Long-Wavelength Ion-Acoustic Waves*

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The time dependence of current and temperature in a spatially uniform plasma, and the nature of the ion heat sink as a function of density and ion temperature, have a profound effect on the stability of ion-acoustic waves with wavelengths much longer than the electron mean free path.

The instability of ion-acoustic waves with wavelengths much longer than an electron mean free path, $kv_{\text{th}\,e}/\nu_{ei} \ll 1$, has been the subject of considerable interest as a possible mechanism for anomalous resistivity and turbulent energy transport.[1,2] Since the wave frequency in this regime is less than even the ion-ion collision frequency, the properties of the wave can be predicted from the moment equations, provided that all the dominant transport effects are included.[3] Rognlien and Self[1] (RS) predict instability over a limited range of long wavelengths driven by only moderate values of the temperature ratio, $T_i/T_e \lesssim 0.6$, even in the absence of a relative drift (background current). However, we find that there is *no* instability for the same range of wavelengths for *any* temperature ratio or *any* prerunaway drift *if* the background is evolving freely without heat sources or sinks. Alternatively, if a steady state is established with heat sinks (assumed here to be functions of density and temperatures) the stability of these waves depends sensitively upon the functional form of the ion heat sink. A physically realistic model for a heat sink would depend upon the external environment of the plasma—but the resulting two- or three-dimensional boundary-value problem[4] is beyond the scope of this paper. It suffices to point out the importance of heat sinks and of time dependence in the background state.

For plane, electrostatic, ion-acoustic waves propagating parallel to a magnetic field in a spatially uniform plasma, the stability calculation proceeds as follows: The system is characterized by two densities, velocities, and temperatures satisfying the moment equations and charge neutrality ($Zn_i = n_e \equiv n$) as written in a one-dimensional form by RS. To the right-hand side of the heat-transfer equations [Eq. (3) in RS] are added heat-sink (or source) terms, $-3\sigma^s(n_e, T_e, T_i)/M$ ($s = e$ or i, $M \equiv m_i/m_e \gg 1$, $m_e = 1$ henceforth).[5]

The "background state" of the plasma is taken to be the spatially uniform solution of these equations. For this background state it is assumed that the density is constant; that there is no net momentum, $v_e + Mv_i/Z = 0$; and that either the applied electric field E or the relative drift $U = v_e - v_i = (1 + Z/M)v_e$ is held fixed. The equations for the background state are (to leading order in Z/M)

$$\dot{U} = -eE - C_{er}\nu_e U, \tag{1}$$

$$-M\dot{T}_e/2\nu_e T_e = 1 - R + \Sigma^e - \tfrac{1}{3}\alpha, \tag{2}$$

$$M\dot{T}_i/2Z\nu_e T_e = 1 - R - \Sigma^i, \tag{3}$$

where

$$R \equiv T_i/T_e, \quad \Sigma^{e,i} \equiv \sigma^{e,i}/nT_e\nu_e,$$

$$\alpha \equiv C_{er}MU^2/T_e, \quad C_{er} = 0.51 \text{ for } Z = 1,$$

$$\nu_e = [4(2\pi)^{1/2}\ln\Lambda e^4/3\sqrt{m_e}]ZnT_e^{-3/2}.$$

Since the moment equations are nonlinear, one must rigorously satisfy the equations for the background state before investigating the evolution of a perturbation.

The linearized equations for the perturbation generally have time-dependent coefficients. However, it is convenient to use the exponential representation ($n \to n + \tilde{n}$, etc.),

$$\tilde{n}(x, t) \equiv \tilde{n}\exp\{\textstyle\int^t dt'\,[i\omega(t') + \gamma(t')] - ikx\},$$

$$\tilde{T}(x, t) \equiv \tau(t)\tilde{n}(x, t)/n,$$

so that the results reduce to a normal-mode form in the special case of constant coefficients. Then, the linearized moment equations become

$$(\omega/k)^2 = [T_+ + \operatorname{Re}\tau_+ + 4\gamma(\eta_e + \eta_i)/3n - U^2]ZM^{-1} + (\gamma^2 + \dot{\gamma})k^{-2}, \tag{4}$$

$$\gamma = -\dot{\omega}/2\omega - \{\operatorname{Im}\tau_+ + 4[(\eta_e + \eta_i)\omega - (\eta_e - Z\eta_i/M)ku](3n)^{-1}\}Zk^2(2\omega M)^{-1}, \tag{5}$$

$$[K^2C_{e\chi}T_e+\tfrac{3}{2}i(W-i\Gamma-Ku)-\tfrac{3}{2}M^{-1}(1-3R-2\Sigma_e{}^e-\alpha)]\tau_e-3M^{-1}(1-\Sigma_i{}^e)\tau_i$$
$$=T_e[i(W-i\Gamma-Ku)-\alpha/M-3M^{-1}(1-R+\Sigma_n{}^e-\Sigma^e)]-\tfrac{3}{2}\dot{\tau}_e/\nu_e, \qquad (6)$$

$$\frac{3Z}{2M}[1-3R+2\Sigma_e{}^i]\tau_e+\left[\tfrac{3}{2}i\left(W-i\Gamma+\frac{KuZ}{M}\right)+K^2C_{i\chi}\frac{T_e\sqrt{2}R^{5/2}}{Z^2\sqrt{M}}+\frac{3Z}{M}(1+\Sigma_i{}^i)\right]\tau_i$$
$$=T_i\,i\left(W-i\Gamma+\frac{KuZ}{M}\right)+\frac{3Z}{M}T_e(1-R-\Sigma_n{}^i+\Sigma^i)-\frac{3}{2}\frac{\dot{\tau}_i}{\nu_e}, \qquad (7)$$

where

$$W-i\Gamma\equiv(\omega-i\gamma)/\nu_e,\quad K\equiv k/\nu_e,\quad T_\pm\equiv T_e\pm T_i/Z,\quad u\equiv V_e,\quad \Sigma_{e,i}{}^{e,i}\equiv(\partial\sigma^{e,i}/\partial T_{e,i})(n\nu_e)^{-1},$$
$$\Sigma_n{}^{e,i}\equiv(\partial\sigma^{e,i}/\partial n)(T_e\nu_e)^{-1},\quad \Sigma^{e,i}=\sigma^{e,i}/nT_e\nu_e,$$

and the other symbols are defined as in RS.[1]

An asymptotic solution for these equations can be found for

$$M^{-1}\ll K^2T_e\ll M^{-1/2}. \qquad (8)$$

The first inequality permits iterative solution of the differential equations (6) and (7) while the second inequality makes viscosity effects in Eq. (5) negligible. We must further require that

$$MU^2/T_e\lesssim O(1) \qquad (9)$$

so that Ohmic heating does not drive the system on a time scale faster than $(M\nu_e)^{-1}$.

For the special case where there are no heat sinks or sources ($\sigma^e=\sigma^i=0$) the solution of the background equations (1)–(3) always evolves in time (there are no nontrivial steady-state solutions) given an initial temperature difference with no current or given any initial conditions together with an applied electric field or an applied current driving the system. The damping rate ($\gamma=\nu_e\Gamma$) for ion-acoustic waves under assumptions (8) and (9) is

$$\frac{-\gamma M}{\nu_e}\simeq\frac{\frac{2}{3}R}{1+\frac{5}{3}R/Z}+\frac{C_{er}}{6}\left(\frac{U}{V}\right)^2+\frac{Z}{2C_{e\chi}}\left(\frac{1-U}{V}\right), \qquad (10)$$

where

$$V^2\equiv\left(\frac{W}{K}\right)^2=\frac{Z}{M}T_e\left(1+\frac{5}{3}\frac{R}{Z}\right). \qquad (11)$$

This stability criterion disagrees fundamentally with the results of Rognlien and Self and of Coppi and Mazzucato. In particular, for plasmas with $Z=1$ ($C_{er}=0.51$ and $C_{e\chi}=3.16$), Eq. (10) indicates that the ion-acoustic waves described by Eq. (8) are *stable* ($\gamma<0$) for *all* values of $R\equiv T_i/T_e$ and for *all* values of the drift velocity within the bounds specified by Eq. (9). The effect due to Ohmic heating [second term in Eq. (10)] overwhelms the two-stream-like instability [last term in Eq. (10)].

Instability is possible for plasmas with a larger value of Z. If $R=0$, $Z\geqslant4$ is needed for instability over a limited domain of drift velocities —provided that such conditions are achieved by a solution of the background equations (1)–(3). For $Z\to\infty$ ($C_{er}\to0.29$, $C_{e\chi}\to12.5$) any nonzero drift yields instability for all $R\sim O(1)$.

Consider now possible *steady-state* solutions of the background equations. In order to maintain a temperature difference with no current, an electron heat source and an equal ion heat sink are needed (for $R<1$). If current is present, Ohmic heating must be balanced by heat sinks. Once a steady state is established, it must be determined whether or not the state is stable to a spatially uniform "runaway" mode. The conditions for runaway stability are found by perturbing the steady-state solutions of Eqs. (1)–(3) holding n fixed and $k=0$. By assumption, the coefficients of the perturbed equations are constant so that the necessary and sufficient stability conditions can be derived exactly:

$$Z(1+\Sigma_i{}^i)-(\tfrac{1}{2}-\tfrac{3}{2}R-\Sigma_e{}^e-\tfrac{3}{2}\alpha)\geqslant2\alpha\delta_E, \qquad (12)$$

$$(1-\Sigma_i{}^e)(\tfrac{1}{2}-\tfrac{3}{2}R+\Sigma_e{}^i)-(1+\Sigma_i{}^i)(\tfrac{1}{2}-\tfrac{3}{2}R-\Sigma_e{}^e-\tfrac{3}{2}\alpha)\geqslant(1+\Sigma_i{}^i)2\alpha\delta_E, \qquad (13)$$

where

$$\alpha\equiv C_{er}MU^2/T_e,\quad \delta_E=\begin{cases}1 \text{ if } E \text{ is held fixed,}\\ 0 \text{ if } U \text{ is held fixed.}\end{cases}$$

Both conditions and the steady-state parts of Eqs. (1)–(3) must be satisfied for stability. For example, Rognlien and Self implicitly use σ^e = const and σ^i = const. This choice is stable only if the current is held fixed and not the electric field. This condition is also necessary for the runaway stability of the three-dimensional heat-transfer models investigated by Furth, Rutherford, Rosenbluth, and Stodiek.[4]

If the plasma is runaway unstable, then the stability of long-wavelength ion-acoustic waves must be computed from the time-dependent formalism with energy sinks and/or sources included. However, if the plasma is in a runaway-stable steady state, then the simpler time-dependent formalism yields the stability criterion

$$1 - \tfrac{1}{2}\Sigma_n{}^i - (\tfrac{4}{3} + \tfrac{1}{3}\Sigma_i{}^i)R \leq (1 - u/V)(Z/4C_{e\chi})(1 + \tfrac{5}{3}R/Z) \qquad (14)$$

for the wavelength range

$$M^{-1} \ll K^2 T_e \ll M^{-1/2} R^{-5/2} \ll 1.$$

Note that stability depends critically on the nature of the ion heat sink. Rognlien and Self's results are reproduced if σ^i is independent of n and T_i. However, a physically more reasonable example follows from the scaling of the classical heat transfer across a plasma slab; one finds $\sigma^i \sim n^2 T_i^{1/2}$, $\sigma^e \sim n^2 T_e^{1/2}$; this is runaway stable only if the current is held fixed and is stable to the above range of ion-acoustic waves for all values of R as long as $U < V$. Clearly the results change dramatically with different heat sinks.

In summary, it is shown here that the stability of long-wavelength ion-acoustic waves is determined not only by the temperature ratio and relative drift, but also by time dependence of the uniform background state and by the nature of the ion heat sink as a function of density and ion temperature.

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[5]Because of time dependence, it is inconvenient to use a nondimensional form. However, the notation simplifies if we take $m_e = 1$. Then T has units of velocity squared, etc.

Photoemission Studies of the Layered Dichalcogenides $NbSe_2$ and MoS_2 and a Modification of the Current Band Models*

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Ultraviolet photoemission measurements have been used to determine the electronic structure of two of the layered transition-metal dichalcogenides, $NbSe_2$ and MoS_2, which show trigonal prismatic coordination. Although the electronic structures of the two materials have rather different character, the results are consistent with a loose interpretation of a rigid-band model. The observed energy of the Fermi level above the valence-band maximum in MoS_2 indicates that the splitting between the nonbonding d bands is in excess of 1 eV, a value considerably larger than some previous suggestions.

The layered transition-metal dichalcogenides (TX_2) form a structurally and chemically similar family of compounds having a wide range of electrical properties. These layered materials are of particular interest because of their "two-dimensional" superconducting properties which can be changed by intercalating them with organic molecules[1] or alkali-metal atoms.[2] Wilson and Yoffe[3] have proposed a rigid-band model for the TX_2 compounds in which the electrical properties are determined by the degree of filling of narrow nonbonding d bands lying in the basic