and at $t=0$ the modulation begun. Note that the response of the modulation frequency is linear in the modulation amplitude z, whereas the response at the natural frequency shows threshold behavior near the theoretical value of z. The amplification saturates at a low level possibly because of detuning produced by nonlinear frequency shift.

The linearly driven oscillations produced by the growth-rate modulation exhibit a resonance behavior near $\Omega=\omega_0$, the natural frequency. Linearizing with respect to z and neglecting the nonlinear sources, we find the amplitude of the oscillation of H at the modulation frequency to be

$$|\Delta H_\Omega| = \frac{g_0^2 z}{2[(\Omega^2 - g_0 + R)^2 + \Omega^2(S+R/g_0)^2]^{1/2}}$$

and thus $|\Delta H_\Omega|_{max} \cong g_0 z/[2(S+R/g_0)]$ which can be very large.

In conclusion, by analyzing the model equations (6) and (7) we find many interesting features regarding the saturation states, relaxation oscillations and their damping. Although our model equations represent a somewhat idealized situation they will be a good first approximation for a number of different problems in which mode coupling between growing and damped waves plays an important role. However, our equations do not include the effects of harmonic generation. Since harmonically generated waves are coherent, it is inappropriate to include them in H. Inclusion of this effect will be left to a future investigation.

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Coherent Quasiparticle Excitation in a Type-II Superconductor in Crossed Electric and Magnetic Fields

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Structures are observed in the current-voltage characteristics of a type-II superconductor in crossed fields. This effect can be explained as due to coherent quasiparticle excitation when the electromagnetic fields satisfy excitation conditions.

In previous reports the author showed that in a longitudinal magnetic field some structures are observed in the magnetic field dependence of the superconducting current threshold[1,2] and in the current-voltage characteristics of the resistive superconducting state[2] of a type-II superconductor. In this Letter we would like to show that in a transverse magnetic field, structures are also observed in I-V curves, and that the origin of the structures can be attributed to coherent quasiparticle excitations which occur when certain conditions for the fields are satisfied.

We investigated the properties of the resistive superconducting state of Nb-25% Zr wire specimens of 0.025 cm diam at 10°K in a transverse magnetic field H and pulsed electric field E. Figure 1 shows a typical example of the current-voltage curve. Steplike structures appear in the curves similar to those seen in the longitudinal magnetic field.[2] In Fig. 2 the values of the fields E and H where the structures appear are plotted. It is seen from the figure that the values of the

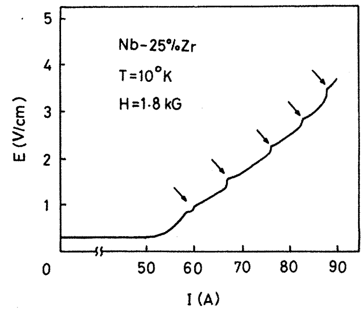

FIG. 1. An example of the structures which appear in the current-voltage curve of Nb-Zr wire specimens in a transverse magnetic field.

fields satisfy the relation

$$(E/E^*)^2 - [H/(2m+1)H^*]^2 = (2n)^{-2} \quad (m, n \text{ integers}) \tag{1}$$

with $E^* \approx 13$ V/cm, $H^* \approx (c/v_F)E^* \approx 0.9$ kG, $v_F \approx 1.5\times 10^6$ cm/sec.

In the following we consider the condition for the quasiparticle excitation in the crossed fields on the basis of the Bogoliubov equation[3]:

$$\begin{aligned} \epsilon u(\vec{r}, \sigma) &= (2m)^{-1}(p^2 - p_F^2)u(\vec{r}, \sigma) + \textstyle\sum_\mu \Delta(\vec{r})\rho_{\sigma\mu} v(\vec{r}, \mu), \\ -\epsilon v(\vec{r}, \sigma) &= (2m)^{-1}(p^2 - p_F^2)v(\vec{r}, \sigma) + \textstyle\sum_\mu \Delta^*(\vec{r})\rho_{\sigma\mu} u(\vec{r}, \mu), \end{aligned} \tag{2}$$

with

$$\rho = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix};$$

u and v are defined by the unitary transformation of the one-electron annihilation operator

$$\psi(\vec{r}, \sigma) = \textstyle\sum_s [u_s(\vec{r}, \sigma)\gamma_s + v_s^*(\vec{r}, \sigma)\gamma_s^\dagger],$$

with spin σ at point $\vec{r}$; γ_s is the annihilation operator of a quasiparticle in a quantum state s, ϵ and p are the energy and momentum of the quasiparticle, respectively, $p_F = mv_F$ is the Fermi momentum, and $\Delta(\vec{r})$ is the pair potential.

Using the relation

$$(2m)^{-1}(p^2 - p_F^2) \approx v_F(|p| - p_F),$$

and putting

$$\begin{bmatrix} u(\vec{r}, \uparrow) \\ u(\vec{r}, \downarrow) \\ v(\vec{r}, \uparrow) \\ v(\vec{r}, \downarrow) \end{bmatrix} = \begin{bmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \chi_4 \end{bmatrix} \exp(iv_F p_F t/\hbar),$$

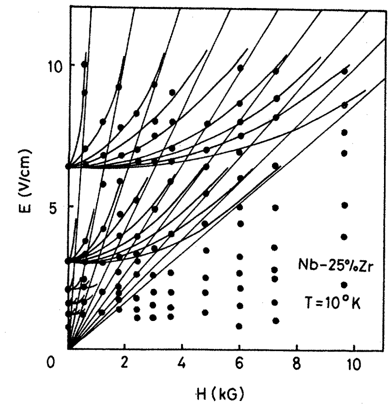

FIG. 2. Plot of the values of the fields E and H where the structures are observed. The solid curves show Eq. (1).

we find from Eq. (2) that

$$(\epsilon^2 - v_F^2 p^2 - |\Delta|^2)\chi_j = 0 \quad (j = 1, 2, 3, 4). \tag{3}$$

This equation can be expressed in a form which is linear in the four-dimensional momentum operator, like Dirac's relativistic wave equation,[4]

$$[\epsilon/v_F - \rho_1 \vec{\sigma}\cdot\vec{p} - \rho_3|\Delta|/v_F]\chi_j = 0, \tag{4}$$

where

$$\sigma_1 = \begin{bmatrix} 0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0 \end{bmatrix}, \quad \sigma_2 = \begin{bmatrix} 0&-i&0&0\\i&0&0&0\\0&0&0&-i\\0&0&i&0 \end{bmatrix},$$

$$\sigma_3 = \begin{bmatrix} 1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&-1 \end{bmatrix}, \quad \rho_1 = \begin{bmatrix} 0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0 \end{bmatrix},$$

$$\rho_3 = \begin{bmatrix} 1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1 \end{bmatrix}.$$

It is easy to verify that any solution of Eq. (4) is also a solution of Eq. (3) when there is no electromagnetic field. When the electromagnetic potentials are involved, however, a solution of Eq.

(4) is no longer that of Eq. (3), but satisfies an equation

$$\left[\left(\frac{i\hbar}{v_F}\frac{\partial}{\partial t}-\frac{e}{v_F}\varphi\right)^2-\left(\frac{\hbar}{i}\nabla-\frac{e}{c}\vec{A}\right)^2-\frac{|\Delta|^2}{v_F^2}-\frac{\hbar e}{c}\vec{\sigma}\cdot\vec{H}+i\rho_1\frac{\hbar e}{v_F}\vec{\sigma}\cdot\vec{E}\right]\chi_j=0. \tag{5}$$

It is well known that, in the mixed state of superconductors in crossed electric and magnetic fields, the fluxoids move with flow velocity $v_L = cE/H$.[5] The maximum velocity of the fluxoid may be the Fermi velocity v_F.[6] When $cE/H > v_F$, a degradation of superconductivity can be expected due to the quasiparticle excitation. We investigate the condition for the electromagnetic field in which the excitation is allowed by solving Eq. (5) in the crossed fields $\vec{E}=(0,E,0)$ and $\vec{H}=(0,0,H)$. We choose the electromagnetic potentials as $\varphi=0$ and $\vec{A}=(0, Hx-cEt, 0)$, and put $w=x-(cE/H)t$. Assuming that χ_j is a function of w, y, and z, we can rewrite Eq. (5) as

$$\left\{\left[-g^2\frac{\partial}{\partial w^2}+\left(\frac{\partial}{\partial y}-i\frac{eH}{\hbar c}w\right)^2\pm ig\frac{eH}{\hbar c}\right]+\left[\frac{\partial^2}{\partial z^2}-\frac{|\Delta|^2}{v_F^2}\right]\right\}\chi_j=0, \tag{6}$$

with

$$g=[(cE/v_FH)^2-1]^{1/2}.$$

Looking for a solution of this equation in the form

$$\chi_j=\exp(ik_yy)F(w)G(y),$$

we separate Eq. (6) into (w,y) and z parts:

$$\left[g^2\frac{\partial^2}{\partial w^2}+\left(k_y-\frac{eH}{\hbar c}w\right)^2\pm ig\frac{eH}{hc}-K\right]F(w)=0, \tag{7}$$

$$[\partial^2/\partial z^2-1/\xi^2-K]\,G(y)=0, \tag{8}$$

where the coherence length $\xi=|\Delta|/v_F$ and K is a constant. Equation (7) has physically acceptable solutions only when

$$K=(eH/\hbar c)g(2l+1\pm1)i \quad (l \text{ an integer}). \tag{9}$$

In the range of excitation energy around $|\Delta|$, each quasiparticle is regarded as a mixture of an electron and a hole. Putting the spatial extension of the wave packet of the quasiparticle as $\sqrt{2}\,\alpha\xi$, and assuming that the constant K takes an imaginary value

$$K=i(\sqrt{2}\,\alpha\xi)^{-2},$$

we have from Eq. (9)

$$\alpha^2(E^2/E_{c2}^2-H^2/H_{c2}^2)=(2l+1\pm1)^{-2}, \tag{10}$$

where

$$H_{c2}=(c/v_F)E_{c2}=\hbar c/2e\xi^2.$$

Comparing Eq. (10) with the observed result Eq. (1) for the case $m=0$ and with use of the observed value of the upper critical field $H_{c2}=8$ kG, we get $\alpha\approx3$ and $E_{c2}\approx120$ V/cm. On the other hand, Eq. (8) has a solution

$$G(z)=\exp[\pm(z/\xi)(1+i/2\alpha^2)^{1/2}]\approx\exp[\pm z/\xi].$$

From the solutions of Eqs. (7) and (8), an Abrikosov-type function[7] can be constructed for the excited quasiparticle field with $l=0$:

$$\chi_j=\exp(-|z|/\xi)\exp(-ieHw^2/2\hbar cg)\vartheta_3(k_1(y+w/g)(1+i)/\sqrt{2}\,|1),$$

with

$$k_1=(geH/\hbar c)^{1/2}.$$

This expression shows that the stripes of the excitation density wave move in the direction $\tan\theta=y/x=g$ with velocity v_F. The electric field in the superconductor may be maintained by the polarization of the quasiparticles.

The existence of structures when $cE/H<v_F$ [$m>1$ in Eq. (1)] can be explained as follows. The pair potential around a fluxoid will have an expression

$$\Delta(\vec{r})=|\Delta|\exp(2i\vec{q}\cdot\vec{r}),$$

with the quantization condition for a flux quantum

$$\oint\vec{q}\cdot d\vec{r}=\pi.$$

Since BCS wave functions can be built by pairing electrons $(\vec{k}+\vec{q}, \uparrow)$ and $(-\vec{k}+\vec{q}, \downarrow)$, we must use one-electron states satisfying the quantization condition around the fluxoid

$$(2j+1)\pi = \oint \vec{k}\cdot d\vec{r} \approx 2\pi\xi k \quad (j \text{ an integer})$$

or

$$k \approx (j+\tfrac{1}{2})\xi^{-1}. \tag{11}$$

When the condensed state moves with the velocity $\vec{v}_s(\vec{r})$, the local excitations may have a shifted BCS spectrum

$$\epsilon(\vec{k}, \vec{r}) = |\Delta(\vec{r})| + \hbar\vec{v}_s\cdot\vec{k}. \tag{12}$$

From Eqs. (11) and (12) we see that the excitation spectrum becomes gapless when

$$v_s \geqslant |\Delta|/\hbar k \approx v_F/(2j+1).$$

In the crossed fields the order-parameter variation moves with the fluxoid velocity cE/H. Therefore, the critical state of the quasiparticle excitation is

$$cE/H = v_F/(2j+1).$$

This relation teaches us that Eq. (5) may be appropriate in this case if the magnetic field H in the equation is replaced by $H/(2j+1)$. Then the excitation condition (10) is rewritten as

$$\alpha^2[E^2/E_{c2}^{\ 2} - H^2/(2j+1)^2 H_{c2}^{\ 2}] = (2l+1\pm 1)^{-2},$$

which coincides with the observed result Eq. (1).

The voltage steps in the current-voltage characteristics have also been observed for superconductive thin wires and whiskers with diameter comparable to the coherence length ξ.[8-10] On the basis of our discussion, these phenomena may be interpreted as phenomena due to coherent quasiparticle excitations in the absence of a magnetic field.

Certain characteristics of these phenomena may have applications in high-speed–computer elements and high-frequency oscillators.

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Thermoelectric Anomaly Near a Critical Point

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New measurements on β-brass reveal an anomaly in the thermopower in the vicinity of the order-disorder phase transition. A simple analysis, which is applicable to a wide class of phase transitions, suggests that the anomaly arises from the scattering of conduction electrons from short-range critical fluctuations and that the thermopower can, in principle, provide a direct measure of correlation functions.

A number of advances have been made recently in our understanding of transport properties in metallic systems near critical points.[1] One of the more important of these was the clarification by Fisher and Langer[2] of the vital role that short-range spin fluctuations play in the scattering of conduction electrons. We suggest that the Fisher-Langer approach can be extended in a simple way to describe the anomaly in the temperature dependence of the thermopower for a wide class of