[3]E. A. Freiman, Astrophys. J. 120, 18 (1954).
[4]D. Layzer, Astrophys. J. 122, 1 (1955).
[5]G. Birkhoff and D. Carter, J. Math. Mech. 6, 769 (1957).
[6]P. R. Garabedian, Proc. Roy. Soc., Ser. A 241, 423 (1957).
[7]H. W. Emmons, C. T. Chang, and B. C. Watson, J. Fluid Mech. 7, 177 (1960).
[8]G. Birkhoff, in *Proceedings of the Thirteenth Applied Mathematics Symposium* (American Mathematical Society, Providence, R. I., 1960), p. 55.
[9]F. H. Harlow and J. E. Welch, Phys. Fluids 9, 842 (1966).
[10]B. J. Daly, Phys. Fluids 10, 297 (1967), and 12, 1340 (1969).
[11]E. Ott, to be published.
[12]G. I. Taylor, Proc. Roy. Soc., Ser. A 201, 192 (1950).
[13]Indeed a number of examples exist where problems which appear badly nonlinear in an Eulerian formulation have been simply solved by resorting to a Lagrangian formulation [J. M. Dawson, Phys. Rev. 113, 383 (1959); S. L. Ossakow, Ph. D. thesis, University of California at Los Angeles, 1966 (unpublished); R. C. Davidson and P. P. J. M. Schram, Nucl. Fusion 8, 183 (1968)].
[14]G. Birkhoff, D. P. MacDougall, and E. M. Pugh, J. Appl. Phys. 19, 563 (1948).
[15]J. M. Walsh, R. G. Shreffler, and F. J. Willig, J. Appl. Phys. 24, 349 (1953).

Anomalously Rapid Current Penetration in a Toroidal Turbulent Heating Experiment*

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The behavior of the plasma-current penetration in a toroidal device has been investigated for a wide range of plasma density ($10^{11} \lesssim n \lesssim 10^{13}$ cm^{-3}). The observed rapid penetration cannot be attributed to the enhancement of plasma resistivity. A possible explanation is proposed in terms of electrostatic instabilities driven by electron velocity gradients.

We report an experimental observation of anomalous skin effects in a toroidal system,[1] a schematic of which is shown in Fig. 1. An argon plasma with an average density $10^{10} \lesssim n \lesssim 10^{13}$ cm^{-3} is initially formed with the help of rf breakdown and preheating. The argon neutral pressure is typically 0.3 mTorr. A betatron-type electric field E ($\lesssim 100$ V/cm) is then applied to the plasma along a toroidal magnetic field B ($\lesssim 2$ kG). The maximum current induced in the plasma is about 20 kA at the density $n \simeq 10^{13}$ cm^{-3}. The electron temperature, measured by an orbit analyzer probe[2] and a diamagnetic probe, reaches several keV within 1 μsec starting from an initial temperature of the order of 10 eV. Poloidal magnetic fields produced by the plasma current were measured using small (5 mm diam) movable magnetic probes; one measures vertical magnetic fields and the other measures horizontal fields. It has been found that there is no significant difference in behavior between these two fields. Measurements were performed in the early stages of the current pulses, namely, up to 300 nsec, where skin effects, if any, should be most pronounced since the plasma resistivity is expected to be small.

The behavior of the electron acceleration by the electric field depends on the plasma density. At densities in the range $10^{10}-10^{11}$ cm^{-3}, the electrons, as previously reported,[3] are freely accelerated up to about 100 nsec, when the electron drift velocity overtakes the electron thermal ve-

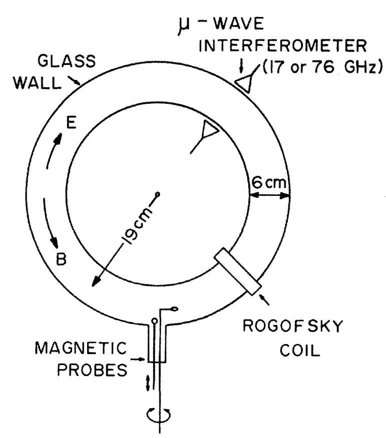

FIG. 1. Schematic of the device and major diagnostic tools. One magnetic probe measures vertical magnetic fields and the other measures horizontal fields.

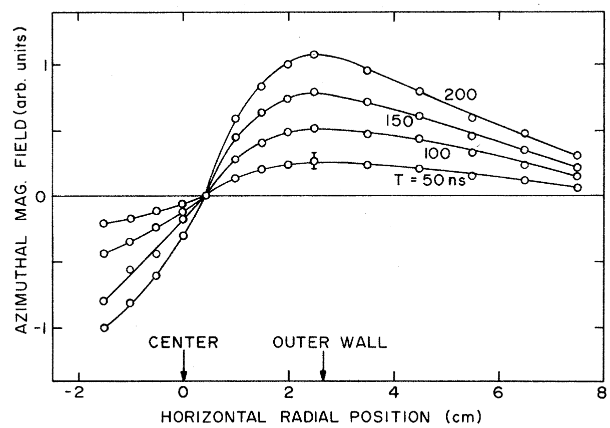

FIG. 2. Results of magnetic-probe measurements at the plasma density 1×10^{12} cm^{-3} (vertical magnetic field).

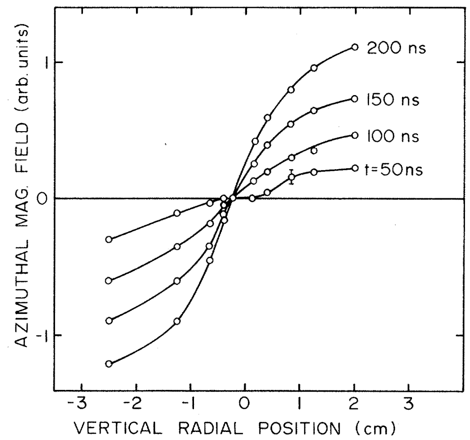

FIG. 3. Horizontal magnetic field variation at $n \simeq 1 \times10^{13}$ cm^{-3}.

locity. The departure from free acceleration was attributed to the presence of an initial inhomogeneity of the plasma density.

The presence of electron free acceleration would suggest that the skin depth of the field penetration should be approximately given by c/ω_{pe}, where c is the speed of light and ω_{pe} is the electron plasma frequency. For plasma densities $10^{10} \leqslant n \lesssim 10^{12}$ cm^{-3}, for which an initial free acceleration has now been observed, the value of c/ω_{pe} ranges from 5 mm to 5 cm. This is smaller than, or comparable to, the effective plasma radius (2.5 cm).

Results of magnetic-probe measurements for $n = 10^{12}$ cm^{-3} are shown in Fig. 2. It can be seen that no skin effect exists beyond our time resolution ($\simeq$ 20 nsec). For lower plasma densities ($n \simeq 10^{11}$ cm^{-3}), almost the same behavior of the current penetration was observed.

Figure 3 shows magnetic-probe measurements for $n \simeq 10^{13}$ cm^{-3}, at which density the free acceleration is no longer observable. Instead, the total plasma current varies in proportion to the electric field, indicating that the plasma resistivity is well defined. The conductivity was found to be about 30 mho/cm, corresponding to an effective electron collision frequency by a factor of more than 50.

Since the plasma is resistive at $n = 10^{13}$ cm^{-3}, and the (classical) penetration depth given by $(2\nu/\omega)^{1/2}c/\omega_{pe}$ ($\nu \gg \omega$, where $\omega = 6\times10^5$/sec is the frequency of the induction field) is about 3 cm, the plasma current is expected eventually to penetrate into the plasma core. However, the time constant to establish this quasi steady state is given by

$$4\pi\sigma R^2/c^2G,$$

where σ is the plasma conductivity (in esu), R is the plasma radius (cm), and G is a numerical factor ($G \simeq 6$ for cylindrical geometry). For our experimental conditions, τ is found to be about 400 nsec, using $G = 6$. Therefore, if the above classical penetration time were valid, we should see no current flowing inside the plasma column during our measurement time (up to 300 nsec). Figure 3, however, clearly indicates that the plasma current can penetrate deeply into the plasma much faster than the classical skin time. The penetration time may be estimated to be about 50 nsec.

We note that theoretical models of anomalous skin effects have been proposed by several authors. The well-known formula of anomalous skin depth,[4]

$$\delta \simeq (c/\omega_{pe})(\omega_{pe}\beta_e/\omega c)^{1/3},$$

cannot be applied to our case because the toroidal magnetic field smears out the Landau resonance. [Here, ω is the frequency of external fields or the inverse of the rise time for transient cases, and $\beta_e = (T_e/m)^{1/2}$ is the electron thermal velocity.] Neither can the collisional skin depth

$$\delta = (c/\omega_{pe})(2\nu/\omega)^{1/2} \quad (\nu \gg \omega)$$

be applied since at lower densities the collision frequency in the early stages is vanishingly small, and at higher densities the penetration time is the dominant skin effect. An extremely large anomalous skin depth $\delta = c/\omega_{pi}$ has been proposed by Breizman, Mirnov, and Ryutov[5] and Robson,[6] who assumed that the electron drift velocity is limited by the ion-acoustic velocity $(T_e/M)^{1/2}$. This, however, cannot be applied here either because the electron drift velocity exceeds the ion-acoustic velocity.

Recently, Hirose and Alexeff[7] and Horton[8] have suggested the possibility that steep electron drift-velocity gradients across an external magnetic field, which are associated with skin effects, can drive fast-growing electrostatic instabilities with growth rates $\gamma > \omega_{pi}$. It is expected that these instabilities tend to reduce the velocity gradient and thus help the plasma current penetrate into the plasma core. The skin depth enhanced by the instabilities may be deduced from the instability criterion.[7,8] The minimum velocity gradient required for the instability is given by

$$\delta^{-1} \equiv \frac{1}{u}\frac{\partial u}{\partial x} = \frac{4\omega_{pe}\beta_e}{c^2}\left(1+\frac{\omega_{pe}^2}{\omega_{ce}^2}\right)^{1/2}.$$

Thus, after the instability develops, the classical, collisionless skin depth c/ω_{pe} can be enhanced by a factor

$$\tfrac{1}{4}(c/\beta_e)(1+\omega_{pe}^2/\omega_{ce}^2)^{-1/2}.$$

If we substitute our experimental values for which electron free acceleration is observed ($n \lesssim 10^{12}$ cm^{-3}, $\beta_e \simeq 10^8$ cm/sec, $B = 1.5$ kG), the skin depth actually becomes larger than the plasma radius ($\gtrsim 15$ cm compared with 2.5 cm), and thus the skin depth should not be detectable, consistent with the experimentally observed initial free streaming of essentially all the electrons in the plasma column.

At higher plasma densities ($n \simeq 10^{13}$ cm^{-3}), we should consider the skin time instead of the skin depth since no electron free acceleration is observed. (We cannot exclude the possibility of free acceleration during an initial interval of ~20 nsec when the small current is not clearly discernible because of noise.) As mentioned earlier, the classical, collisional penetration time is much longer than the observed penetration time. This suggests that the electron collisions responsible for the plasma resistivity do not play a major role for the current penetration even though at higher densities the collision frequency is not negligible. Instead, we again consider the influence of velocity-gradient instabilities which can greatly enhance the effective electron viscosity.[9] Assuming that the kinematic viscosity[10] $\mu \simeq$(effective mean free path)^2/(collision time) $\equiv$ (radial correlation length)^2/(correlation time) is of the order of, or greater than, $\omega_{pi}L^2$ (L = characteristic length of the velocity gradient), we can estimate the current penetration time from

$$\tau \simeq \tfrac{1}{10}\,\omega_{pe}^2 R^4 L^{-2}/c^2\omega_{pi},$$

where the factor $\frac{1}{10}$ comes from the cylindrical geometry chosen for the analysis. (The ion plasma frequency is the minimum growth rate of the velocity-gradient instabilities.) If we choose $L \simeq R$, we get $\tau \simeq 40$ nsec for our experimental conditions, which is in reasonable agreement with the observed penetration time.

Anomalous skin effects not due to an enhancement of plasma resistivity are favorable in terms of the thermal energy transport. The instabilities driven by a velocity gradient in general tend only to redistribute the spatial current profile and are not expected to enhance the resistivity greatly.

We believe that our experimental results suggest that such a process may be occurring. The extreme case (Fig. 2), in which the plasma resistivity is practically zero, clearly indicates that the anomalous skin effect cannot be explained by the resistivity enhancement.

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[1]H. M. Skarsgard and J. V. Gore, Rev. Sci. Instrum. 36, 1807 (1965).

[2]D. J. Loughran, L. Schott, and H. M. Skarsgard, Can. J. Phys. 45, 3055 (1967).

[3]A. Hirose, S. Q. Mah, H. M. Skarsgard, A. R. Strilchuk, and D. W. A. Whitfield, Phys. Rev. Lett. 28, 1185 (1972).

[4]E. S. Weibel, Phys. Fluids 10, 741 (1967), and references therein.

[5]B. N. Breizman, V. V. Mirnov, and D. D. Ryutov, Zh. Tekh. Fiz. 39, 1817 (1969) [Sov. Phys. Tech. Phys. 14, 1365 (1970)].

[6]A. E. Robson, University of Texas Center for Plasma Physics and Thermonuclear Research Report CCPT No. 30, 1971 (unpublished).

[7]A. Hirose and I. Alexeff, Phys. Rev. Lett. 27, 949 (1971).

[8]W. Horton, Jr., Phys. Rev. Lett. 28, 1506 (1972).

[9]The concept of the anomalous electron viscosity caused by ion-acoustic turbulence has been proposed by J. Schmidt and Y. Yoshikawa [Phys. Rev. Lett. 26, 753 (1971)] to explain the observed absence of skin effects in tokamaks. L. A. Artsimovich *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. 10, 130 (1969) [JETP Lett. 10, 82 (1969)].

[10]S. I. Braginskii, in *Reviews of Plasma Physics*, edited by M. A. Leontovich (Consultants Bureau, New York, 1965), Vol. I, p. 228.