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$\alpha^2 \ln\alpha^{-1}$ Recoil Corrections to the Hydrogen Hyperfine Structure and the Proton Polarizability*

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(Received 21 August 1972)

We report a calculation of the order $(m_e/m_p)\alpha^2 \ln\alpha^{-1}$ corrections to the hyperfine splitting (ν_H) of the hydrogen ground state. The result

$$\Delta\nu_H = \tfrac{8}{3}(m_e/m_p)^2(1+m_e/m_p)^{-5}c\alpha^4 R_\infty \ln\alpha^{-1}[9+7\kappa(1+\kappa)-(m_e/m_p)\kappa(3+14\kappa)]$$
$$= 0.0016 \text{ MHz (1.1 ppm)}$$

is used in conjunction with the corresponding formula for the muonium hyperfine interval to obtain an estimate of the proton polarizability.

Comparison of the experimental and theoretical determinations of ν_H, the hydrogen ground-state hyperfine splitting (hfs), has traditionally served as an important check on quantum electrodynamics (QED). However, given the many other successful applications of QED and the theoretical uncertainties attributable to hadron dynamics, the role of the hydrogen hfs has shifted to one of serving as a probe of the proton's structure.[1] The proton structure enters the expression for ν_H through recoil correction terms, δ_p, which arise from the proton's finite size and mass, and polarizability correction terms, δ_p', whose presence is due to the existence of proton excited states. While the former corrections can be calculated very reliably,[2] calculations of the latter are quite model dependent[3] since they require a knowledge of the spin-dependent virtual Compton scattering amplitude. Under these circumstances, the practice has been to extract δ_p' directly from the hyperfine data. With precision measurements of the muonium hyperfine interval,[4] ν_{Mu}, and the muon-to-proton magnetic-moment ratio,[5] μ_μ/μ_p, now available, the ratio[6] ν_{Mu}/ν_H seems to be the best source for a value of δ_p'. This ratio is independent of radiative corrections to the electron moment and uncertainties in the fine-structure constant α. In addition to δ_p (and δ_p'), ν_{Mu}/ν_H contains the muonium recoil correction δ_μ, which has been calculated[7,8] to relative order $(m_e/m_\mu)\alpha^2\ln\alpha^{-1}$. For a consistent determination of δ_p', δ_p should be evaluated to the same relative order, and this was the motivation for our calculation.

Our result for the frequency correction $\Delta\nu_H$ of order $(m_e/m_p)\alpha^2\ln\alpha^{-1}$ is

$$\Delta\nu_H = \tfrac{8}{3}(m_e/m_p)^2(1+m_e/m_p)^{-5}c\alpha^4 R_\infty \ln\alpha^{-1} \times[9+7\kappa(1+\kappa)-(m_e/m_p)\kappa(3+14\kappa)]$$
$$= 0.0016 \text{ MHz (1.1 ppm)}, \quad (1)$$

where $\kappa \simeq 1.79$ denotes the proton anomalous moment. Note that Eq. (1) reduces to the previous results obtained for muonium[8] and positronium[9] in the appropriate limits. When this contribution is included in the theoretical expression[10] for ν_H, the recoil-dependent portion δ_p has the numerical value

$$\delta_p = -34.6 \pm 0.9 \text{ ppm} + 1.1 \text{ ppm}. \quad (2)$$

The first term on the right-hand side of Eq. (2), which represents the leading size and recoil correction, is well established,[1,2] and the ± 0.9 ppm allows for uncertianties in the proton form factors. The remaining contribution to δ_p, of order $(m_e/m_p)\alpha^2\ln\alpha^{-1}$ (which was computed here) represents a recoil effect arising from low-momentum components of the Bethe-Salpeter wave function. Thus, although we use a proton vertex of the form

$$\Gamma_\mu(p+k,p) = ie[\gamma_\mu - F_2(k^2)\sigma_{\mu\nu}k_\nu/2m_p], \quad (3)$$

only $F_2(0)=\kappa$ contributes to the result[11] Eq. (1). As a consequence, no further uncertainty is introduced into the expression for δ_p.

The calculation is performed using a perturbation treatment[12] of the Bethe-Salpeter equation, with the wave function being obtained by a single

Table I. Contributions to $\Delta\nu_H$ from the various diagrams, where η denotes the coefficient of the square bracket in Eq. (1), $M=(m_e+m_p)$, and μ is the reduced mass.

	One-photon exchange	Two-photon exchange	
		Ladder	Crossed
Charge	$\eta(2M/\mu)$	$\eta(9+M/\mu)$	$-\eta(3M/\mu)$
Linear dipole	$\kappa\eta\left(\frac{2M}{\mu}+1-\frac{m_e}{m_p}\right)$	$\kappa\eta\left(10+2\frac{m_e}{m_p}\right)$	$-\kappa\eta\left(\frac{2M}{\mu}+4+4\frac{m_e}{m_p}\right)$
Quadratic dipole		$\kappa^2\eta\left(9-12\frac{m_e}{m_p}\right)$	$-\kappa^2\eta\left(2+2\frac{m_e}{m_p}\right)$

iteration of the Salpeter[13] equation. Our approach differs from the muonium and positronium $\alpha^2\ln\alpha^{-1}$ calculations[8,9] in that we use the Lorentz gauge rather than the Coulomb gauge for the photon propagator. We obtain contributions from diagrams involving the exchange of one and two photons, and the results are summarized in Table I. Using the recommended value for the fine-structure constant,[14]

$$\alpha^{-1}=137.036\,02(21), \tag{4}$$

the theoretical value for ν_H is[14]

$$\nu_H{}^{\text{theor}}=1420.4041(1+\delta_p')\pm 0.0045\ \text{MHz}\ (3.2\ \text{ppm}), \tag{5}$$

where the uncertainty is the vector sum of the uncertainties in α^2 (3.0 ppm), δ_p (0.9 ppm), and the estimated uncalculated radiative corrections[10] (0.6 ppm).

We can obtain a value for δ_p' which is free of the uncertainties in α by expressing ν_{Mu}/ν_H as[3,6]

$$\nu_{Mu}/\nu_H=(\mu_\mu/\mu_p)(1+m_e/m_p)^3\times(1+m_e/m_\mu)^{-3}(1+\delta_\mu-\delta_p-\delta_p'), \tag{6}$$

where δ_μ is the muon recoil correction (-179.7 ppm + 5.6 ppm). Using the average value of ν_{Mu} as determined by the University of Chicago and Yale University groups,[4] $\nu_{Mu}=4463.3023(35)$, the University of Washington–Lawrence Radiation Laboratory result[5,15] $\mu_\mu/\mu_p=3.183\,346\,7(82)$, and the value of δ_p obtained from Eq. (2), Eq. (6) gives

$$\delta_p'=5.6\pm 2.8\ \text{ppm}. \tag{7}$$

This result for δ_p' is consistent with that obtained directly from Eq. (5), $\delta_p'=1.2\pm 3.2$ ppm.

Finally, it is worth noting that the ratio of the (order $\alpha^2\ln\alpha^{-1}$) correction term in Eq. (2) to the corresponding muonium result (1.1/5.6) is very nearly identical to the ratio of the leading (order α) corrections (34.6/179.7). Moreover, in both cases, the logarithmic corrections are well approximated by simply scaling the leading terms by $\alpha\ln\alpha^{-1}$. These facts suggest that the logarithmic corrections are not unusually large and provide a means of estimating the uncalculated (order α^2) recoil corrections. Such a simple scaling of the $\alpha^2\ln\alpha^{-1}$ correction changes the uncertainty in Eq. (7) to $\simeq\pm 3.0$ ppm.

*Research supported in part by the National Science Foundation.

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Anomalous Temperature Dependence of Recoilless Emmission: ^{161}Dy in Molybdenum

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The Mössbauer recoilless fraction is found to show anomalous temperature dependence for a ^{161}Dy source in a molybdenum metal host, decreasing by almost a factor of 5 as the temperature decreases from 300 to 25°K, then increasing again at 4.2°K.

We have determined the Mössbauer absorption intensity for the 26-keV γ ray in ^{161}Dy under the following conditions: The source consisted of ^{161}Dy in a host composed of metallic molybdenum, satisfying the "thin-source" condition with negligible self-absorption, fastened to a vibrating mount in a cryostat which allowed the temperature to be varied between 300 and 4.2°K; the absorbers consisted alternatively either of Dy_2O_3 powder enriched to 90% ^{161}Dy, or DyF_3 powder with ^{161}Dy present in its natural (19%) abundance, held at room temperature throughout the measurements. This arrangement makes it possible to determine the recoilless fraction f_s for the ^{161}Dy molybdenum emitter as a function of temperature.

Molybdenum was chosen for the host because it has a high Debye temperature (459° at 4.2°K), no known phase transitions in the entire temperature range of the measurements, and, according to its crystal symmetry, no crystalline magnetic field or electric field gradient. It is expected to provide a substantial recoilless fraction, together with nuclear energy levels with small hyperfine splitting, and hence a somewhat broadened single-line source. In fact, it does.

The source was made by implanting 7-day ^{161}Tb in molybdenum foil with an isotope separator, an essential step in the experiment, for which we wish to express special thanks to Jerry Lerner of Argonne National Laboratory. We found no chemical method of inducing terbium to penetrate into molybdenum. The source was not subjected to any annealing process. The atomic concentration of ^{161}Tb at the start was approximately 0.3%, corresponding to a total activity of the order of 1 mCi. Data were acquired over a period of 2.5 half-lives for the molybdenum host, and 8 half-lives in another host. In no case was there any detectable change with time in the shape of the absorption spectra taken under corresponding temperature and absorber conditions. There appear to be no concentration-related effects.

The absorption spectrum at each temperature consisted of a single broad peak containing unresolved hyperfine components. With the oxide absorber, the full width at half-maximum (FWHM) increased from 14.2 mm/sec at 300° to 19 mm/sec at 4.2°. No large splitting of the kind reported, for example, for ^{166}Er in Zr occurred.[1] For the fluoride absorber, the observed FWHM was 6.4 mm/sec at 300°, increasing to approximately 12 mm/sec at 4.2°. For both absorbers, decreasing the source temperature from 300° to 4.2° resulted in an increased of the absorption resonance width by about 5.7 mm/sec. In each measurement, the velocity range was carried well beyond the absorption region, in order to be sure that no part of the spectrum was being overlooked, and also to provide a secure off-resonance base line for the f-value computations. In the case of the oxide absorber, with the source at 4.2°, for example, the velocity range chosen was ±65 mm/sec.

Every run included a simultaneous calibration, operation, and stability check with ^{57}Fe. The