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## Optical-Model Information Provided by Scattering from Aligned Targets\*

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Data on the scattering of  $\alpha$  particles from aligned  $^{165}\text{Ho}$  was recorded over the energy range  $14 \leq E_\alpha \leq 23$  MeV. Optical-model calculations are presented. It is shown that the data provide a clear resolution of the optical-model ambiguities found in previous analyses of  $\alpha$  scattering from rare-earth nuclei.

The parameter ambiguities which arise in optical-model fits to scattering data for strongly absorbed particles are well known and have been discussed by several authors.<sup>1,2</sup> At low bombarding energies where only the tail of the nuclear potential is probed, the principal source of ambiguity results from changes in the parameters which leave the potential in this region unaltered. The tail region itself, however, is well determined.<sup>1,3</sup> At higher energies ambiguities of a discrete nature are found as well as a continuous  $VR^n$  ambiguity.<sup>4,5</sup> It has been shown<sup>6</sup> that at very high energies, near 80 MeV in the case of  $^{24}\text{Mg}$ , only the continuous type of ambiguity remains. In this case, a good fit to the data can be obtained with almost any strength potential by suitable adjustment of the other parameters. These characteristics are consistent with the assumption that the scattering is only sensitive to a portion of the nuclear surface.

The resolution of these ambiguities has been the subject of much investigation. Initially, it was believed that including inelastic scattering data with a coupled-channel analysis would be of value. Austern and Blair<sup>7</sup> have shown, however, that optical potentials which reproduce the elastic scattering must be expected to reproduce the inelastic angular distributions. Several authors<sup>8</sup> have demonstrated that partial resolution of the ambiguities is obtained by including large-angle scattering data. Simultaneous fitting of data from several target nuclei at various energies, constraining the parameters to vary smoothly, offers another means of reducing the ambiguities.<sup>4,5</sup>

In the present Letter we propose that scattering data from aligned targets offer additional information which can lead to better determination of the optical parameters. The potential for a permanently deformed nucleus can be written in the form

$$V(r, \theta) = -(U + iW)[1 + \exp(\{r - R_0[1 + \beta Y_2^0(\theta)]\}/a)^{-1} + V_c(r, \theta)], \quad (1)$$

where  $V_c(r, \theta)$  is the potential due to the charge distribution and  $\beta$  is the nuclear deformation parameter.

For  $r \gg R$  the real part of the potential can be expanded as

$$V(r, \theta) = -U[1 + \beta(R_0/a)Y_2^0(\theta)] \exp(-r/a) \exp(R_0/a) = A(U, R_0/a, \beta, \theta) \exp(-r/a), \quad (2)$$

where the potential has been written as the product of a strength and a form factor. In the case of scattering from an unoriented target,  $\theta$  is averaged over all directions. If  $\beta$  is determined by another method, it is possible to determine at most two of the three remaining parameters uniquely. Data on the scattering from an aligned target, which select a particular value of  $\theta$ , are capable, in principle, of resolving this ambiguity. Although the argument is an oversimplification, it demonstrates that aligned-target data offer more than a consistency check of data from an unaligned target.

Aponick *et al.*<sup>9</sup> have made a careful search for parameters which describe the scattering of 27.0-, 30.0-, and 32.5-MeV  $\alpha$  particles from <sup>148</sup>Sm and other nuclei in the rare-earth region. Two completely different parameter sets were found which gave equally good fits to all the data. We present data on the scattering of  $\alpha$  particles from aligned and unaligned <sup>165</sup>Ho and show that the data establish a clear choice between the two parameter sets of Aponick *et al.*

The experimental apparatus involving the aligned <sup>165</sup>Ho target was similar to that employed previously.<sup>10</sup> The target, a single crystal of holmium metal, 1 cm  $\times$  0.7 cm  $\times$  0.35 mm, was attached with Cd-Bi eutectic solder to the mixing chamber of a <sup>3</sup>He-<sup>4</sup>He dilution refrigerator with the *c* axis parallel to the direction of the  $\alpha$ -particle beam. At low temperatures, the crystal structure of holmium results in a nuclear alignment which is negative and symmetric with respect to the *c* axis of the crystal.<sup>11</sup> The temperature of the mixing chamber was monitored by a carbon resistor (Speer grade 1002, 100  $\Omega$ ) and was held at 200°K during the aligned-target measurements by suitably adjusting the beam intensity to correspond to a heat input of 1440 erg/sec. A temperature drop of 0.060°K between the target surface and the copper mixing chamber was estimated from the known thermal conductivity of holmium metal at low temperatures<sup>12</sup> and separate measurements on the conductivity of holmium-copper solder junctions. This led to the value (0.260  $\pm$  0.020)°K for the temperature of the target surface.

$\alpha$ -particle beams of energies 20 and 24 MeV were provided by the Stanford University FN tandem accelerator. As in Ref. 10, a beam-diffusing foil was inserted 184 cm upstream from the

target to produce a stable and uniform beam at the target of a few picoamperes intensity. The scattered  $\alpha$  particles were detected by a matrix of four intrinsic Ge detectors<sup>13</sup> at an average scattering angle of 160°. The beam intensity was monitored by detecting the scattering from a nickel foil which also served as the entrance window to the cold section of the cryostat. The intrinsic Ge detectors were maintained at a temperature of 10°K and gave reliable operation throughout the course of the experiment showing a resolution of 50 keV or better to 5.48-MeV  $\alpha$  particles from an <sup>241</sup>Am source.

The aligned cross section was not measured directly, since it was more convenient to make a comparison between the relative cross sections at two different temperatures, 0.26 and 2.15°K. The corresponding values of the nuclear alignment parameter  $B_2/B_2(\text{max})$  are -0.291 and -0.010. Since the nuclear alignment is symmet-

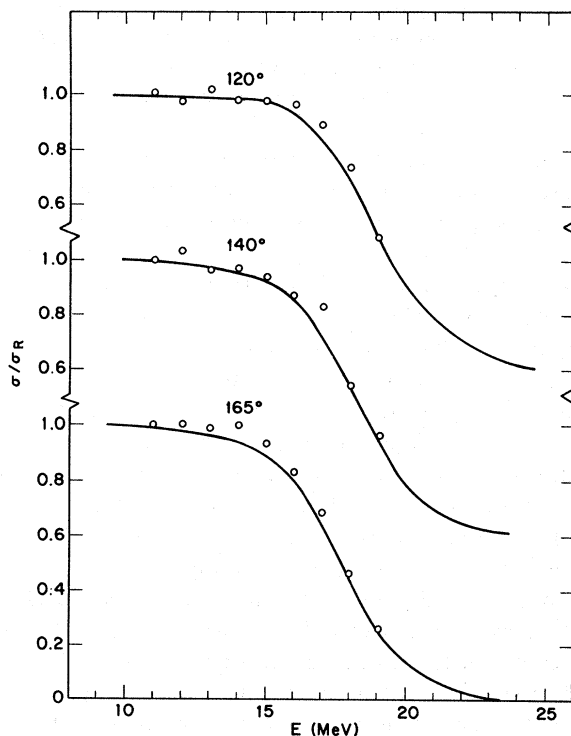


FIG. 1. Differential cross-section data for the scattering of  $\alpha$  particles by an unaligned <sup>165</sup>Ho target. The target was a polycrystalline foil 300  $\mu\text{g}/\text{cm}^2$  thick. The curves were calculated with the optical parameters of Ref. 9. Parameter sets I and II gave essentially identical fits.

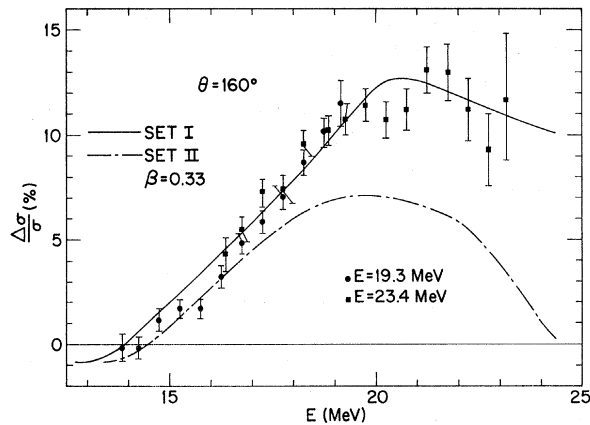


FIG. 2. Aligned-target cross-section effect as a function of  $\alpha$  bombarding energy. The incident beam energies 19.3 and 23.4 MeV correspond to the beam energies 20.0 and 24.0 MeV corrected for energy degradation in the foils. The calculated curves were obtained with parameter sets I and II of Ref. 9. Each point has an associated energy spread of  $\pm 250$  keV, and an angular spread which depends upon the thickness of target which was traversed. This angular spread was  $\pm 15^\circ$  in the worst case.

ric with respect to the beam direction, the difference in cross section at the two temperatures can be expressed in the form

$$\frac{\sigma_{0.26^\circ\text{K}} - \sigma_{1.75^\circ\text{K}}}{\sigma_{1.75^\circ\text{K}}} = \frac{\Delta\sigma}{\sigma} = \sum_{M>0} A_M \sigma_M(\theta), \quad (3)$$

where  $\sigma_M(\theta)$  is the cross section for scattering at angle  $\theta$  by a nucleus in magnetic substate  $M$ . The coefficients  $A_M$  can be calculated from knowledge of the target temperature, the holmium crystal structure, and the parameters of the nuclear hyperfine Hamiltonian. In the present case, their values were

$$A_{7/2} = -0.146, \quad A_{5/2} = -0.049,$$

$$A_{3/2} = 0.068, \quad A_{1/2} = 0.127.$$

The cross sections  $\sigma_M(\theta)$  were computed in the coupled-channel formalism for an optical potential of the form (1).

Figure 1 shows cross-section data measured in a separate experiment with an unaligned polycrystalline foil  $300 \mu\text{g}/\text{cm}^2$  thick. In Fig. 2 the thick-target data on  $\Delta\sigma/\sigma$  are presented, organized into energy intervals approximately 500 keV wide; i.e., each cross-section point represents an average over an energy interval  $\pm 250$  keV. There is also an angular spread associated with the points in Fig. 2 due to the finite detector size and to small-angle multiple scattering which

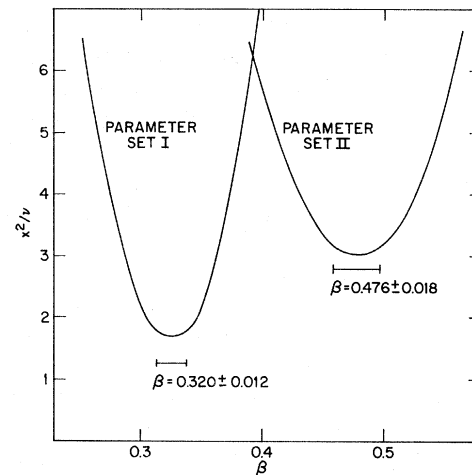


FIG. 3. Plots of  $\chi^2$  versus  $\beta$  for the fits to the aligned-target cross section. The unaligned-target data are insensitive to the value of  $\beta$ , and have not been included in the above curves.

occurs as the beam traverses the thick target. This angular spread is typically of the order of  $\pm 10^\circ$ . The energy range from 14 to 23 MeV was covered by choosing incident beam energies of 20 and 24 MeV which produce overlapping data points in the region 16.5 to 19.0 MeV. The good agreement between the data taken at the two bombarding energies serves as a consistency check on the data reduction and also indicates that there is no appreciable degradation in the nuclear alignment due to imperfections at the surface of the crystal.

Both parameter sets I and II give equally good fits to the data of Fig. 1; the two calculated curves are indistinguishable on the scale of the figure. Only parameter set I gives an acceptable fit to the aligned-target data, however, as can be seen from Fig. 2. The nuclear deformation parameter,  $\beta$ , was set to the value 0.33 obtained from Coulomb excitation<sup>14</sup> in the above calculations. The result of treating  $\beta$  as a free parameter is shown in Fig. 3. Although the fit obtained with parameter set II is improved, it is still clearly inferior to the fit obtained with parameter set I. The best value of  $\beta$  using parameter set I,  $0.320 \pm 0.020$ ,<sup>15</sup> is in excellent agreement with the results of Coulomb excitation, 0.33, and photoneutron cross-section measurements,  $0.319 \pm 0.003$ .<sup>16</sup> It is worth noting that the sign of  $\beta$  is uniquely determined since the curves of Fig. 2 would have the opposite sign for an oblate deformation.

The results of the present experiment and anal-

ysis demonstrate that scattering from aligned targets offers additional information for the determination of nuclear characteristics. Specifically, within the framework of the optical model, it is shown that the data resolve the discrete ambiguity in the optical parameters found by Aponick *et al.*<sup>9</sup> A more detailed analysis investigating the continuous ambiguities would be useful. The results of Brissaud *et al.*<sup>17</sup> seem to indicate that a real potential strength in excess of 100 MeV is required to describe the scattering of 166-MeV  $\alpha$  particles from Sn and Pb, whereas the real potential in parameter set I is only 50 MeV. It is therefore of great interest to extend aligned target measurements to higher bombarding energies.

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## Two-Nucleon Transfer with Heavy Ions\*

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A distorted-wave Born-approximation calculation of two-particle transfers between heavy ions is presented which does not treat the transferred nucleons as a cluster. Application is made to ( $O^{18}, O^{16}$ ) reactions on the nuclei  $Mo^{92,94,96,98}$ .

The transfer of one or more nucleons in reactions involving heavy-ion projectiles has attracted considerable attention recently.<sup>1-6</sup> Direct-reaction codes have been constructed and applied<sup>7,8</sup> to the transfer of a single nucleon between heavy ions. These single-particle codes have been used to analyze multinucleon transfer in terms of a cluster expansion.<sup>4</sup> The present Letter describes a proper distorted-wave calculation of the cross section for two-nucleon transfer between heavy ions. In this calculation the nucleon coordinates enter separately, and it is therefore possible to

insert the full two-particle spectroscopy into the transfer amplitude. The resultant two-particle code is used to analyze a series of ( $O^{18}, O^{16}$ ) reactions on molybdenum isotopes<sup>2</sup> with apparent success. One important consequence of treating the wave function of each transferred nucleon correctly, i.e., an avoidance of any cluster assumption, is a surprisingly strong state dependence.

The reactions considered are of the type  $(A + 2N) + B \rightarrow A + (B + 2N)$ , where  $2N$  stands for two like nucleons and  $A, B$  are inert cores. The "exact" distorted-wave Born-approximation (DWBA) am-