PHYSICAL REVIEW LETTERS

VOLUME 29 | 30 OCTOBER 1972 | NUMBER 18

Parity Unfavoredness and the Distribution of Photofragments*

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(Received 11 September 1972)

The angular distribution of photodisintegration products, previously resolved into contributions with different angular momentum transfers $\vec{j}_t = \vec{j}_\gamma - \vec{l}$, is analyzed further according to the parity of j_t. Odd values of $j_t + j_\gamma + l$ correspond to parity-unfavored transitions which result from effectively pseudotensor interactions with the unobserved fragments. In electric dipole processes, these transitions yield an angular distribution with the fixed asymmetry parameter $\beta_{\rm unf} = -1$ while the distribution of parity-favored transitions consists of three standard functions weighted by dynamical factors.

An earlier paper[1] on angular distribution theory has stressed the role of the angular momentum transfer

$$\vec{j}_t = \vec{j}_\gamma - \vec{l} = \vec{J}_B - \vec{J}_A, \tag{1}$$

where $\vec{j}_\gamma$ is the angular momentum of the incident radiation and $\vec{l}$ the orbital momentum of an observed reaction product, while $\vec{J}_A$ and $\vec{J}_B$ are the total angular momenta of *all* other reactants in the initial and final states, respectively. (The unobserved spin of the observed reaction product is thus included in $\vec{J}_B$.) The differential cross section for ejection of particles with momentum $\vec{k}$ resolves into a sum of terms

$$d\sigma/d\hat{k} = \sum_{j_t} d\sigma(j_t, \vec{k})/d\hat{k}, \tag{2}$$

provided only that no observation is made bearing on the orientations of $\vec{J}_A$ and $\vec{J}_B$.

This note points out that alternate terms of (2), with odd values of

$$j_t + l + j_\gamma, \tag{3}$$

correspond to *parity-unfavored* transitions for which $d\sigma(j_t, \vec{k})/d\hat{k}$ vanishes in certain directions.[2] In particular, for electric dipole photoprocesses $A(\gamma, \varphi)B$ produced by linearly polarized radiation, the parity-unfavored transitions are restricted to $j_t = l$; they contribute no fragment φ in the direction of polarization $\hat{E}$ and yield a partial cross section

$$\left.\frac{d\sigma(j_t)}{d\hat{k}}\right|_{j_t = l} \propto 1 - P_2(\hat{E}\cdot\hat{k}) \propto |\hat{E}\times\hat{k}|^2. \tag{4}$$

Note that the parity of l (though not its magnitude) is fixed by the parities of A, γ, and B.

Parity-unfavored contributions to a reaction have effectively pseudovector or, more generally, pseudotensor character. That is, they occur when the reaction imparts to the unobserved fragment B a parity $-(-1)^{j_t}$ along with the angular momentum $\vec{j}_t$, in addition to the parity and momentum initially possessed by A. Their role has experimentally observable consequences. Consider, for example, the photoionization process $\mathrm{Xe}(\gamma, e)\mathrm{Xe}^+$ with Xe^+ in the doublet levels ${}^2P_{3/2,1/2}{}^\circ$. Here we have $j_\gamma = 1$, the photoelectron has $l = 0$ or 2, and $j_t = 1$ for parity-favored or 2 for parity-unfavored transitions. The latter can be shown[3] to yield a small but measurable difference between the angular distributions of the photoelectron groups with energies corresponding to the two levels of the ion; they involve both electrostatic quadrupole interactions and pseudovector spin-orbit coupling. Another example is afforded by the inelastic collision process of Ref. 2, namely, He(e,e')He** with He** in $2p^{2}\,{}^3P^e$; here the parity-unfavored character results from a Coulomb interaction leading to spin exchange.

These examples indicate that parity-unfavored transitions can arise from a variety of dynamical mechanisms. Our main purpose here is to point out the existence and operational definition of these transitions and the observability of their manifestations. We are not quite able to characterize their dynamical origin precisely and all-inclusively. We know, however,[3] that they can arise not only from pseudotensor interactions but also from secondary angular momentum transfers between photofragment and residue whose vector addition alters the magnitude of j_t by an amount unrelated to parity. We also know that the approximations made in simple models of photoionization[4] have the effect of excluding parity-unfavored transitions, by disregarding not only spin-orbit coupling—a weak force—but also the stronger electrostatic repulsions that separate different terms of the same configuration and yield secondary momentum transfers. Thus we anticipate that manifestations of parity unfavoredness should be readily obserable in the distribution of photoelectrons.

According to Eq. (14) of Ref. 1, the cross section for photoemission in a direction $\hat{k}$ can be expressed as

$$\frac{d\sigma}{d\hat{k}}=\frac{3\pi\lambda\!\!\!^{-2}}{2J_A+1}\sum_{j_t}\sum_{l,l'}(J_Bl|\bar{S}(j_t)|J_Aj_\gamma)(J_Aj_\gamma|\bar{S}(j_t)^\dagger|J_Bl')\Theta(j_t;j_\gamma m_\gamma,ll';\theta), \quad (5)$$

where $\lambda\!\!\!^-$ is the photon wavelength, $\bar{S}(j_t)$ is reduced S-matrix operator, and l and l' are alternative values of l. For an electric dipole ($E1$) process we have $j_\gamma=1$ and $m_\gamma=0$ with respect to the polarization direction $\hat{E}$. The geometrical functions Θ are then given by

$$\Theta(j_t;10,ll';\theta)=(-1)^{j_t}\frac{[(2l+1)(2l'+1)]^{1/2}(2j_t+1)}{4\pi}\sum_{K=0,2}\begin{Bmatrix}1&1&K\\ l&l'&j_t\end{Bmatrix}(l0,l'0|K0)P_K(\cos\theta)(K0|10,10), \quad (6)$$

where $\cos\theta=\hat{E}\cdot\hat{k}$ and the condition $K=0, 2$ derives from the triangular and symmetry properties of the Wigner coefficient $(K0|10,10)$; this condition embodies Yang's theorem on angular correlations.[5]

For parity-unfavored transitions, we obtain special properties of the function Θ of Eq. (6) through the following argument. At $\cos\theta=\pm1$, the value $P_K(\pm1)=1$ can be factored out of the sum over K, after which the sum is carried out analytically[6] to yield

$$\Theta(j_t;10,ll';0\text{ or }\pi)=(4\pi)^{-1}(2j_t+1)(10,j_t0|l'0)(l0|10,j_t0). \quad (7)$$

The Wigner coefficients in (7) vanish for odd values of $l+1+j_t$, that is, when $j_t=l=l'$. The fact that Θ vanishes at $\theta=0$ and $\theta=\pi$, combined with its integral value obtained from the $K=0$ term, gives the complete angular dependence

$$\Theta(j_t;10,ll';\theta)=\tfrac{1}{12}\pi^{-1}(2j_t+1)[1-P_2(\cos\theta)]=\tfrac{1}{8}\pi^{-1}(2j_t+1)|\hat{E}\times\hat{k}|^2, \quad (8)$$

for all parity-unfavored $E1$. [The result (7) can be generalized to other processes.]

Parity-favored transitions yield instead two alternative functions Θ for processes diagonal in l,l', i.e., with $l=l'=j_t\pm1$,

$$\Theta(j_t;10,j_t+1\,j_t+1;\theta)=\tfrac{1}{12}\pi^{-1}(2j_t+1)[1+(j_t+2)(2j_t+1)^{-1}P_2(\cos\theta)], \quad (9)$$

$$\Theta(j_t;10,j_t-1\,j_t-1;\theta)=\tfrac{1}{12}\pi^{-1}(2j_t+1)[1+(j_t-1)(2j_t+1)^{-1}P_2(\cos\theta)], \quad (10)$$

and one interference function,

$$\Theta(j_t;10,j_t+1\,j_t-1;\theta)=-\tfrac{1}{4}\pi^{-1}[j_t(j_t+1)]^{1/2}P(\cos\theta). \quad (11)$$

The complete angular distribution of $E1$ photodisintegration fragments is thus a superposition of the four basic distributions (8)–(11). The dynamics of the process determines only the relative weights of these four distributions. The phases of the scattering matrix elements of Eq. (5) affect only the weight of the interference term (11). *In the absence of parity-unfavored transitions* and *for each value of* j_t one obtains from the weighting a distribution $1+\beta(j_t)P_2(\cos\theta)$ with

$$\beta(j_t)_{\text{fav}}=\frac{(j_t+2)|\bar{S}_+|^2+(j_t-1)|\bar{S}_-|^2-3[j_t(j_t+1)]^{1/2}[\bar{S}_+\bar{S}_-^\dagger+\bar{S}_+^\dagger\bar{S}_-]}{(2j_t+1)[|\bar{S}_+|^2+|\bar{S}_-|^2]}, \quad (12)$$

where $\bar{S}_\pm$ stands for the matrix elements of $\bar{S}(j_t)$ with $l = j_t \pm 1$. Formulas with the structure of (12) have been obtained in the photoionization literature for a number of special examples and dynamical models. For example, the Bethe-Cooper-Zare formula[4] pertains to spin-independent processes without any noncentral electron-core interaction which would result in secondary momentum transfers; thus j_t coincides with the initial orbital momentum l_i and the formula results by setting in (5) and (12)[3]

$$(l|\bar{S}(l_i)|l_i) = i^{-l} e^{i\delta_l} (l\|C^{[1]}\|l_i) R(l, l_i), \tag{13}$$

where R is a radial dipole matrix element and the other symbols have standard meaning.

As a further illustration, let us consider the photoionization of Xe,

$$\mathrm{Xe}(^1S_0) + \gamma(E1) \to e(l = 0,2) + \mathrm{Xe}^+(^2P_{3/2,1/2}{}^\circ), \tag{14}$$

in somewhat greater detail. Here we have $J_A = 0$ and $j_\gamma = 1$. The orbital quantum numbers $l = 0,2$ are selected by parity and angular momentum conservation and the photoelectron's spin combines with the angular momentum of Xe^+ to yield the unobserved $\vec{J}_B$, with values 0, 1, or 2. The allowed values of j_t are 1 (parity favored) and 2 (parity unfavored) because $j_t = 0$ and ≥ 3 do not satisfy the triangular conditions implied by Eq. (1); the value $j_t = 2$ occurs *only* for transitions to the lower level of the Xe^+ doublet, with $j = \frac{3}{2}$. Thereby the angular distributions of the two groups of photoelectrons differ at least by an amount corresponding to the parity-unfavored transitions resulting from breakdown of L-S coupling; this breakdown is thus far known to be non-negligible in Xe from other spectroscopic evidence.[7] For each group of photoelectrons, the asymmetry parameter can be cast into the form

$$\beta = \frac{\sigma(j_t = 1)\beta(j_t = 1) - \sigma(j_t = 2)}{\sigma(j_t = 1) + \sigma(j_t = 2)}, \tag{15}$$

where $\sigma(j_t)$ is the integrated cross section for each j_t value, $\beta(j_t = 1)$ has the form (12), and $\sigma(j_t = 2)$ vanishes for the $j = \frac{1}{2}$ group.

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*Work supported by the U. S. Atomic Energy Commission under Contract No. C00-1674-70.

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