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Dependence of X-Ray Yields in Argon, Krypton, and Xenon upon the Charge State of Fluorine Ions at 35.7 MeV*

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In high-velocity ion-atom single collisions, a strong dependence of target x-ray production cross sections upon the ionic charge state has been observed. Experiments were performed in thin gas targets of argon, krypton, and xenon with 35.7-MeV fluorine ions incident in charge states +5 to +9. Production cross sections for Ar *K*, Kr *L*, and Xe *L* characteristic lines increase by as much as a factor of 5 with increasing incident charge state but cannot be fitted by a q^2 dependence.

In this Letter we report that there is a strong dependence of target x-ray production cross sections upon the charge state of energetic heavy ions that produce vacancies in single collisions with target atoms. To our knowledge no such considerations of the effect of projectile charge state on target ionization cross sections (or possibly on fluorescence yields) have been made previously either experimentally or theoretically. At low energies where inner-shell promotion along molecular orbitals is important during the collision, it has been recognized that x-ray yields might be influenced by the ionic charge state of the projectile[1-4]; however, in this experiment the projectile and orbital-electron velocities are comparable and Coulomb excitation should be the dominant ionization mechanism. In this limit, different electron screening for different charge states may account for the observed cross-section dependence, but the importance of electron capture processes which are also strongly dependent upon ionic charge state cannot be ruled out. The x-ray yields from these single collisions vary by more than a factor of 5 for the ionization states studied and this charge-state dependence must be considered if any meaningful comparison is to be made between heavy-ion–induced vacancies and theoretical calculations.

The observation of characteristic x rays and satellite lines produced from collisions of energetic heavy ions incident upon solid targets has been reported by an increasing number of workers in recent years.[5] The extraction of inner-shell ionization cross sections for such collisions has been made by numerous authors using observed thin-target x-ray yields corrected for self-absorption and with estimates made for the fluorescence yields that may or may not be pertinent to the highly stripped collision products. Ionization cross sections which are somewhat less reliable have been determined from thick-target yields with heavy-ion stopping power formulas used to unfold the data. Inner-shell ionization cross sections derived from experiments[6,7] have been compared with the theoretical results calculated with the Born approximation[8] or the binary-encounter model[9] and the agreement is within an order of magnitude except at low velocities. The overall energy dependence of the experimental cross sections is reproduced theoretically at intermediate energies where the cross section rises rapidly to a peak near matching velocity, but at lower velocities does not agree as well. In attempts to reduce the discrepancy between theory and experiment below the peak of the excitation function, Brandt, Laubert, and

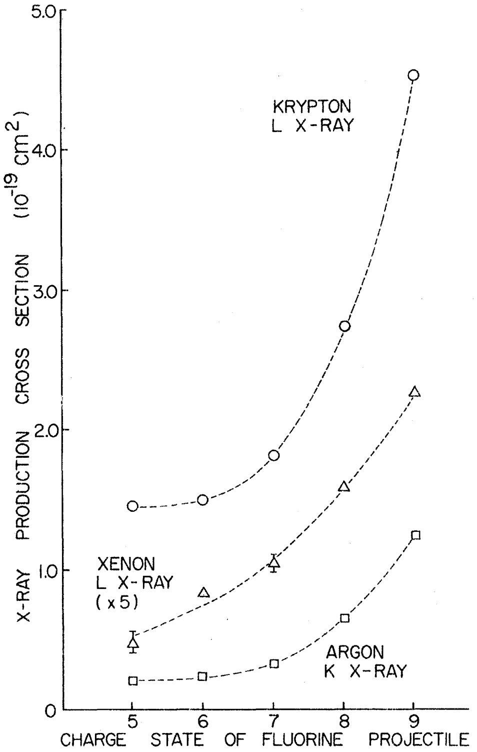

FIG. 1. X-ray production cross sections for 35.7-MeV fluorine ions of charge states +5 to +9 in single collisions with argon, krypton, and xenon.

Sellin[10] have included projectile deflection and binding-energy corrections and obtained good agreement with the heavy-ion ionization cross sections. At lower velocities, the importance of electron-promotion processes dominates[11] and calculations of ionization cross sections using Coulomb excitation are not applicable.

In experimental tests of the dependence of ionization cross sections on the projectile nuclear charge, it has been found[12, 13] that the predicted Z^2 dependence is violated by up to 30% even at energies in excess of 1 MeV/amu. This result has been obtained for H, He, and Li projectiles and has been interpreted[13] in terms of increased binding and initial-state polarization phenomena caused by the presence of the projectile nucleus within the target atom. In no previous work has the influence of electron screening related to the charge state of the projectile nucleus been considered.

In our experiment, we have measured absolute cross sections that are shown in Fig. 1 and presented in Table I for the yield of *K* x rays of argon and *L* x rays of krypton and xenon in single collisions of 35.7-MeV fluorine ions with atoms in thin gas targets. The incident fluorine charge states were from +5 to +9 and the gas targets were kept thinner than 10^{15} atoms/cm^2. Less than 5% of the incident beam underwent charge exchange in the interaction region. In addition, we have measured the charge-exchange cross sections for the same projectile-target combinations (presented in Table II) so that the criterion on the charge purity of each charge state has been rigidly maintained.

In performing this experiment, fluorine beams in charge states +5, +6, and +7 were accelerated to about 36 MeV in the Kansas State University tandem Van de Graaff. To obtain the 35.7-MeV fluorine ions of charge states +5 to +9, the analyzed beam was passed through a thin carbon foil and then the beam switching magnet was used to direct separately each charge state of the equilibrium distribution up the beam line through the gas cell. The beam either was collected in a large suppressed Faraday cup or was passed through a magnetic spectrometer into a position-sensitive detector for charge-state analysis. X rays produced in the 2-cm-long interaction region of the gas cell at target pressures less than 5 μm were detected in an 80-mm^2 Si(Li) detector mounted inside the gas cell and having a resolution of about 200 eV at 5.9 keV. The details of the determination of absolute x-ray yields[14] and

TABLE I. X-ray and ionization cross sections of argon (*K*), krypton (*L*), and xenon (*L*) by 35.7-MeV fluorine ions.

	X-ray production cross section (10^{-20} cm^2/atom)					Ionization cross section Experimental (10^{-19} cm^2/atom)					Ionization cross section Calculated (10^{-19} cm^2/atom)
q	9	8	7	6	5	9	8	7	6	5	
Ar *K*	12.3	6.52	3.25	2.24	2.14	10.1	5.34	2.67	1.83	1.75	3.11
Kr *L*	45.3	27.4	18.1	14.9	14.4	190	115	76.2	62.8	60.4	64.8
Xe *L*	4.50	3.16	2.08	1.66	0.92	4.37	3.07	2.02	1.61	0.89	3.06

TABLE II. Charge-exchange cross sections for 35.7-MeV fluorine ions in argon, krypton, and xenon.

	Total electron capture (10^{-17} cm^2/atom)				Total electron loss (10^{-17} cm^2/atom)			
q	9	8	7	6	8	7	6	5
Ar	3.1	2.1	1.2	0.6	0.1	1.1	1.5	3.3
Kr	5.0	4.1	1.8	0.7	0.1	1.4	3.7	7.0
Xe	3.4	2.6	1.2	0.4	0.2	1.7	3.9	9.3

the charge-exchange cross sections[15] in the gas target are reported elsewhere.

In the x-ray production cross section data given in Table I, the relative experimental uncertainties are approximately the size of the data points in Fig. 1 except in the two cases where error bars are shown on the points. The relative errors range from 3 to 15% and include uncertainties from statistics, beam collection, and gas pressure fluctuations. The accuracy of the absolute cross sections is estimated to be 20% for the largest cross sections with each gas and this is limited by gas pressure calibration, gas-cell flow correction, solid angle determination, and detector efficiency. Uncertainties in the measured charge-exchange cross sections given in Table II are approximately 20% except for the data with charge state 6 which have an error of up to 50%.

The very strong dependence of the x-ray production cross sections upon the charge state of the incident fluorine ions is evident in the data plotted in Fig. 1. Because the vacancy production with these high-velocity ions is dominated by Coulomb excitation processes, one might use a point-charge approximation to include electron screening as a reduction of the nuclear charge and expect that the ionization cross section would increase as the square of the ionic charge. Although this might be the case for the xenon L x rays, for which the approximately 15% relative uncertainty between the data points for the three lowest charge states masks the form of the charge dependence, it definitely does not describe the krypton L and argon K x-ray production.

For both the Ar K and Kr L x-ray lines, the production cross section rises considerably more rapidly than the second power of the charge for the higher charge states (where the point-charge approximation might be appropriate) and somewhat less rapidly for the lower charge states. A shell effect does not describe adequately the charge dependence shown in Fig. 1 for the production of these lines. In fact, a good one-parameter empirical fit to the data for these lines can be obtained by considering that each additional electron added to the ion screens the bare nucleus with diminishing returns. The form of the fit is given by the following equation:

$$\sigma_n = \sigma_0\left(1 - \alpha\sum_{p=1}^{n}\frac{1}{p\,!}\right), \qquad (1)$$

where $n = Z - q$ is the number of electrons screening the projectile nucleus, the fitting parameter α is 0.48 for argon and 0.40 for krypton, and σ_n is the ionization cross section determined from the experimental yields with incident fluorine ions having n electrons about the nucleus. The fit of the data with Eq. (1) is shown in Fig. 2 and although this equation is strictly empirical the goodness of fit might suggest a model to describe the projectile charge dependence in these collisions. The cross section σ_0 obtained with the fully stripped projectiles is used for normalization in Fig. 2 and Eq. (1), and within experimental error it is found that $\sigma_0 = 3\sigma_C$ where σ_C is the calculated ionization cross section for Coulomb excitation.[8] The factor of 3 discrepancy between σ_0 and σ_C is indicative of an inadequacy of the calculation of the ionization cross section or the atomic fluorescence yield[16] being used directly in considering inner-shell ionization for these heavy-ion collisions. Equation (1) does not fit the data for the xenon L line, either in the form of the charge dependence or in the magnitude of the cross section.

From Table II it is evident that the equilibrium charge of 35.7-MeV fluorine ions in these gases is near $q = 7$ and the calculated ionization cross section given in Table I is remarkably close to the value for this charge state. This suggests that experiments in thick targets with fluorine ions can be expected to be in rather good agreement with Coulomb excitation theory. However, any single-collision interpretation of such data

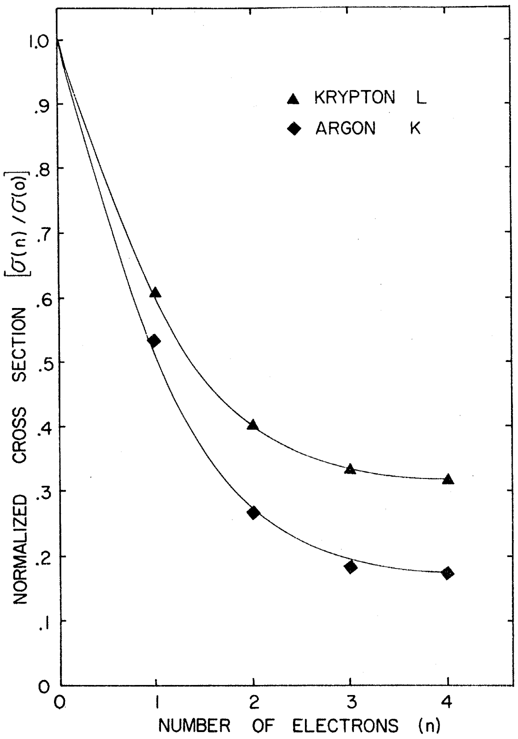

FIG. 2. Empirical fit with Eq. (1) of ionization cross sections of argon and krypton by 35.7-MeV fluorine ions with *n* electrons. The data is normalized to σ_0, the cross section obtained with the fully stripped fluorine nuclei.

must be somewhat suspect because of the charge-state dependence determined in this experiment.

Although there is a monotonic increase of both the ionization cross section shown in Table I and the electron capture cross section shown in Table II with the charge state of the incident ions, the capture cross section is between 1 and 2 orders of magnitude greater than the ionization cross section and no quantitative correlation between the two has yet been found. However, electron capture from inner shells can contribute significantly to single and multiple capture processes[17] and an experimental examination of this correlation with inner-shell target ionization may provide understanding of the strong charge dependence of the ionization cross sections. It should be pointed out that there is an anomaly in the electron capture cross sections in the different gases; the magnitude of the cross section in xenon is surprisingly lower than in krypton. However, the loss cross sections do increase with the atomic number of the target.

In conclusion, we have measured a strong dependence of cross sections for Ar *K*, Kr *L*, and Xe *L* x-ray production in single collisions upon the charge state of incident fluorine ions at 35.7 MeV. Such a charge-state dependence must be taken into account in any comparison of experimental and theoretical ionization cross sections.

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$\alpha^2 \ln\alpha^{-1}$ Recoil Corrections to the Hydrogen Hyperfine Structure and the Proton Polarizability*

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We report a calculation of the order $(m_e/m_p)\alpha^2 \ln\alpha^{-1}$ corrections to the hyperfine splitting ($\nu_{\rm H}$) of the hydrogen ground state. The result

$$\Delta\nu_{\rm H} = \tfrac{8}{3}(m_e/m_p)^2(1+m_e/m_p)^{-5}c\alpha^4 R_\infty \ln\alpha^{-1}[9+7\kappa(1+\kappa)-(m_e/m_p)\kappa(3+14\kappa)]$$
$$= 0.0016 \text{ MHz } (1.1 \text{ ppm})$$

is used in conjunction with the corresponding formula for the muonium hyperfine interval to obtain an estimate of the proton polarizability.

Comparison of the experimental and theoretical determinations of $\nu_{\rm H}$, the hydrogen ground-state hyperfine splitting (hfs), has traditionally served as an important check on quantum electrodynamics (QED). However, given the many other successful applications of QED and the theoretical uncertainties attributable to hadron dynamics, the role of the hydrogen hfs has shifted to one of serving as a probe of the proton's structure.[1] The proton structure enters the expression for $\nu_{\rm H}$ through recoil correction terms, δ_p, which arise from the proton's finite size and mass, and polarizability correction terms, δ_p', whose presence is due to the existence of proton excited states. While the former corrections can be calculated very reliably,[2] calculations of the latter are quite model dependent[3] since they require a knowledge of the spin-dependent virtual Compton scattering amplitude. Under these circumstances, the practice has been to extract δ_p' directly from the hyperfine data. With precision measurements of the muonium hyperfine interval,[4] $\nu_{\rm Mu}$, and the muon-to-proton magnetic-moment ratio,[5] μ_μ/μ_p, now available, the ratio[6] $\nu_{\rm Mu}/\nu_{\rm H}$ seems to be the best source for a value of δ_p'. This ratio is independent of radiative corrections to the electron moment and uncertainties in the fine-structure constant α. In addition to δ_p (and δ_p'), $\nu_{\rm Mu}/\nu_{\rm H}$ contains the muonium recoil correction δ_μ, which has been calculated[7,8] to relative order $(m_e/m_\mu)\alpha^2\ln\alpha^{-1}$. For a consistent determination of δ_p', δ_p should be evaluated to the same relative order, and this was the motivation for our calculation.

Our result for the frequency correction $\Delta\nu_{\rm H}$ of order $(m_e/m_p)\alpha^2\ln\alpha^{-1}$ is

$$\Delta\nu_{\rm H} = \tfrac{8}{3}(m_e/m_p)^2(1+m_e/m_p)^{-5}c\alpha^4R_\infty\ln\alpha^{-1} \times[9+7\kappa(1+\kappa)-(m_e/m_p)\kappa(3+14\kappa)] = 0.0016 \text{ MHz } (1.1 \text{ ppm}), \quad (1)$$

where $\kappa \simeq 1.79$ denotes the proton anomalous moment. Note that Eq. (1) reduces to the previous results obtained for muonium[8] and positronium[9] in the appropriate limits. When this contribution is included in the theoretical expression[10] for $\nu_{\rm H}$, the recoil-dependent portion δ_p has the numerical value

$$\delta_p = -34.6 \pm 0.9 \text{ ppm} + 1.1 \text{ ppm}. \quad (2)$$

The first term on the right-hand side of Eq. (2), which represents the leading size and recoil correction, is well established,[1,2] and the ±0.9 ppm allows for uncertianties in the proton form factors. The remaining contribution to δ_p, of order $(m_e/m_p)\alpha^2\ln\alpha^{-1}$ (which was computed here) represents a recoil effect arising from low-momentum components of the Bethe-Salpeter wave function. Thus, although we use a proton vertex of the form

$$\Gamma_\mu(p+k,p) = ie[\gamma_\mu - F_2(k^2)\sigma_{\mu\nu}k_\nu/2m_p], \quad (3)$$

only $F_2(0)=\kappa$ contributes to the result[11] Eq. (1). As a consequence, no further uncertainty is introduced into the expression for δ_p.

The calculation is performed using a perturbation treatment[12] of the Bethe-Salpeter equation, with the wave function being obtained by a single