with x_R^0). Similarly, a potential-scattering phase shift involves a simple quadrature over $q_R(x)$. Because such quadratures and the angles θ_L and θ_R are very monotonic functions of the energy, they can be accurately collocated from computed values at a few arbitrarily chosen energies, and interpolation used for very rapid computation of an entire eigenvalue spectrum.

For calculation of one solution to the Schrödinger equation [for some given $k^2(x)$], our method is typically 10 times faster than direct integration of Eq. (1) by standard methods, and becomes faster with shorter de Broglie wavelengths. For complete eigenvalue or phase shift spectra, formulas like (15) yield results about 100 times faster.

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*Based in part on a thesis for the M. Sc. degree in Physics, The University of Alberta, Edmonton, Alberta, Canada, 1972.

†Present address: Center for Radiophysics and Space Research, Cornell University, Ithaca, N. Y. 14850.

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Dip Structures in $\pi^+ + p \to \rho^+ + p$ at 1.55-1.84 GeV/c*

Y. Williamson, S. Y. Fung, A. Kernan, U. Mehtani,† T. L. Schalk, and B. C. Shen
Department of Physics, University of California, Riverside, California 92502

and

W. Michael, R. W. Birge, and G. E. Kalmus‡
Lawrence Berkeley Laboratory, Berkeley, California 94720

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Results of a high-statistics study of $\pi^+ + p \to \rho^+ + p$ at 1.55–1.84 GeV/c are consistent with dominance of π and ω exchange close to threshold. A pronounced dip in $\rho_{00}{}^s d\sigma/dt$ at $-t \simeq 0.4$ GeV^2 may be attributed to pion exchange with strong absorption.

An intriguing question at present is whether the structures at fixed t in two-body scattering reactions are inherent in the t-channel Regge-exchange amplitude, or are related to absorption of low partial waves in the s channel. We report here a study of $\pi^+ + p \to \rho^+ + p$,[1] and the observation of structure at $-t \simeq 0.4$ GeV^2 in the pion-exchange cross section, which can be attributed to strong absorption.

The results come from a hydrogen-bubble-chamber exposure at the Bevatron which yielded 9000, 5900, and 6000, and 3400 events in the channel $\pi^+ + p \to \pi^+ + p + \pi^0$ at momenta 1.55, 1.67, 1.77, and 1.84 GeV/c, respectively.[2]

In the momentum interval 1.55–1.84 GeV/c, the reaction $\pi^+ + p \to \pi^+ + p + \pi^0$ is dominated by the quasi-two-body channels $\pi^0\Delta^{++}(1236)$, $\pi^+\Delta^+$, and ρ^+p.[1] The technique used to determine differential cross sections and density-matrix elements for $\pi^+ + p \to \rho^+ + p$ was to make a maximum-likelihood fit of the $\pi^+p\pi^0$ events assuming the following set of amplitudes in the $\pi^+ + p \to \pi^+ + p + \pi^0$ channel: ρ^+p, $\pi^0\Delta^{++}$, $\pi^+\Delta^+$, $\pi^+N^+(1500)$, and $\pi^+N^+(1680)$. The four independent variables characterizing each event at a given momentum were simultaneously fitted to this set of amplitudes with the assumption that the amplitudes are incoherent.

At each momentum the fitting process was performed in two states: (i) All $\pi^+p\pi^0$ events were fitted by the above hypothesis. The parameters in the fit, a total of nineteen, were the relative amounts of each quasi-two-body state, the masses and widths of ρ^+, Δ^{++}, and Δ^+, and the helicity-frame density-matrix elements ρ^s of ρ, Δ^{++}, and Δ^+.[3] This fit gave the total number of ρ^+p events: 2900, 2300, 3000, 1900, and the corresponding ρ^+p cross sections: 3.7 ± 0.3, 4.0 ± 0.3, 3.7 ± 0.4, and 3.2 ± 0.3 mb at 1.55, 1.67, 1.77, and 1.84 GeV/c, respectively. (ii) To determine $d\sigma/dt$ and ρ^+ density-matrix elements over a given t_{pp} interval, the $\pi^+p\pi^0$ events in that t interval were fitted with the same hypothesis.

Density-matrix elements in the Gottfried-Jack-

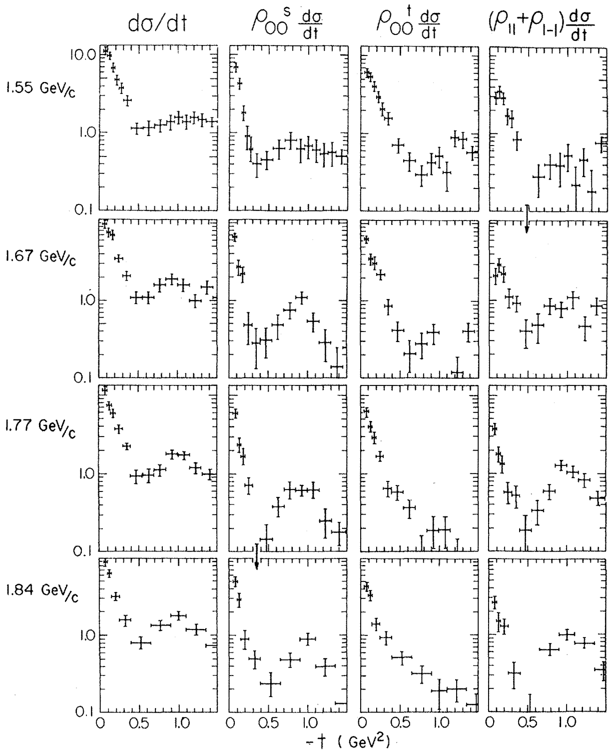

FIG. 1. $d\sigma/dt$, $\rho_{00}^{s} d\sigma/dt$, $\rho_{00}^{t} d\sigma/dt$, and $(\rho_{11}+\rho_{1-1})\times d\sigma/dt$ for $\pi^+ + p \rightarrow \rho^+ + p$ at 1.55, 1.67, 1.77, and 1.84 GeV/c.

son frame, ρ^t, were computed by the method of moments, with each event assigned a weight equal to the probability for that event to be $\rho^+ p$ according to the results of the maximum-likelihood fit.

Figure 1 shows the $d\sigma/dt$, $\rho_{00}^{s} d\sigma/dt$, $\rho_{00}^{t} d\sigma/dt$, and $(\rho_{11}+\rho_{1-1}) d\sigma/dt$ distributions for $\pi^+ + p \rightarrow \rho^+ + p$ at 1.55–1.84 GeV/c for $-t > 0.05$ GeV^2. Over this momentum interval, the value of $-t_{\rm min}$, $(-t_{\rm max})$ for production of $\rho(765)$ ranges from 0.05 (1.47) GeV^2 to 0.03 (2.01) GeV^2. Figure 2 shows the density-matrix elements ρ_{00}^{s}, ρ_{00}^{t}, $\rho_{11}+\rho_{1-1}$, and Reρ_{10}^{s}. We note the following features of these distributions:

(1) The differential cross section $d\sigma/dt$ shows the strong forward peak and the dip at $-t \simeq 0.5$ GeV^2, which are characteristic features of this reaction at higher energies.[4]

(2) Both ρ_{00}^{s} and $\rho_{00}^{s} d\sigma/dt$ show a pronounced dip at $-t \simeq 0.4$ GeV^2, whereas there are no fixed-t structures in ρ_{00}^{t} and $\rho_{00}^{t} d\sigma/dt$. The quantity $\rho_{00}^{s} d\sigma/dt$ ($\rho_{00}^{t} d\sigma/dt$), which is the cross section for producing ρ^+ with zero s- (t-) channel helicity, receives contributions only from unnatural parity exchanges in the t channel.[5]

(3) $(\rho_{11}+\rho_{1-1}) d\sigma/dt$ dips strongly at $-t \simeq 0.5$ GeV^2. The quantity $(\rho_{11}+\rho_{1-1}) d\sigma/dt$ isolates the cross section for natural parity exchange in the t channel; the superscript s,t is omitted because $\rho_{11}+\rho_{1-1}$ is invariant under a transformation between the Gottfried-Jackson and helicity frames.[5]

(4) Reρ_{10}^{s} passes through zero at $-t \sim 0.35$–0.55 GeV^2.

Discussion.—(A) The cross-section behavior suggests that the t-channel exchanges which dominate this reaction at high energies are still opera-

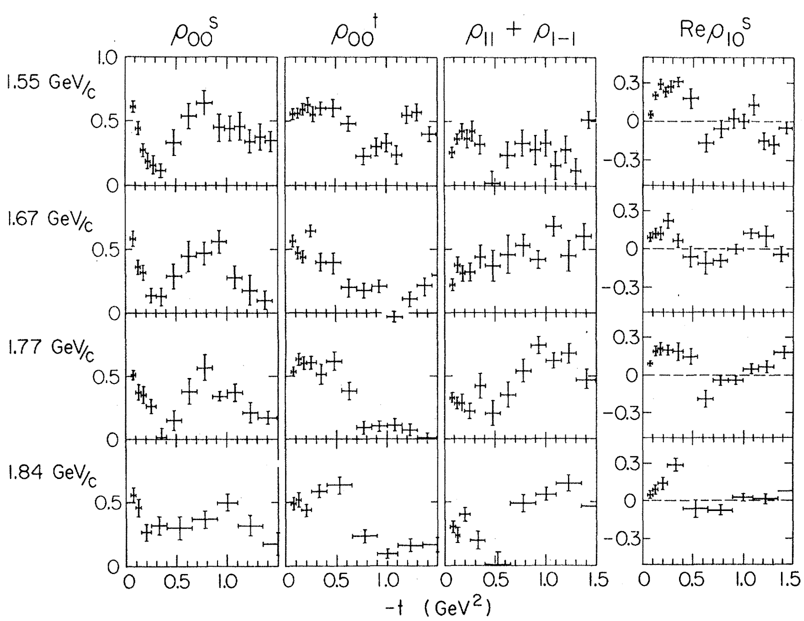

FIG. 2. ρ_{00}^{s}, ρ_{00}^{t}, $\rho_{11}+\rho_{1-1}$, and Reρ_{10}^{s} for $\pi^+ + p \rightarrow \rho^+ + p$ at 1.55, 1.67, 1.77, and 1.84 GeV/c.

tive near threshold. This is consistent with the lack of variation in $d\sigma/dt$ over the momentum interval 1.55–1.84 GeV/c, indicative that s-channel resonances in the mass interval 1955–2090 MeV are not strongly coupled to ρ^+p. It is known that for $F_{37}(1950)$, the leading s-channel resonance in this region, the dominant inelastic decay mode is $\pi\Delta$.[6]

The well-established t-channel trajectories for $\pi^+ + p \to \rho^+ + p$ are π, ω, and A_2. Below, we attempt to understand the features (2), (3), and (4) in terms of these exchanges. Of the variety of possible exchange mechanisms we consider two: (a) single Regge-pole exchange, where the Regge poles have the conventional zero structures,[7] and (b) single Regge-pole exchange modified by strong absorption; the Regge-pole amplitude is smooth. This is the "strong-cut Reggeized absorption model" (SCRAM).[8]

In model (a) the quantities $\rho_{00}\,d\sigma/dt$ and $(\rho_{11}+\rho_{1-1})\,d\sigma/dt$ isolate the cross sections for π and ω, A_2 exchange, respectively. In model (b) absorption spoils these identifications because it generally mixes amplitudes. However, if, as appears likely, elastic scattering conserves helicity in the s channel,[9] these separations work in the s channel, even with absorption.

(B) The dip in $\rho_{00}{}^s\,d\sigma/dt$ at $-t \simeq 0.4$ GeV^2 has not been reported previously. It is, however, predicted by SCRAM.[8] In SCRAM and other absorptive models,[10] structures at fixed t in inelastic reactions arise from absorption of the lower partial waves in the s channel. The s-channel helicity amplitudes are then proportional to $J_n(b\sqrt{-t})$, where n is the net helicity flip in the s channel, and b is the impact parameter of the dominant partial waves. The net s-channel helicity flip is defined as $n = |n_1 - n_2|$, where n_1 and n_2 are the helicity changes at the meson and baryon vertices, respectively. For b of 1 F, the s-channel flip-1 amplitude has, in general, a zero at $-t \simeq 0.6$ GeV^2; for pion exchange, the zero is shifted down to $-t = 0.4$ GeV^2.[8]

It may easily be seen that $\rho_{00}{}^s\,d\sigma/dt$ isolates the pion exchange amplitudes having net helicity flip 1 in the s channel. The πNN vertex has $n_2 = 1$,[8,10] and $n_1 = 0$ since $\rho_{00}{}^s\,d\sigma/dt$ selects ρ^+ with s-channel helicity zero. The observed dips in $\rho_{00}{}^s$ and $\rho_{00}{}^s\,d\sigma/dt$ are thus explicable in terms of a zero in the s-channel helicity flip-1 amplitude $f_{0\,-1/2,0\,1/2}{}^s$.

(C) The dip in $(\rho_{11}+\rho_{1-1})\,d\sigma/dt$ at $-t \simeq 0.5$ GeV^2 has been seen in studies of $\pi^+ + p \to \rho^+ + p$ at higher energies.[4] This structure is predicted for ω exchange both by the conventional Regge-pole model and by the strong absorption model.

(D) Within the framework of SCRAM the zero in $\mathrm{Re}\rho_{10}{}^s$ has the same origin as the dip in $\rho_{00}{}^s\,d\sigma/dt$, namely, a zero in the s-channel helicity amplitude $f_{0\,-1/2,0\,1/2}$ for π exchange. Thus,

$$\rho_{10}{}^s \sum |f^s|^2 = f_{0\,1/2,0\,1/2}{}^{s*}(f_{1\,1/2,0\,1/2}{}^s - f_{-1\,1/2,0\,1/2}{}^s) + f_{0\,-1/2,0\,1/2}{}^{s*}(f_{1\,-1/2,0\,1/2}{}^s - f_{-1\,-1/2,0\,1/2}{}^s).$$

In SCRAM we have $f_{1\,1/2,0\,1/2}{}^s = f_{-1\,1/2,0\,1/2}{}^s$, but $f_{1\,-1/2,0\,1/2}{}^s \neq f_{-1\,-1/2,0\,1/2}{}^s$ since $n = 2$ and 0 for these two amplitudes, respectively. Then a zero in $f_{0\,-1/2,0\,1/2}{}^s$ at $-t \simeq 0.4$ GeV^2 gives rise to a zero in $\mathrm{Re}\rho_{10}{}^s$. Observations (2) and (4) are thus compatible with the predictions of SCRAM.

In conventional Regge-pole exchange of π, ω, and A_2 [model (a)], the expectation is that $\mathrm{Re}\rho_{10}{}^t = 0$. In this experiment $\mathrm{Re}\rho_{10}{}^t$ (not shown) $\simeq -\mathrm{Re}\rho_{10}{}^s$, and is therefore nonzero.

In conclusion, results (1) through (4) are consistent with dominance of π and ω exchange close to threshold in $\pi^+ + p \to \rho^+ + p$. The dip and zero at $-t \simeq 0.4$ GeV^2 in $\rho_{00}{}^s\,d\sigma/dt$ and $\mathrm{Re}\rho_{10}{}^s$, respectively, can be accounted for by the π-exchange amplitude in the strong-absorption model. The dip in $(\rho_{11}+\rho_{1-1})\,d\sigma/dt$ is expected for ω exchange in either the strong-absorption or the conventional Regge-pole model. The quantities $\rho_{00}{}^s\,d\sigma/dt$ and $(\rho_{11}+\rho_{1-1})\,d\sigma/dt$ account for about 40 and 35%, respectively, of the forward cross section for $\pi^+ + p \to \rho^+ + p$.

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†Presently at Argonne National Laboratory, Argonne, Ill. 60439.

‡Presently at Rutherford High Energy Laboratory, Chilton, Didcot, Berkshire, England.

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Measurement of Muon-Pair Photoproduction in the Deep Inelastic Region*

J. F. Davis, S. Hayes,† R. Imlay,‡ P. C. Stein, and P. J. Wanderer
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850

(Received 15 September 1972)

The inclusive cross section for low-mass muon pair photoproduction has been measured in the deep inelastic region. The measured cross section is much larger than that expected for one-photon (Bethe-Heitler) processes. This result may be additional evidence for pointlike structure (partons) in nucleons. The experimental results are compared with the parton model of Bjorken and Paschos. The value of the cross section predicted by this model is too low.

Electron-scattering experiments in the deep inelastic region[1,2] have shown that the cross sections for excitation of nucleons in the continuum region scale and are large. The existence of pointlike charged particles (partons) within the nucleon has been proposed to explain these results. Bjorken and Paschos[3] have suggested that deep inelastic two-photon processes may be a test of some aspects of the parton models. Such processes include inelastic Compton scattering[4] and its analog, the inclusive photoproduction of low-mass muon pairs in the inelastic region. The latter process must be separated from the usual Bethe-Heitler production of muon pairs. Bethe-Heitler production is calculable[5] from the nucleon electromagnetic structure functions. An excess in the cross section can be attributed to the Compton diagram.

This paper reports a measurement of the photoproduction of low-mass muon pairs in the deep inelastic region. Photons produced by bremsstrahlung of 11.7-, 10-, and 8.5-GeV electrons at the Cornell University Wilson Synchrotron Laboratory were incident on a beryllium target. Targets of 9.4 and 14.1 g/cm^2 were used. The technique for the detection of the muons was similar to that employed in previous experiments.[6] It makes use of the difference between the absorption lengths of muons and other particles. Muons passed through absorber A, hodoscope H_F, twelve scintillation counters R alternated with iron, and hodoscope H_R, and stopped in a thick iron pack before reaching counters V (Fig. 1). The apparatus detected muons produced between 10° and 15° with respect to the incident beam and with energies between 2.58 and 3.49 GeV. Hodoscope H_F defined the solid-angle acceptance (0.0154 sr) of the experiment. The R counters

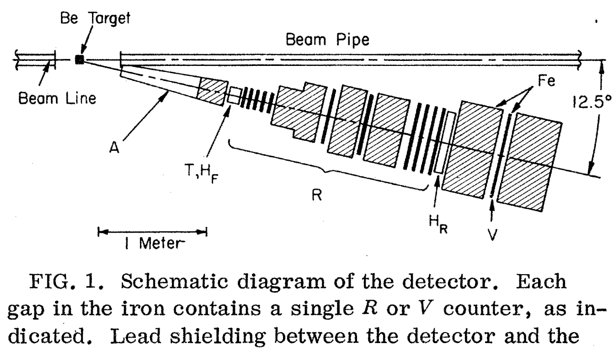

FIG. 1. Schematic diagram of the detector. Each gap in the iron contains a single R or V counter, as indicated. Lead shielding between the detector and the photon beam is not shown. The beam enters from the left.