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[9]Note that the dipole energy is proportional to minus an expression identical to F, but with Λ_{ij} replaced by $\delta_{ij}-3k_ik_j/k^2$.

[10]As well as an NMR shift comparable to the AM state.

[11]In fact, there is probably a continuous transition from one type of state to the other as the field increases, but our argument still goes through qualitatively.

[12]A further possibility is that particle-particle scattering may upset the stability of the BW state relative to the AM state.

[13]The ratio $\nu_\parallel{}^2/(\nu_L{}^2-\nu_S{}^2)$, when $\nu_\parallel$ is the frequency at which rf radiation parallel to the external field is absorbed, should be an important indicator of the precise nature of the state (its l value etc.). For an isotropic state it is clearly 1; for an AM-type P state with $R(k)=1$ it is $\frac{64}{15}$, and this ratio is likely to be far less sensitive to the precise form of $R(k)$ than the absolute magnitude of $\nu_L{}^2-\nu_S{}^2$.

Precise Measurement of Effective Mass of Positive and Negative Charge Carriers in Liquid Helium II

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The resonance of charge carriers trapped under the surface of He II is observed at 200 MHz. The effective masses are found to be $m_+{}^*=(45\pm2)m_4$ and $m_-{}^*=(76\pm2)m_4$, where m_4 is a bare ^{4}He mass. The ionic radii are deduced to be $R_+=6.0\pm0.1$ Å and $R_-=11.4\pm0.1$ Å.

We have used a resonance technique capable of an accuracy of better than 1% to measure the effective masses of negative and positive ions in liquid He II at temperatures below 0.8 K. The results are consistent with the bubble[1] and snowball[2] models for these ions, although the radius deduced for negative ions is rather low compared with other estimates. The technique is complementary to that of Dahm and Sanders[3] as it depends on having a very narrow-frequency response function for the electric susceptibility, whereas theirs requires a very broad response (they made a radiofrequency loss measurement far in the tail of the response function).

Ions can be trapped beneath a surface of He II by the combination of their image force and an electric field E applied perpendicular to the surface.[4] If x is the distance below the surface and ϵ the dielectric constant of the liquid, the potential may be written

$$V(x)=A/x+eEx, \quad A=e^2(\epsilon-1)/4\epsilon(\epsilon+1). \tag{1}$$

At sufficiently low temperatures the ion vibrates about the potential minimum with a frequency ω_0 determined by the second derivative,

$$\omega_0{}^2=2(eE)^{3/2}/m^*A^{1/2}, \tag{2}$$

where m^* is the effective mass for oscillatory motion of the ion in the liquid. Figure 1, appropriate to the positive ion, illustrates the orders of magnitude. The sharpness of the resonance is determined by $\omega_0\tau$, where τ is the momentum relaxation time; from the known mobilities[5] we see that, for $\omega_0/2\pi=200$ MHz, $\omega_0\tau>3$ below 0.9 K for both signs of ion and can reach $\omega_0\tau>100$ below 0.6 K. The experiment consists in finding the applied field E at which the system resonates with an exciting x-directed radio-frequency field of about 200 MHz. As ϵ is known to good precision,[6] the relation (2) determines m^* directly. This method, which has also been proposed by Shikin,[7] permits a very precise measurement of m^* when $\omega_0\tau\gg1$.

We measure the absorption as a function of E with a 200-MHz superheterodyne spectrometer[8]

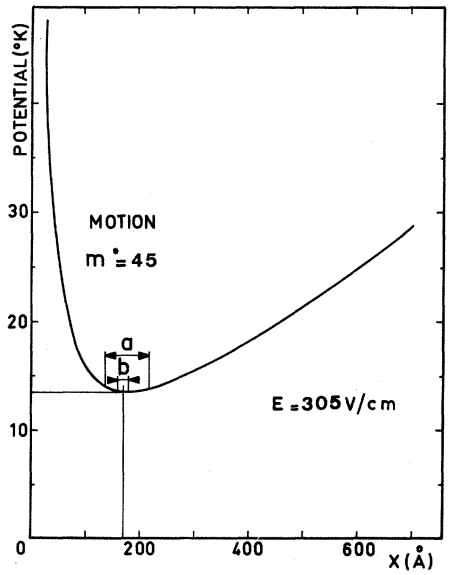

FIG. 1. Potential for a positive ion at a distance x under the liquid-vapor surface showing (a) rms thermal (0.7K) and (b) zero-point motions.

whose resonant circuit is illustrated in Fig. 2. The sample chamber of gold-plated copper contains the capacitive part of the LC resonant circuit and forms the lower end of the pot of ^{3}He by which it can be cooled in a one-shot fashion to better than 0.5 K for the typical rf power of 0.3 mW. The three-electrode plate P_2, grid G, and source S are insulated from the main body so that their dc potentials may be freely adjusted. The rf is coupled to the capacitance P_2-G via the large capacitances P_1-P_2 and G body. P_1, P_2, and G are gold plated, while S is plated with a tritiated Ti β source of 1 Ci. The liquid half-fills the space P_2-G. On applying the appropriate potentials between P_2 and G and between G and S, the potential well is charged to the point where the surface charge annuls the electric field between G and the surface (typical ion densities are $\sim 3 \times 10^7$ cm^{-2}); reversing the sign of $V_{G\text{-}S}$ then stops further charging independent of the magnitude of $V_{G\text{-}P}$ which governs E. At our temperatures and field, ions of either sign have trapped lifetimes much longer than the scan times (up to 10 min).[9,10] A low-frequency (683-Hz) electric-field, modulation is superimposed on the scan of $V_{P_2\text{-}G}$ so that phase-sensitive detection can be employed. The derivative of the absorption signal is thus plotted against $V_{P_2\text{-}G}$ on the X-Y recorder.

Figure 3 shows the experimental resonance curves observed at $T = 0.7$ K and $\nu = 208$ MHz. The analysis of these curves is contained in

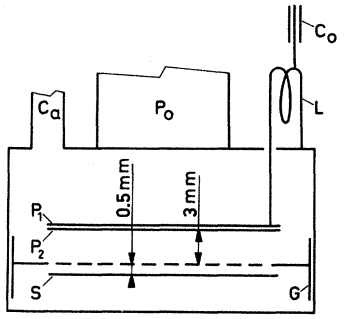

FIG. 2. Schematic representation of resonant circuit with sample chamber. C_0, coaxial cable carrying 200 MHz. C_a, ^{4}He-filled cappillary. P_0, ^{3}He pot. P_1, P_2, G, and S, electrodes (see text). L, self-inductance and and autotransformer.

Table I.

E_{res} is the observed electric field at resonance, m_{harm}* the effective mass calculated from (2), m_{corr} the effective mass corrected for anharmonicity, ΔE_{obs} the observed linewidth, ΔE_{hom} and ΔE_{anharm} the calculated linewidth deduced from the mobility and the anharmonicity of the well, respectively. The uncertainty of our results has been evaluated from the error in the evaluation of the electric field, i.e., the distance between the electrodes P_2 and G. The potential (1) is not perfectly harmonic so that the thermal spread of the ions about the minimum has three consequences: a line shift which gives a correction of about 4% to the mass; a broadening also by about 4% which is not, however, sufficient to explain the observed width; and the appearance of the first harmonic at a field $E' = E/2^{4/3} = E/2.51$ with a relative intensity of about 7%.

We observe that the negative-ion linewidth

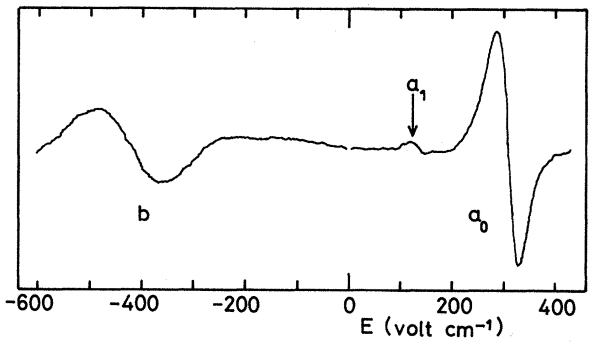

FIG. 3. Resonance lines of (a_0) positive and (b) negative ions at $\nu = 208$ MHz, $T = 0.7$ K. In the case of positive ions the first harmonic (a_1) can be seen.

TABLE I. Observed positions (E_{res}) and widths (ΔE_{obs}) of ion resonances with uncorrected m_{harm}* calculated from formula (2), m_{corr}* corrected for anharmonicity, width contribution ΔE_{hom} calculated from mobility values, and ΔE_{anharm} calculated from anharmonicity.

	E_{res} (V/cm)	m_{harm}*	m_{corr}*	ΔE_{obs} (V/cm)	ΔE_{hom} (V/cm)	ΔE_{anharm} (V/cm)
+ion	305	$47m_4$	$(45\pm2)m_4$	39	4	17
−ion	427	$78m_4$	$(76\pm2)m_4$	90	19	20

varies with the density of trapped ions, while that of the positive ions does not. At this time neither the absolute value of the linewidth nor its temperature variation (rather linear and not exponential as the homogen linewidth would be) are explained. From our data we can deduce the negative and positive ionic radii. At low temperature, $T \ll T_\lambda$, we may calculate the effective mass of a sphere of radius R using hydrodynamics,[12] but neglecting the contribution of normal fluid:

$$m_+ = \tfrac{4}{3}\pi\rho_{sol}R_+^3 + \tfrac{2}{3}\pi\rho_{liq}R_+^3,$$

$$m_- = \tfrac{2}{3}\pi\rho_{liq}R_-^3.$$

Table II gives the comparison of our results with preceding experiments and a theoretical evaluation.

We conclude that our results are consistent with the theoretical model of the ball of solid helium for positive ions and the bubble model for negative ions.

We are now undertaking a series of experiments to elucidate the observed linewdith, which may shed light on the coupling of the mode of vibration of the ions with other excitations.

We should like to express our thanks to all our colleagues of the magnetic-resonance group and particularly to Mr. A. Abragam, Mr. G. Delmas, Mr. M. Goldman, Mr. A. Landesman, and Mr. J. Winter.

TABLE II. Comparison of radii for both signs of the ion deduced from this experiment, from microwave loss (Ref. 3), vortex escape (Ref. 13), mobility in phonon dominated region (Ref. 14), and theoretical estimations (Refs. 15–17).

R_+ (Å)	R_- (Å)
6.0 ±0.1 (this expt)	11.4±0.1 (this expt)
5.8 ±0.2 [a]	13 ±4 [a]
6.44±0.1 [b]	14.5±0.4 [b]
5.0 ±0.1 [c]	
6.3 [d]	12.5 [e]

[a] Ref. 3.
[b] Ref. 13.
[c] Ref. 14.
[d] Ref. 15.
[e] Ref. 16, 17.

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