Consistent Description of Higher-Spin Fields*

William J. Hurley†
Department of Physics, Syracuse University, Syracuse, New York 13210
(Received 17 April 1972)

A generalization of the scalar product of the Dirac theory is proposed which together with a recently described generalization of the Dirac equation leads to a simple description of a massive ($m>0$), spin-s relativistic field. This theory avoids the difficulties of parity doubling, indefinite metric, and negative energies in the second-quantized formalism, while preserving the consistency and causality of the wave equation in an external electromagnetic field.

It has recently been rediscovered[1] that a relativistic, massive ($m>0$) particle with arbitrary spin may be described by a wave equation of the form[2]

$$(i\beta_\epsilon{}^\mu \partial_\mu - m)\varphi_\epsilon(x) = 0 \quad (\epsilon = +\text{ or }-), \tag{1}$$

where $\varphi_\epsilon(x)$ is a $(6s+1)$-component wave function which transforms as $\varphi_\epsilon'(x') = S_\epsilon(\Lambda)\varphi_\epsilon(x)$. Here $x' = \Lambda x + \alpha$, Λ is an element of the homogeneous Lorentz group, α is a space-time translation, and $S_+(\Lambda)$ $[S_-(\Lambda)]$ is the $(s,0)\oplus(s-\frac{1}{2},\frac{1}{2})$ $[(0,s)\oplus(\frac{1}{2},s-\frac{1}{2})]$ representation of the Lorentz group. The $\beta_\epsilon{}^\mu$ are $(6s+1)$-dimensional matrices which are generalizations of the Dirac matrices. In a representation where $\beta_\epsilon{}^0$ is diagonal, they may be written as

$$\beta_\epsilon{}^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$\vec{\beta}_\epsilon = \frac{\epsilon}{s}\begin{bmatrix} 0 & \vec{S} & \vec{K}^\dagger/\sqrt{2} \\ -\vec{S} & 0 & \vec{K}^\dagger/\sqrt{2} \\ -\vec{K}/\sqrt{2} & -\vec{K}/\sqrt{2} & 0 \end{bmatrix}, \tag{2}$$

where the spin matrices $\vec{S}$ are the generators of the $(2s+1)$-dimensional irreducible representation of the rotation group, $D^{(s)}(R)$, and the $\vec{K}$ are $(2s-1)\times(2s+1)$-dimensional rectangular matrices with the property that

$$R\vec{K}_{\alpha'\beta} = D_{\alpha'\gamma'}{}^{(s-1)}(R^{-1})\vec{K}_{\gamma'\delta}D_{\delta\beta}{}^{(s)}(R),$$

where $\alpha',\gamma' = 1,\ldots,2s-1$; $\beta,\delta = 1,\ldots,2s+1$.

For either value of ϵ, this equation has the following properties: It is form invariant under the proper Poincaré group and describes particles with a unique spin and a unique mass without requiring the use of any subsidiary conditions. There are $2(2s+1)$ independent components and $2s-1$ dependent components which may be immediately eliminated in terms of the independent components (no secondary constraints). In the nonrelativistic limit each equation reduces to the satisfactory Galilei-covariant arbitrary-spin equations,[3] and for the case $s=\frac{1}{2}$ each equation is equivalent to the Dirac equation. The Hamiltonian form is easily obtained. Most importantly, however, when coupled to an external electromagnetic field ($\partial_\mu \to \partial_\mu + ieA_\mu$) each equation remains consistent and causal, thus avoiding the external field difficulties peculiar to many higher-spin formulations.[4]

However, each of these equations by itself admits neither a parity symmetry nor a "Hermitizing" matrix which would permit the construction of a Lagrangian in the usual manner. One remedy is to simply take the direct sum of the two equations, thus eliminating these shortcomings. This procedure leads to further difficulties, however (even in the free case), in the form of (1) a parity doubling of components [$4(2s+1)$ independent components] and, when second-quantized, (2) wrong-sign (anti)commutation relations leading to a negative metric in the underlying Fock space, and (3) negative-energy states which cannot be eliminated by conventional methods.[5]

In the following we propose that spin-s particles be described by the undoubled formalism but that a new scalar product be used on the solution space of Eq. (1). This scalar product in turn leads to a second-quantized formalism which avoids the three difficulties of the parity-doubled case while preserving the desirable properties of Eq. (1).

Plane-wave solutions.—We seek solutions of Eq. (1) with the matrices given by (2) of the form $\varphi_\epsilon(x) = u_\epsilon(p)e^{-ip\cdot x}$. It is easily verified that $u_\epsilon(p)$

may be written as

$$u_\epsilon(\vec{p},\pm,\sigma)=\frac{1}{2m}\begin{bmatrix}[m\pm E-(\epsilon/s)\vec{S}\cdot\vec{p}]f_\sigma\\ [m\mp E+(\epsilon/s)\vec{S}\cdot\vec{p}]f_\sigma\\ \sqrt{2}(\epsilon/s)\vec{K}\cdot\vec{p}f_\sigma\end{bmatrix},\quad \sigma=-s,\dots,s, \tag{3}$$

where we have explicitly noted the sign of the energy and the spin state. f_σ is an eigenstate of the spin matrix S_3 and $E=(p^2+m^2)^{1/2}$ is always positive. As the notation indicates, the top $2(2s+1)$ components are eigenstates of the Hamiltonian with eigenvalue $\pm E$.

The plane waves satisfy the relations

$$\bar{u}_+(\vec{p},\pm,\sigma)u_-(\vec{p},\pm,\sigma')=\pm(E/m)\delta_{\sigma\sigma'},\quad \bar{u}_+(\vec{p},\mp,\sigma)u_-(\vec{p},\pm,\sigma')=0,$$
$$\sum_{\sigma=-s}^{s}[u_+{}^\alpha(\vec{p},+,\sigma)\bar{u}_-{}^\beta(\vec{p},+,\sigma)-u_+{}^\alpha(\vec{p},-,\sigma)\bar{u}_-{}^\beta(\vec{p},-,\sigma)]=(E/m)\delta^{\alpha\beta},\quad \alpha,\beta=1,\dots,4s+2, \tag{4}$$

where $\bar{u}_\epsilon(\vec{p},\pm,\sigma)=u_\epsilon{}^\dagger(\vec{p},\pm,\sigma)\beta^0$. Expanding a general solution to (1) in terms of these plane waves, we get

$$\varphi_\epsilon(x)=\sum_\sigma\int d^3p\,(2\pi)^{-3/2}(m/E)^{1/2}[a_\epsilon(\vec{p},\sigma)u_\epsilon(\vec{p},\sigma)e^{-ip\cdot x}+b_\epsilon{}^\dagger(\vec{p},\sigma)v_\epsilon(\vec{p},\sigma)e^{ip\cdot x}], \tag{5}$$

where $u(\vec{p},+,\sigma)\equiv u(\vec{p},\sigma)$ and $u(\vec{p},-,\sigma)\equiv v(-\vec{p},\sigma)$.

Scalar product.—Suppose that a spin-s particle is described by the $(6s+1)$-component solutions $\varphi_+(x)\equiv\varphi(x)$. To each $\varphi(x)$ we associate a $\varphi_-(x)\equiv\varphi_c(x)$ such that $a_-(\vec{p},\sigma)=a_+(\vec{p},\sigma)$ and $b_-(\vec{p},\sigma)=(-1)^{2s}b_+(\vec{p},\sigma)$. Now consider the object

$$(\varphi,\varphi')\equiv\int d\sigma_\mu\,\varphi_c{}^\dagger(x)\beta^\mu\varphi'(x), \tag{6}$$

where σ is an arbitrary spacelike surface. It is easily checked that this object has the following properties: (a) It is invariant under the proper Poincaré group. (b) It is independent of σ $[\partial_\mu\times(\varphi_c{}^\dagger(x)\beta^\mu\varphi'(x))=0$ by virtue of the equations of motion]. (c) $(\varphi,\varphi')=(\varphi',\varphi)^*$. (d) $(\varphi,\alpha\varphi_1+\beta\varphi_2)=\alpha(\varphi,\varphi_1)+\beta(\varphi,\varphi_2)$. (e) $\|\varphi\|\equiv(\varphi,\varphi)$ is positive definite for s equal to a half odd integer and positive (negative) definite for the positive (negative) energy solutions when s equals an integer. (f) It reduces to the usual Dirac scalar product for $s=\frac{1}{2}$. Equation (6) therefore defines a scalar product for the solutions of Eq. (1).

Second quantization.—We define the field densities of the second-quantized theory in terms of the integrand of the scalar product as defined above. The free Lagrangian density may be taken to be $\mathcal{L}(x)=\varphi_c{}^\dagger(x)(i\beta_\mu\partial^\mu-m)\varphi(x)+\text{H.c.}$ and the canonical quantization procedure may be applied.

If we take the Fock space (anti)commutation relations to be

$$[a(\vec{p},\sigma),a^\dagger(\vec{p}',\sigma')]_\pm=\delta(\vec{p}-\vec{p}')\delta_{\sigma\sigma'},$$
$$[b(\vec{p},\sigma),b^\dagger(\vec{p}',\sigma')]_\pm=\delta(\vec{p}-\vec{p}')\delta_{\sigma\sigma'}, \tag{7}$$

and all others equal to zero, then we may define in the usual way an occupation-number space which will have a positive-definite metric for fermions as well as for bosons.

Starting from the relations (7) and the expansion (5) it may be seen that the theory is local:

$$[\varphi^\alpha(\vec{x},t),\bar{\varphi}_c{}^\beta(\vec{x}',t')]_\pm=i(H^{\alpha\beta}+i\partial_t)\Delta(x-x')=\delta^{\alpha\beta}\delta(\vec{x}-\vec{x}')\text{ for }t=t',\quad \alpha,\beta=1,\dots,4s+2, \tag{8}$$

where H is the Hamiltonian. The free particle energy is found to be

$$H=(\varphi,H\varphi)=\sum_\sigma\int d^3p\,E(\vec{p})[a^\dagger(\vec{p},\sigma)a(\vec{p},\sigma)+(-1)^{2s}b(\vec{p},\sigma)b^\dagger(\vec{p},\sigma)], \tag{9}$$

which, when normal ordered, leads to positive-definite energies for both bosons and fermions.

Thus one may avoid the difficulties of parity doubling, indefinite metric, and negative energies while still maintaining locality.

Although the wave equation does not admit a parity symmetry, parity may be introduced on the second-quantized level according to $\mathcal{P}a(\vec{p},\sigma)\mathcal{P}^{-1}=a(-\vec{p},\sigma)$ and $\mathcal{P}b(\vec{p},\sigma)\mathcal{P}^{-1}=(-)^{2s}b(-\vec{p},\sigma)$. It leaves the (anti)commutation relations invariant and commutes with H. Since $\mathcal{P}\varphi(\vec{x},t)\mathcal{P}^{-1}=\varphi_c(-\vec{x},t)$, we have $\mathcal{P}\mathcal{L}(\vec{x},t)\mathcal{P}^{-1}=\mathcal{L}(-\vec{x},t)$, $\mathcal{P}j^0\mathcal{P}^{-1}=j^0(-\vec{x},t)$, and $\mathcal{P}\vec{j}(\vec{x},t)\mathcal{P}^{-1}=-\vec{j}(-\vec{x},t)$, where $j^\mu(x)\equiv\varphi_c{}^\dagger(x)\times\beta^\mu\varphi(x)+\text{H.c.}$ is the free-particle current. Likewise, charge conjugation and time reversal may also be introduced.

We have shown that a particular generalization of the Dirac equation along with a generalization of its scalar product has led to a simple description of relativistic particles with any spin. This description preserves the consistent and causal nature of the wave equation while avoiding the difficulties of the parity-doubled theory when second quantized. In the present report we have attempted to present only the essential features of this approach. A more detailed and extended discussion will appear elsewhere.[6]

The author is grateful to Professor F. Rohrlich and Professor J. Weinberg for numerous conversations. He wishes also to thank Professor Rohrlich for his critical reading of the manuscript.

*Work supported by the National Science Foundation.

†Present address: Center for Particle Theory, University of Texas, Austin, Tex. 78712.

[1]W. J. Hurley, Phys. Rev. D 4, 3605 (1971).

[2]We use the metric $g = \mathrm{diag}(1,-1,-1,-1)$ and natural units.

[3]W. J. Hurley, Phys. Rev. D 3, 2339 (1971).

[4]A. S. Wightman, in *Proceedings of the Fifth Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, 1968,* edited by A. Perlmutter, C. A. Hurst, and B. Kursunoğlu (Benjamin, New York, 1968); G. Velo and D. Zwanziger, Phys. Rev. 186, 1337 (1969), and 188, 2218 (1969); B. Schroer, R. Seiler, and A. Swieca, Phys. Rev. D 2, 2927 (1970).

[5]Relativistic wave equations with the transformation properties $(s,0)\oplus(s-\frac{1}{2},\frac{1}{2})$ were studied by F. Cap [Z. Naturforsch. 8a, 740, 748 (1953)] and by H. Donnert [Z. Naturforsch. 8a, 745 (1953), and Acta Phys. Austr. 11, 321 (1957)]. See also J. S. Dowker, Proc. Roy. Soc., Ser. A 297, 351 (1967). The parity doubling seems to have been first explicitly studied by J. D. Harris, Ph. D. thesis, Purdue University, 1955 (unpublished). K. Johnson and E. C. G. Sudarshan [Ann. Phys. (New York) 13, 126 (1961)] noted the indefinite-metric difficulty for the case $s=\frac{3}{2}$. The parity doubling and indefinite energy were noted by W. K. Tung, Phys. Rev. Lett. 16, 763 (1966), and Phys. Rev. 156, 1385 (1967), and by S. J. Chang, Phys. Rev. Lett. 17, 1024 (1966).

[6]W. J. Hurley, to be published.

Evidence for a Neutral Meson near 1033 MeV*

A. F. Garfinkel
Purdue University, Lafayette, Indiana 47907

and

E. W. Hoffman
Argonne National Laboratory, Argonne, Illinois 60439

and

R. W. Jacobel, D. L. Cheshire,† R. C. Lamb, and F. C. Peterson
Ames Laboratory—U. S. Atomic Energy Commission and Department of Physics, Iowa State University, Ames, Iowa 50010

(Received 23 August 1972)

In a scintillation-counter experiment on the reaction $\pi^- + p \to M^0 + n$ at 2.4 GeV/c, we have obtained evidence for a neutral meson denoted $M^0(1033)$ with a mass of 1032.6 ± 2.3 MeV and a width of $16.2^{+4.8}_{-7.5}$ MeV.

In a scintillation counter experiment at the Argonne zero-gradient synchrotron we have observed neutrons and charged particles from the reaction $\pi^- + p \to \text{anything} + n$. The apparatus has been described previously.[1-3] A pion beam incident on a liquid-hydrogen target produced neutrons which were detected in the nearly forward direction by an array of twenty plastic scintillators. In the case of meson-plus-neutron final states, the events correspond to forward meson production. At a mass of 1033 MeV the range in four-momentum transfer squared (t) between the incident pion and the missing mass for the accepted events is $0.00005 \lesssim |t - t_{\min}| \lesssim 0.0010$ (GeV/c)^2. The effective mass of the particles produced with the neutron is determined primarily by the measurement of the velocity of the recoiling neutron. Information from the charged-particle detector surrounding the hydrogen target is used only in the data analysis to subdivide the missing-mass spectrum into categories corresponding to various topologies of the final state. The charged-particle detector consisted of an array of sixteen scintillators arranged to form a cylinder coaxial with the beam, an array of seventeen scintillators ("front array") at the downstream end of the cylindrical array, and two scintillators at the tupstream end of the cylindri-