sorption model (Michigan[9]) prediction lies well below the data, and both the Argonne[10] and mixed model of Gault[11] predict more structure than is seen experimentally. The models of Argyres[12] (dual absorptive model) and Worden[13] (nonsense zeros and strong cuts) are in best agreement with the data. There is very little energy dependence in the prediction of any of the models mentioned above (at least between 4.7 and 8.2 GeV).

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†Present address: University of Lancaster, Lancaster, England.

‡Present address: Argonne National Laboratory, Argonne, Ill. 60439.

§Present address: DESY, Hamburg, Germany.

[1]W. T. Meyer *et al.*, Phys. Rev. Lett. 28, 1344 (1972).
[2]G. C. Bolon *et al.*, Phys. Rev. Lett. 27, 964 (1971).
[3]J. C. Young *et al.*, Nucl. Instrum. Methods 68, 333 (1969).
[4]R. J. Kurz *et al.*, UCRL Report No. UCRL-11339 (unpublished) (program modified by M. Hauser, Princeton Univ.).
[5]R. L. Anderson *et al.*, SLAC Report No. SLAC-PUB 925 (unpublished).
[6]W. Braunschweig *et al.*, Nucl. Phys. B20, 191 (1970).
[7]B. Wiik, in *Proceedings of the Fifth International Symposium on Electron and Photon Interactions at High Energies, Ithaca, New York, 1971,* edited by N. B. Mistry (Laboratory of Nuclear Studies, Cornell Univ., Ithaca, N. Y., 1972).
[8]J. Froyland, Nucl. Phys. B11, 204 (1969).
[9]R. C. Arnold and M. L. Blackmon, Phys. Rev. 176, 2082 (1968).
[10]F. Henley *et al.*, Phys. Rev. 182, 1579 (1969).
[11]F. Gault, private communication.
[12]E. N. Argyres, private communication.
[13]R. Worden, private communication.

Evidence for the Onset of Semi-inclusive Scaling in Proton-Proton Collisions in the 50–300-GeV/c Momentum Range*

P. Slattery
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
(Received 19 October 1972)

Evidence is presented to support the onset, in the 50–300-GeV/c region of incident momentum, of the asymptotic prediction of Koba, Nielsen, and Olesen regarding the scaling behavior of the charged-prong multiplicity distribution in proton-proton collisions. Lower-energy data, typified by data at 19 GeV/c, are found not to demonstrate this limiting behavior.

One of the simplest and most direct measurements which can be made with a bubble chamber is the determination of charged-particle multiplicities. The present availability of bubble-chamber facilities at Serpukhov, U. S. S. R., and at Batavia, U. S. A., has consequently made available for the first time accurate measurements of topological cross sections for very high-energy proton-proton collisions (50–300 GeV/c). In this note we wish to examine the energy variation of these partial cross sections, and to point out how this variation supports the onset, in this unexpectedly low-energy domain, of the asymptotic prediction of Koba, Nielsen, and Olesen (KNO) regarding semi-inclusive scaling.[1]

The semi-inclusive scaling prediction for the asymptotic behavior of topological cross sections may be summarized in terms of the following limit:

$$\sigma_n/\sigma_{\text{inel}} \xrightarrow[s\to\infty]{} \langle n\rangle^{-1}\psi(n/\langle n\rangle). \tag{1}$$

Here, σ_n is the partial cross section for the reaction $pp \to n$ charged particles, σ_{inel} is the total inelastic pp cross section (throughout this Letter σ_2 does not include the elastic channel), $\langle n\rangle$ is the average number of charged particles produced at a particular value of squared center-of-mass energy s, and ψ is an energy-independent function.

In Fig. 1 we examine this prediction in the 50–300-GeV/c range of incident momenta by plotting $\langle n\rangle(\sigma_n/\sigma_{\text{inel}})$ versus $n/\langle n\rangle$ for experimental data consisting of topological cross sections measured at the following incident momenta: 50 and 69

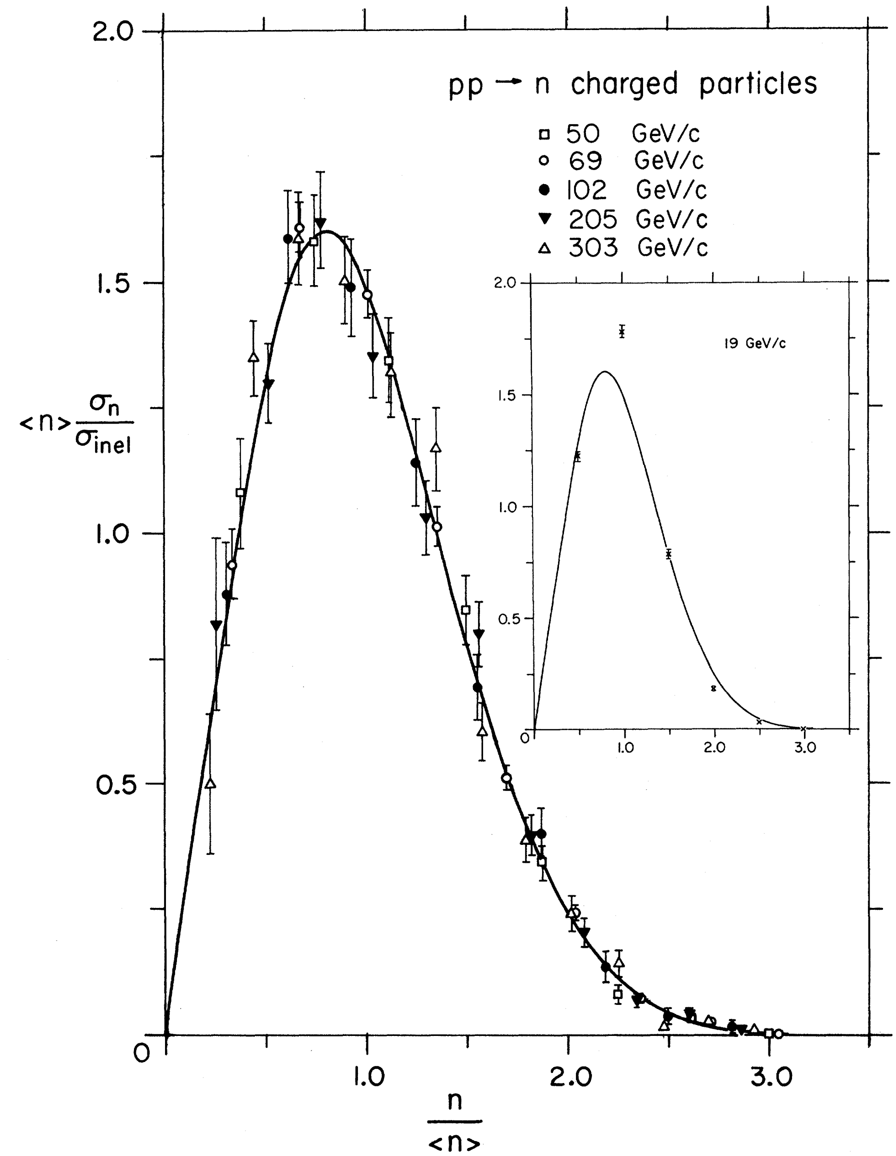

FIG. 1. Plot of $\langle n\rangle(\sigma_n/\sigma_{\rm inel})$ versus $(n/\langle n\rangle)$ for the reaction $pp\to n$ charged particles at incident momenta of 50, 69, 102, 205, and 303 GeV/c. The curve is an empirical fit to the data; for the details of this fit see the text. The insert compares lower energy data at 19 GeV/c with this fitted curve.

GeV/c^2 taken in the Mirabelle chamber at Serpukhov, and 102,[3] 205,[4] and 303 GeV/c[5] taken in the National Accelerator Laboratory–Argonne National Laboratory 30-in. chamber at the National Accelerator Laboratory. The fact that a universal curve may be drawn through all of the data points with a satisfactory χ^2 is a dramatic indication that the semi-inclusive scaling idea is experimentally useful in this range of incident momentum. The particular parametrization of the function $\psi(n/\langle n\rangle)$ which yields the curve shown on the figure was chosen as described below.

By virtue of the normalization condition $\sum_n\sigma_n=\sigma_{\rm inel}$, the function $\psi(z=n/\langle n\rangle)$ must satisfy the following two constraints:

$$\int_0^\infty \psi(z)\,dz=\int_0^\infty z\,\psi(z)\,dz=2. \qquad (2)$$

(The value 2 reflects the fact that charged particles must be created in pairs in particle collisions.) If, in addition, we require that higher moments, defined by $\int_0^\infty z^q\psi(z)\,dz$, be defined for all q, then a convenient parametrization for the function $\psi(z)$ is

$$\psi(z)=e^{-\beta z}\sum_{m=0}^{M}\alpha_m z^m. \tag{3}$$

A function of this form, satisfying constraints (2), was fitted to the data displayed in Fig. 1, and β and $\{\alpha_m,\ m=0,1,\ldots,M\}$ chosen so as to minimize the overall χ^2. Since it was empirically observed that a functional form employing only odd powers of z, and hence passing through zero at $z=0$, was sufficient to fit all of the data with a satisfactory χ^2, such a parametrization is employed in this Letter. It should be noted, however, that there are at present no data below $z=0.2$, and consequently a nonzero limiting value for $\psi(z)$ at $z=0$ is certainly not excluded. The curve displayed in Fig. 1 is described by the formula

$$\psi(z=n/\langle n\rangle)=(3.79z+33.7z^3-6.64z^5+0.332z^7)\times\exp(-3.04z). \tag{4}$$

The overall χ^2 between this function and the data shown in Fig. 1 is 47 for a five-parameter fit to fifty data points plus two constraints.[6] In contrast, lower-energy data are not consistent with this fitted curve. For example, data at 19 GeV/c,[7] shown in the insert to Fig. 1 but not included in the fit, yield a χ^2 of 205 for seven data points when compared with the curve defined by Eq. (4). This is, of course, somewhat surprising since $\langle n\rangle$ for this experiment (4.02) is not very much lower than the range of $\langle n\rangle$ values for the higher-energy experiments (5.32–8.86), but nevertheless the high-energy data do appear to exhibit scaling behavior absent at lower energies. The numerical coefficients in the above expression for $\psi(n/\langle n\rangle)$ are presented without errors since correlation effects cannot justifiably be neglected in evaluating the fit. (The uncertainties are typically in the second decimal place.) The formula is presented in the expectation that it will prove useful in extrapolating the 50–300-GeV/c results to higher energies (once $\langle n\rangle$ is either measured or estimated for these energies).

From the theoretical point of view, however, despite its apparent experimental applicability in the 50–300-GeV/c range of incident momenta, the KNO semi-inclusive scaling prediction is based upon arguments which are explicitly asymptotic in nature, and involves approximations which are probably not justified in this energy regime. For example, the same arguments leading to Eq. (1) also imply that $\langle n\rangle=a\ln s+b$, and for the 50–300-GeV/c data the best fit between the experimental data and this hypothesis yields a χ^2 of 8.2 for a two-parameter fit to five data points. Consequently, the observed experimental agreement between the 50–300-GeV/c data and the KNO semi-inclusive scaling prediction is probably best viewed at present as an unexplained empirical observation which, if it persists at still higher energies, has important implications for current models of strong interaction dynamics.[8]

I am pleased to acknowledge helpful discussions with Paul Olesen, and I wish to thank my colleague Tom Ferbel for stimulating this note by initially bringing the KNO analysis to my attention, and for critically reading this paper.

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[1]Z. Koba, H. B. Nielsen, and P. Olesen, Nucl. Phys. B 40, 317 (1972).

[2]Soviet-French Collaboration, contribution submitted to the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Illinois, September 1972 (to be published).

[3]J. W. Chapman *et al.*, University of Michigan–Rochester Collaboration, contribution submitted to the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Illinois September 1972 (to be published), and University of Rochester Report No. UR-395, 1972 (unpublished).

[4]G. Charlton *et al.*, Phys. Rev. Lett. 29, 515 (1972).

[5]F. T. Dao *et al.*, National Accelerator Laboratory-University of California, Los Angeles Collaboration, report submitted to the Sixteenth International Conference on Hign Energy Physics, National Accelerator Laboratory, Batavia, Illinois, September 1972 (to be published).

[6]To investigate the energy dependence of the proton-proton charged-prong multiplicity distribution, it is important that all of the data be handled in a self-consistent fashion. Since each of the references 2 through 5 presents the actual number of events in each topology, this was possible. In carrying out the fit described in the text, the errors on the numbers of the events having four or more charged tracks were treated as entirely statistical, while the error on the number of two-prongs was regarded as predominantly systematic. This is probably not a completely justified assumption since for very high-multiplicity events systematic errors can also be important, but, in terms of evaluating the overall consistency of the data, it was regarded as being preferable to using overly inflated errors. (The authors of Refs. 2 through 5 include systematic effects in their error estimates).

[7]H. Bøggild *et al.*, Nucl. Phys. B27, 285 (1971).

[8]For a more detailed analysis of the charged-prong multiplicity distributions in the 50–300-GeV/c region of incident momentum, see P. Slattery, University of Rochester Report No. UT-409, 1972 (to be published). This paper also includes a discussion of the impact of this observation on the Mueller-Regge viewpoint.

pp Interactions at 303 GeV/c: Multiplicity and Total Cross Section

F. T. Dao, D. Gordon, J. Lach, and E. Malamud
National Accelerator Laboratory, Batavia, Illinois 60510*

and

T. Meyer, R. Poster, and W. Slater†
University of California, Los Angeles, Los Angeles, California 90024
(Received 26 October 1972)

In an exposure of the 30-in. hydrogen bubble chamber to a 303-GeV/c proton beam, 2245 interactions have been observed. The measured total cross section is 39.0 ± 1.0 mb and the average charged particle multiplicity $\langle n_{ch} \rangle = 8.86 \pm 0.16$.

We present data on the charged-particle multiplicity produced in 303-GeV/c proton-proton collisions. The shape of this distribution as well as the energy dependence of its moments are of fundamental importance to strong interaction theory.

The primary proton beam was extracted from the National Accelerator Laboratory proton synchrotron and the 30-in. hydrogen bubble-chamber beam line and was tuned using these protons. The intensity of the beam was then suitably attenuated by closing collimators and defocusing magnets, and by insertion of an aluminum target approximately 1 km upstream of the bubble chamber which was viewed at 1.5 mrad. Measurements indicated that the beam-momentum spread was less than 0.5%. The primary proton beam entered the bubble chamber with a very small angular spread, and we believe that within this angular region the fraction of contaminating particles is small.

For this run we used the following bubble-chamber parameters: $B = 27$ kG, 35-mm film, four views, bubble size on film ~ 15 μm, and a bubble density for minimum-ionizing particles 10–12/cm. The entrance window is ~ 18 cm high by ~ 5.5 cm wide.

In the scanning the chamber was imaged at 90% of its true size, and all the film was scanned twice, once by a physicist and once by a professional scanner. In the first scan a decision was made as to whether a picture was acceptable based on two criteria: (1) All hadron tracks entering the chamber had to be parallel to the beam to $\lesssim 1$ mrad. (2) The number of beam tracks entering through the window as projected in view 2 had to be ≤ 15. Frames were also rejected because of chamber malfunction. With these criteria 6201 frames were accepted out of $\sim 16\,000$ frames taken. The scanners were instructed to record the number of entering beam tracks, all events, any secondary interactions, neutron stars, V's, kinks, Dalitz pairs, and stopping protons as well as any unusual features in the picture. The average number of tracks per accepted framed was 4.9. The beam intensity was adjusted to optimize the product of acceptable pictures and tracks.

The rescan consisted of only the 6201 accepted frames. In a third, or conflict scan, every event was carefully re-examined by one or two physicists, if necessary using magnification a factor of $\sim 9\times$ chamber size.

Scanning was done with no fiducial cut and 2750 events were found. A fiducial volume cut reduced this to our final sample of 2245 events. From the first two scans the scanning efficiency was computed to be (99.2 ± 0.2)% for two-pronged events; for other topologies it was even higher. For the 1674 events with the number of charged secondaries $n_{ch} \geq 4$ found in both scans, and assuming the conflict scan topological decision to be correct, 78.2% of the events were correctly identified in both scans, 16.6% of the events were wrongly identified in one scan, and 5.2% were wrongly identified in both scans.

The following corrections were applied to the data:

(I) *Low-t two-prongs.*—The recoil proton has