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Sixth-Order Radiative Corrections to the Electron Magnetic Moment*

T. Kinoshita and P. Cvitanovic
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850
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We have calculated the contribution of fifty Feynman diagrams of the order α^3 to the electron magnetic moment. Our result, $(1.02 \pm 0.04)(\alpha/\pi)^3$, agrees with the result of Levine and Wright, and is about 5 times more accurate. Together with the contribution from the rest of the α^3 diagrams calculated previously, the complete theoretical prediction for the electron anomaly up to the order α^3 is

$$\alpha/2\pi - 0.328\,48(\alpha/\pi)^2 + (1.29 \pm 0.06)(\alpha/\pi)^3,$$

in good agreement with the latest experimental result.

The accuracy of measurement of the electron g factor [or the anomaly $a = (g-2)/2$] has been improved substantially in the last few years. The latest result reported by Wesley and Rich[1] is

$$a^{\text{expt}} = (1\,159\,656.7 \pm 3.5) \times 10^{-9}. \quad (1)$$

Since $(\alpha/\pi)^3 \simeq 12.5 \times 10^{-9}$, it is obvious that a complete calculation of the sixth-order contribution to a is needed in order to have a meaningful comparison of theory and experiment.

There are altogether 72 diagrams contributing to the magnetic form factor of the electron in the sixth order. These diagrams fall naturally into four different groups according to the way the vacuum-polarization subgraphs appear in them.

Group 1.—Diagrams containing the fourth-order vacuum-polarization subgraph. Four diagrams belong to this group. A typical one is shown in Fig. 1(a).

Group 2.—Diagrams containing the second-order vacuum-polarization subgraph. Twelve diagrams belong to this group. A typical diagram is shown in Fig. 1(b).

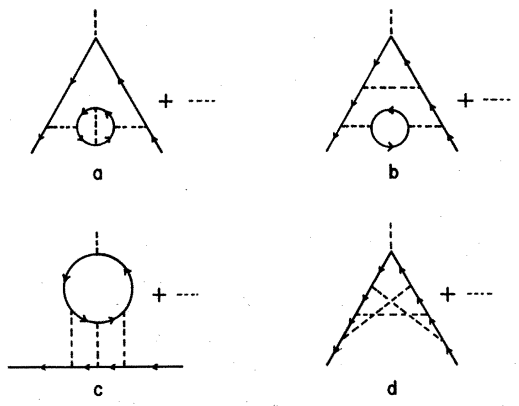

FIG. 1. (a) Typical diagram containing the fourth-order vacuum-polarization subgraph. There are three other diagrams of this type. (b) Typical diagram containing the second-order vacuum-polarization subgraph. There are altogether twelve diagrams of this type. (c) Typical diagram containing the photon-photon scattering subgraph. Six diagrams belong to this group. (d) Typical diagram that contains no vacuum-polarization subgraph. There are altogether fifty diagrams of this type.

Group 3.—Diagrams containing the photon-photon scattering subgraph (vacuum-polarization tensor of fourth rank). Six diagrams belong to this group. One of them is shown in Fig. 1(c).

Group 4.—Diagrams that contain no vacuum-polarization subgraph. This group consists of 50 diagrams of which 28 are distinct. A typical diagram is shown in Fig. 1(d).

By now all 72 diagrams have been evaluated at least once by numerical method.[2-8] Furthermore, the diagrams of group 1 have been evaluated analytically.[9] However, all diagrams of group 3 and most diagrams of group 4 have not yet been double checked. In view of the fundamental importance of the electron g-factor determination, it is highly desirable that all these results are checked by several independent calculations. The purpose of this note is to report the preliminary result of our calculation of the group-4 diagrams.

In our work we have adopted two alternative approaches:

(I) Evaluate the diagrams of group 4 separately and combine the results afterwards.

(II) First, classify the fifty diagrams into ten subgroups, each consisting of five diagrams obtained by insertion of an external magnetic-field vertex in one of the self-energy graphs shown in Fig. 2, combine and simplify the integrands within each subgroup using a version of the Ward identity due to Nakanishi,[10] and then evaluate each subgroup as a unit.

The trace calculation and simplification of the integrands have been carried out with the help of the program SCHOONSCHIP[11] using the CDC-6600 computer at Brookhaven National Laboratory. Some integrands have been checked by hand calculation and also by the PDP-10 version of REDUCE 2.[12] We have also confirmed numerically that the integrands (before renormalization) for all 50 diagrams in approach I are in complete agreement with the corresponding quantities of Levine and Wright[13a] in spite of their strikingly different appearance.

Most diagrams have ultraviolet and/or infrared divergences that must be subtracted or separated out before they are put on the computer. This is carried out by an extension of the technique described in Ref. 2. The numerical integration is then performed using the integration routine RIWIAD, written by Lautrup, Sheppey, and Dufner.

Approach I is used primarily for the study of subtraction procedures in individual integrals. Approach II, which requires evaluation of only eight independent integrals, is then adopted to improve the numerical accuracy as much as possible. The results obtained by these alternative approaches are consistent with each other, although we have not attempted to improve the accuracy of approach I too hard except in one case. One of the subgroups [corresponding to Fig. 2(d)] turned out to have a very complicated integrand which RIWIAD could not handle as easily as other subgroups. Thus, in this case, we have evaluated the integrals for five diagrams of subgroup D separately, and combined the results afterwards. At present our result for group 4 as a whole is $(1.02 \pm 0.04)(\alpha/\pi)^3$, which we shall write as

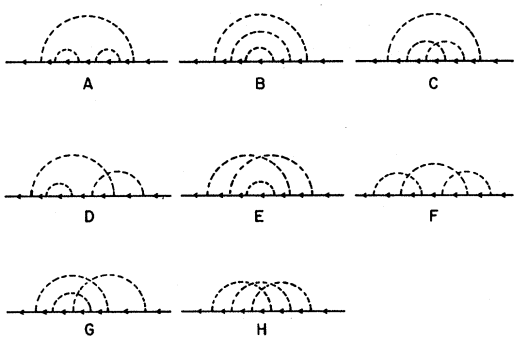

FIG. 2. Eight self-energy graphs of sixth order. There are two more graphs of this type obtained by inversion from graphs D and G. By inserting an external magnetic-field vertex in all possible ways in these ten diagrams, we can generate all fifty diagrams contributing to the sixth-order magnetic form factor.

$$a_4^{(6)} = 1.02(4)(\alpha/\pi)^3. \quad (2)$$

This is to be compared with the result of Ref. 5,[13b]

$$a_4^{(6)} = 1.23(20)(\alpha/\pi)^3. \quad (3)$$

The uncertainty in (2) represents the 90% confidence limits estimated by the integration routine. The errors from independent subgroups have been added quadratically because of the statistical nature of the RIWIAD. The uncertainty in (3) is not statistical but is rather an educated guess based on the behavior of successive Gaussian quadratures.[5]

Details of the calculation by Levine and Wright[5] are as yet unpublished. Thus we have not had occasion to compare our result with theirs. However, the results for ten of the diagrams of group 4 are available.[6-8] They are in good agreement with ours in most cases. The fact that our result (2) agrees with (3) of Levine and Wright who used a technique entirely different from ours suggests that both results are correct. However, further

independent checks are highly desirable.

For completenesss we list the results for other groups: Group 1,

$$a_1^{(6)} = \begin{cases} 0.055\,429(\alpha/\pi)^3 & \text{(Ref. 9)} \\ 0.055\,46(6)(\alpha/\pi)^3 & \text{(Ref. 2).} \\ 0.055(2)(\alpha/\pi)^3 & \text{(Ref. 3)} \end{cases} \quad (4)$$

Group 2,

$$a_2^{(6)} = \begin{cases} -0.153(5)(\alpha/\pi)^3 & \text{(Ref. 2)} \\ -0.151(3)(\alpha/\pi)^3 & \text{(Ref. 3)} \end{cases} \quad (5)$$

Group 3,

$$a_3^{(6)} = 0.36(4)(\alpha/\pi)^3 \quad \text{(Ref. 4).} \quad (6)$$

The overall result for the sixth-order electron anomaly is thus

$$a^{(6)} = 1.29(6)(\alpha/\pi)^3, \quad (7)$$

where we have used the analytic result in (4), the weighted average of the results in (5), and the results (2) and (6). If we use the ac Josephson value of the fine-structure constant[14]

$$\alpha^{-1} = 137.036\,08(26), \quad (8)$$

we obtain

$$a_t^{\text{theor}} = (1\,159\,652.9 \pm 2.4) \times 10^{-9}. \quad (9)$$

The uncertainty arises from two sources; one from the fine-structure constant (± 2.2) and the other from theory (± 0.75). Experiment (1) and theory are thus slightly more than 1 standard deviation apart.

The theoretical uncertainty in (9) is now 3 times smaller than that of α. Thus a slight improvement in the $g-2$ experiment will lead to a value[15] for the fine-structure constant which is more accurate than the value (8), or that determined by fine-structure[16] and hyperfine-structure[17] measurements of hydrogen atoms, as well as that value from the hyperfine splitting of the muonium ground state.[18]

There is no theoretical reason why the calculated accuracy of the electron anomaly cannot be improved by a factor of 10 or more beyond that given by our result (7) since it has no bound-state complication and all conceivable effects such as the breakdown of quantum electrodynamics, the hadronic corrections, etc., will be at most of the order $(\alpha/\pi)^4$ in magnitude. Thus, further improvement in theoretical[19] and experimental values of the electron anomaly will provide the cleanest and most accurate determination of the fine-structure constant. Particularly interesting will be the comparison of the α's determined by the electron $g-2$ measurement and the ac Josephson effect.

The numerical part of this work was carried out at the Brookhaven National Laboratory. We would like to thank Dr. R. F. Peierls for his helpful advice and generous support of this work. We wish to thank Dr. A. C. Hearn, Dr. R. W. Brown, and Dr. D. R. Yennie for useful discussions. The cooperation of the staff of the CDC-6600 computing facility at the Brookhaven National Laboratory is greatly appreciated.

Note added.—After this paper was completed, Dr. J. Wright showed us the improved result of their calculation for the individual diagrams of group 4. For the few diagrams for which immediate comparison can be made, the agreement is satisfactory. For most diagrams, however, detailed comparison requires some further work because of the widely different approaches. We would like to thank Dr. J. Wright for communicating his and Levine's results to us. We would also like to thank Dr. S. J. Brodsky for showing us a paper by himself and R. Roskies[20] in which they have calculated two diagrams of group 4 using the infinite-momentum-frame technique.

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[13b]The numerical value given in Ref. 5 has been modified very recently. The new value is $0.89(10)(\alpha/\pi)^3$. Although this value is still preliminary, awaiting some further checks, it has been included in a report of N. M. Kroll [in Proceedings of the Third International Conference on Atomic Physics, Boulder, Colorado, August 1972, edited by J. S. Smith (Plenum, New York, to be published)]. If this new value and our result (2) are combined statistically (which is a somewhat questionable procedure), $a^{(6)}$ of Eq. (7) will become $1.27(6) \times (\alpha/\pi)^3$ and Eq. (9) will be reduced by 0.25×10^{-9}.

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CHEMICAL EFFECTS ON LINEWIDTHS OBSERVED IN PHOTOELECTRON SPECTROSCOPY. R. M. Friedman, J. Hudis, and M. L. Perlman [Phys. Rev. Lett. 29, 692 (1972)].

In Table I, column 4, the atom site designations NH_4^+ and NO_3^- should be interchanged.