straightforward. The first integral of Eq. (8) is obtained analytically, whereas the second integral of Eq. (8) is evaluated numerically. The value of $\mathfrak{D}$ we obtain for Ge is $3.46\times10^5$ eV. It is interesting to note that the plane-wave part of the wave function $\chi$ only gives a contribution of $1.11\times10^3$ eV to $\mathfrak{D}$. It is, therefore, the core part which gives rise to the major contribution to the two-phonon deformation potential.

While the present calculation can be somewhat improved by accurate numerical OPW calculations now available for various semiconductors, we do not expect the results to differ significantly from our simplified version.

To compare the value of $\mathfrak{D}$ obtained for Ge with the experiment described in I, we made the assumption that the size of the NPO two-optic-phonon deformation potentials for Ge and InSb are nearly the same. This assumption is based on the observation that the NPO one-optic-phonon deformation potentials for Ge and InSb are nearly equal.[4] Then the value of the NPO two-phonon deformation potential $\mathfrak{D}$ obtained here is in good agreement with the experimental data. By good agreement we mean that on comparing these independent calculations of $\mathfrak{D}$, we obtain $p_{\Xi}\approx\frac{1}{38}$, which is certainly within an order of magnitude agreement with what we expect for the value of $p_{\Xi}$. This lends strong support to the correctness of the origin of the physical effects as suggested in I. Finally, we may conclude that the relatively large value of $\mathfrak{D}$ as determined here indicates the importance of the bilinear electron-phonon interaction $H_{ep}^{(2)}$ in realistic physical situations.

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# Further Evidence for the Dominance of Nucleon-Nucleon *P*-Wave Forces in Vector Polarizations in *N-d* Scattering below 15 MeV*

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We have measured vector analyzing powers in deuteron-proton scattering with polarized deuterons incident at 20 and 30 MeV. Our results are in excellent agreement with very recent three-body calculations which clearly show that the nucleon-nucleon *P*-wave interaction is the dominant cause of the vector polarization in nucleon-deuteron scattering at these energies.

During the past eight years the experimental determination of polarization effects in nucleon-deuteron scattering below 50 MeV has shown that measurable polarizations occur at energies as low as 2 MeV and that these rapidly reach substantial values with increasing energy.[1] Theoretical progress in fitting these polarization results has proved to be difficult. The earlier exact three-body calculations,[2,3] which used the Faddeev-Lovelace[4] equations with simple *S*-wave separable nucleon-nucleon potentials, had given fair quantitative agreement with the experimental elastic *N-d* differential cross section data at energies up to about 50 MeV. However, because of the simple *S*-wave forces used, those calculations could not provide the observed polarizations.

It was clear that more realistic, i.e., more complicated, nucleon-nucleon potentials would be required and that the polarization data would provide the important and essential tests of any more refined theory. Later calculations[5-7] with more complicated two-body forces produced polarizations of the required magnitude, but they did not succeed in obtaining quantitative agreement with experiment. These calculations, however, did result in the clear and important conclusion that in any three-body calculation of $N$-$d$ scattering, the $P$-wave nucleon-nucleon interaction is the dominant cause of the nucleon and deuteron vector polarization. Finally, very recent calculations[8,9] that include $P$-wave nucleon-nucleon forces have provided quantitative fits to the nucleon polarizations in $N$-$d$ elastic scattering at energies up to 40 MeV. The agreement with the sparsely available deuteron vector polarization data was less satisfactory. We report here, however, on measurements of the vector analyzing power in $d$-$p$ scattering with polarized deuterons which are in remarkably good agreement with these calculations. These results provide further evidence that the $P$-wave contribution to the two-body amplitudes is the essential factor in producing good quantitative fits to both the nucleon and deuteron vector polarizations.

Pieper and Kowalski[5] used the unitary first-order approximation of Sloan[10] to calculate nucleon polarizations in $N$-$d$ elastic scattering at energies up to 40 MeV. They found that even at 14 MeV the polarization was very sensitive to the contribution from $P$-wave components of the two-nucleon amplitudes used in the calculation, and these components were required to produce polarizations of sufficient magnitude. Although the angular dependence of the calculated polarizations showed little resemblance to the experimental data, they concluded that any further three-body calculation of the nucleon polarization must include $P$-wave two-nucleon amplitudes in a realistic way.

Aarons and Sloan,[6] on the other hand, obtained both nucleon and deuteron (vector and tensor) polarizations from an exact three-body calculation using a noncentral, spin-dependent, two-body force. This two-body force had only two separable terms, corresponding to the $^1S_0$ and $^3S_1-{}^3D_1$ components of the two-nucleon interaction, and so did not include $P$ waves. The calculations over the range 3–23 MeV gave deuteron tensor polarizations in qualitative agreement with experiment, but the nucleon and deuteron vector polarizations were much too small and exhibited an unrealistic angular dependence. They suggested that the inclusion of $P$ waves was necessary for calculating both the nucleon and deuteron vector polarizations. In a similar calculation of the nucleon polarization at 14 MeV, Doleschall[7] obtained the same result.[11] Further, in order to study the effect of the $P$-wave interaction specifically, he also made the calculation with separable terms corresponding to the $^1S_0$, $^3S_1$ (without the tensor force), and $^3P_0$ two-body interactions. This gave polarizations of sufficient magnitude but of the wrong shape, indicating again that the $P$-wave interaction was the dominant cause of the observed nucleon polarization.

Finally, both Doleschall and Pieper have now included the complete $P$-wave set of $^1P_1$, $^3P_0$, $^3P_1$, and $^3P_2$ interactions in recent calculations,[8,9] and the improvement over the previous calculations is dramatic. The nucleon polarizations are in excellent agreement with the experimental data up to 14 MeV and qualitative agreement is achieved beyond that to 40 MeV.[9] The agreement with the deuteron vector polarizations is not as good, although the data are less accurate and less extensive, reaching only to 10.8 MeV.[12]

The purpose of the present work, therefore, was to obtain accurate angular distributions of the vector analyzing power, $iT_{11}$, in $d$-$p$ scattering with polarized deuterons at 20 and 30 MeV, corresponding to proton energies of 10 and 15 MeV. This would provide more accurate data at 10 MeV and extend the measurements to higher energies, thus providing further tests for the calculations. For time-reversal–invariant interactions, the vector analyzing power, $iT_{11}$, in $d$-$p$ scattering is equal to the deuteron vector polarization, $it_{11}$, produced in $p$-$d$ scattering,[13] so the calculated polarizations and measured analyzing powers can be compared directly.

The experiment was performed in a 36-in.-diam scattering chamber, using the axially injected polarized deuteron beam from the Berkeley 88-in. cyclotron. The beam had a vector polarization of 82% of the maximum possible value $p_y=(2/\sqrt{3})it_{11}=\frac{2}{3}$, and the tensor components were zero. A 7.5-cm-diam gas target with a 5-$\mu$m Havor foil window was used at $H_2$ gas pressures ranging from 0.25 to 0.75 atm. Left-right asymmetry data were taken simultaneously at two angles separated by 20°, using pairs of $\Delta E$-$E$ silicon detector telescopes. In order to eliminate instrumental asymmetries, alternate runs were

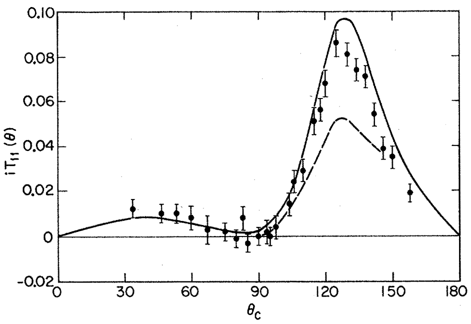


FIG. 1. The angular distribution of the vector analyzing power $iT_{11}(\theta)$ in $d$-$p$ scattering 20 MeV. The solid curve is the theoretical result from Ref. 9. The dashed curve represents the back-angle data from Ref. 12 at 21.7 MeV.

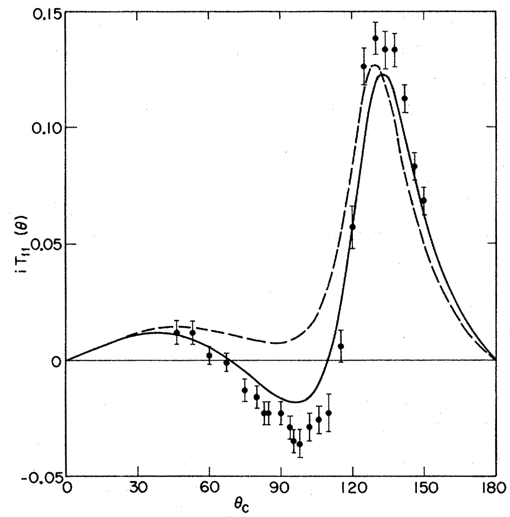


FIG. 2. Angular distribution of the vector analyzing power $iT_{11}(\theta)$ in $d$-$p$ scattering at 30 MeV. The dashed curve is from Ref. 8, the solid curve from Ref. 9, both calculated for $E_d = 28.2$ MeV.

taken with the spin vector of the beam oriented up and down with respect to the scattering plane. The angular resolution, defined by tantalum collimators, was 0.85° and 1.5° (full width at half-maximum) for the forward and backward telescopes, respectively. Two monitor counters were placed left and right of the beam axis at a scattering angle of $\theta \simeq 23°$ and azimuthal angles $\varphi \simeq 70°$ and 110°. A polarimeter, consisting of a smaller scattering chamber containing a gas target and a pair of $\Delta E$-$E$ counter telescopes at equal left and right scattering angles, was placed downstream of the main scattering chamber and provided continuous monitoring of the beam polarization. The analyzer used was $^4$He, whose analyzing power in $d$-$^4$He elastic scattering had been measured in detail previously.[14] Particle identification was used with all detector systems except the monitors. This allowed simultaneous detection of forward scattered deuterons and recoil protons from backward scattered deuterons.

Our 20-MeV data are shown in Fig. 1, where the relative errors include the statistical error and a contribution of $\pm 0.004$ determined from measured asymmetries with the beam polarization set to zero. In addition, there is an overall normalization uncertainty of $\pm 5\%$. The Saclay back-angle data[12] at 21.7 MeV are also indicated in Fig. 1. The shapes of the angular distributions are similar, and the discrepancy of a factor of about 1.7 in overall normalization is most likely due to an uncertainty in the Saclay beam polarization.[15] Also shown in Fig. 1 is the result from Pieper's[9] recent calculation. The agreement is excellent, particularly in view of the fact that there has been no adjustment of the two-body input parameters in order to improve the fit to these data.

Figure 2 shows our 30-MeV data along with the predictions from the calculations of both Doleschall[8] and Pieper[9] at 28.2-MeV deuteron energy. They both fit the backward peak very well, but Pieper also succeeds in providing the negative maximum at $\theta = 95°$. Clearly it would be of interest to identify the feature of the Pieper calculation that produces this result, which is not predicted by Doleschall. Of course, the calculations are not directly comparable. The Doleschall calculation is an exact one using the Faddeev-Lovelace[4] equations with a complete set of $P$-wave interactions, and it also provides good fits to the differential cross section and nucleon polarization data at the same energy.[8] The Pieper calculation is based on a perturbative treatment of three-particle scattering[16] in which the nucleon-nucleon $T$ matrix is taken as the sum of two parts: $t = t^{(s)} + t^{(w)}$. The strong part $t^{(s)}$ is derived from potentials of the Yamaguchi type[17] in the $^1S_0$ and $^3S_1$ partial waves, and it is treated exactly in the Faddeev equations. The weak part $t^{(w)}$ contains all the additional nucleon-nucleon input information and is treated in first-order perturbation theory. Sets of both $P$ and $D$ partial waves are included, so it is possible that the $D$-wave contributions account for the major differences seen in the curves in Fig. 2.

Although the older work showed that the cross sections in $N$-$d$ scattering could be quite well reproduced in three-body calculations using just the $S$-wave two-nucleon forces, it is now very clear that accurate and extensive polarization data have provided the tests that prove the necessity for the inclusion of higher partial waves in the calculations.

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# Perturbations on the Mixmaster Universe*

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Mathematical formalisms for the separation and solution of the tensor perturbation equations in an empty, diagonal, type-IX space is developed, based upon group-symmetry properties of homogeneous spaces. Numerical results in sampling solutions of the "mixmaster universe" show damping amplitudes of perturbations as the universe expands, a behavior in qualitative accordance with earlier results on the Friedmann universe.

In an attempt towards the construction of the general cosmological solution to the Einstein equation, Lifshitz and Khalatnikov have shown that near the singularity, solutions containing matter manifest no features not already found in the vacuum solutions.[1,2] Later they discovered that the asymptotic behavior of the metric near the singularity is at each point of the singular hypersurface described by a mixmaster-type behavior.[3] This discovery has put the mixmaster universes[4,5] among the most viable models for studying the general cosmological solution. To probe deeper into the earlier states of the universe near the singularity, it is important to understand the behavior of perturbations in the mixmaster universe: Given an anisotropic