820–2090 MeV.

Phase-shift analysis in the elastic channel shows that this energy region is dominated by the resonance $F_{37}(1950)$.[1] Other isospin-$\frac{3}{2}$ resonances believed present are $F_{35}(1890)$ and $P_{31}(1910)$; there is also some indication for the existence of $D_{35}(1960)$.[1] Our analysis gives evidence for the coupling of $F_{37}(1950)$ and $F_{35}(1890)$ to the $\pi\Delta$ channel with $(\chi_{\pi N}\chi_{\pi\Delta})^{1/2}$ of 0.43 ± 0.06 and 0.20 ± 0.03, respectively. In addition we find strong evidence for duality constraints in $\Delta \to \pi + \Delta(1236)$ resonance decays.

The data comes from a large bubble-chamber exposure at the Bevatron which gave 35 400 events $\pi^+ + p \to \pi^+ + p + \pi^0$ at six incident π^+ momenta: 1.28, 1.34, 1.42, 1.55, 1.67, and 1.84 GeV/c. Details of data processing and of the determination of the $\pi^+ p \pi^0$ cross section have been given in a previous publication on elastic scattering in this experiment.[2]

The $\pi^+ p \pi^0$ channel is dominated by the final states $\pi^0\Delta^{++}$, $\pi^+\Delta^+$, and $\rho^+ p$. The channel cross sections for $\pi\Delta$ and $\rho^+ p$ were determined at each momentum by a maximum-likelihood fit of the $\pi^+ p \pi^0$ events, assuming the following set of amplitudes in the $\pi^+ + p \to \pi^+ + p + \pi^0$ channel: $\pi^0\Delta^{++}$, $\pi^+\Delta^+$, $\rho^+ p$, $\pi^+ N^+(1500)$, and $\pi^+ N^+(1680)$.[3]

To obtain $\pi^0\Delta^{++}$ angular distributions free from $\rho^+ p$ background, we utilized the linear relationship between $M_{\pi^+\pi^0}{}^2$ and $\cos\delta$ at fixed $M_{\pi^+ p}$; δ is the decay angle of the $(\pi^+ p)$ system in the helicity frame. If the ρ band intersects the Δ^{++} band in the interval $1 \geq \cos\delta > 0$ (or $-1 \leq \cos\delta < 0$), we can obtain unbiased $\pi^0\Delta^{++}$ distributions by taking only Δ^{++} events with $-1 \leq \cos\delta < 0$ (or $1 \geq \cos\delta > 0$). This technique takes advantage of the symmetry of the Δ^{++} distributions about $\cos\delta = 0$, and was used at 1.28, 1.34, 1.42, 1.55, and 1.84 GeV/c. At 1.67 GeV/c, where the ρ^+ band intersects the $\cos\delta = 0$ line, the mass conjugation technique of