cause the phase-space variations are small in our experimental conditions. It cannot be seen at 50 MeV/c because the single-nucleon contribution is too large. At $P_R = 300$ MeV/c (see Table I) preliminary experimental values are in good agreement with the 200 MeV/c one, while the impulse approximation predictions[1] are smaller by an order of magnitude.

The observed width of the structure ($\simeq 35$ MeV) is determined by the convolution of the natural width with that due to the motion of the nucleon pairs, as shown by Eq. (6). A rough evaluation of the latter gives about 40 MeV, showing that the natural width must be smaller than this value.

In relation to this narrow width we have considered possible selection rules inhibiting the decay. We note that on the deuteron (isospin $T=0$), the photon can produce only $T=1$ excited states, whereas in ^{4}He a pair of nucleons can be found in a $T=1$ state and, thus, the photon can excite this pair to a $T=2$ state. To investigate the isospin dependence we asked Piazza, Rossi, and Susinno to reanalyze their data on the reaction $\gamma + d \to \pi^- + p + p$. Since they did not find any corresponding anomaly, it is tempting to attribute our phenomenon to the excitation of $T=2$ states which cannot decay into two nucleons and therefore have a much longer lifetime than those observed in similar two-baryon $T=1$ systems such as occur, for example, in reactions $\pi^+ + d \to p + p$ and $\gamma + d \to p + n$.

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γ-Neutrino Angular Correlations in Muon Capture*

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Doppler-broadened γ-ray transitions have been observed for the first time in the reaction $\mu^- + (Z,A) \to (Z-1,A) + \nu_\mu$, which are suitable for analysis in terms of angular correlations between the neutrino and a de-excitation nuclear γ-ray. The observed transitions are interpreted in terms of $\gamma-\nu$ correlation coefficients which are functions of the weak-interaction coupling constants.

Experimental determinations of the weak-interaction coupling constants in muon capture have principally involved measurements of capture rates to specific final states.[1] The values of the coupling constants extracted from these experiments are very sensitive to the choice of initial- and final-state nuclear wave functions. In a series of recent articles, Popov and co-workers[2-8] and Oziewicz and Pikulski[9] have made theoretical studies of the angular correlations between the emitted neutrino and the de-excitation γ ray which occurs following muon capture to specific excited nuclear states. They find that the correlations in certain allowed transitions are sensitive to the induced pseudoscalar coupling C_P and, to a precision of 10%, should be independent of the nuclear wave functions involved. Grenacs *et al.*[10] have proposed a method to observe these correlations in terms of Doppler broadening of the transition γ ray due to the recoil of the nucleus upon neutrino emission. Using a high-resolution Ge(Li) spectrometer, we have observed several Doppler-broadened transitions in ^{28}Al excited by muon capture in ^{28}Si. Several of the transitions, the first such reported, are suitable for analysis in terms of γ-ν correlations.

The correlation function W^N for Nth forbidden muon capture can be written[9] in terms of the muon spin polarization $\vec{p}$, the neutrino direction $\vec{q}$, and the γ-ray direction $\vec{k}$, and the correlation coefficients α^N, a_l^N, b_l^N, c^N, and d^N, which are functions of the weak-interaction coupling constants, nuclear wave functions, the nuclear-spin sequence, γ-ray multipolarity, and kinematic effects. If we consider only *allowed* muon capture, then in terms of observable quantities

$$W^0 = 1 + (\alpha^0 + \tfrac{2}{3} c_1{}^0)\vec{p}\cdot\vec{k}\,\vec{k}\cdot\vec{q} + (a_2{}^0 + b_2{}^0\vec{p}\cdot\vec{k}\,\vec{k}\cdot\vec{q})P_2(\vec{k}\cdot\vec{q}),$$

where the $P_2(\vec{k}\cdot\vec{q})$ are the second-order Legendre polynomials. The correlation coefficients, following Ref. 9, are themselves defined in terms of quantities M_1 and P_1 which are combinations of the nuclear matrix elements and the weak-interaction coupling constants in the muon capture transition, and a quantity $\Lambda_2{}^{11}$ which is a function of the nuclear spin sequence and of the multipolarity of the γ-ray transition. To a 10% accuracy, however, M_1 and P_1 are functions only of the weak-interaction coupling constants C_A, C_V, and C_P for these allowed transitions.[9] C_A, C_V, and C_P are the axial-vector, vector, and pseudoscalar weak-interaction coupling constants, respectively. If one uses the experimental[11,12] value for C_A/C_V, then a measurement of γ-ν correlation coefficients could provide a determination of the induced pseudoscalar coupling constant.

The experiment was performed at the NASA Space Radiation Effects Laboratory (SREL) 600-MeV synchrocyclotron. Experimental details are discussed elsewhere,[13] and we list only the following experimental considerations: (1) The experiment was performed using muons from the "backward" muon beam from the meson channel. The accidental background associated with this beam was less than that of the beam from forward pion decay. (2) Electronic logic requirements ensured that γ-ray events were rejected if a decay electron was detected within 3 mean lifetimes of a muon-stop signal. (3) γ-ray events were analyzed only for the first 700 nsec following a muon stop in order to enhance the signal/noise ratio. (4) The γ rays were observed with a 50-cm^3 Ge(Li) detector having a resolution of 2.5-keV full width at half-maximum (FWHM) at 1.3 MeV, under beam running conditions. (5) Targets of isotopically pure $^{28}SiO_2$ and metallic natural Si were used. A target of $^{29}SiO_2$ was used to determine that muon capture in the 4.7% component of ^{29}Si in the natural Si target did not cause contamination of the transitions of interest in ^{28}Al. (6) The present experimental results are based on 7×10^{10} μ stops in $^{28}SiO_2$ and 3.2×10^{10} μ^- stops in Si (natural).

Background levels were of major concern in the experiment. Conditions (2) and (3) and the use of the metallic Si rather than the oxide produced a reduction in the signal/noise from $\frac{1}{35}$ to $\frac{1}{4}$ for one of the transitions of interest. Most of the background originated from the beam and target.

The Doppler-broadened γ-ray lines were analyzed with a least-squares fitting technique in which the functional form used contained the correlation coefficients as variable parameters. The Doppler shift of a γ ray due to nuclear recoil from neutrino emission is given by

$$E = E_0(1 + v_0 \cos\theta_{\gamma\nu}),$$

where E and E_0 are the energies of Doppler-shifted γ ray and of the γ ray in the rest frame of the emitting nucleus, respectively, v_0 is the nuclear recoil velocity, and $\theta_{\gamma\nu}$ is the angle between the directions of the γ ray and the neutrino. Using this Doppler distribution, we can write the correlation function W^0 in terms of the energy spectrum of the γ rays directly. Slowing-down effects for the γ-ray lines reported here were negligible, but, in principle, can be included using the methods of Pratt.[14] The effect of the finite resolution of the detector is included by using a convolution integral having a Gaussian instrumental response. The width (FWHM) of the Gaussian was determined from radioactive source calibrations and unbroadened nuclear γ rays. The convolution was integrated numerically. The quantity $\vec{p}\cdot\vec{k}$ in the correlation function is determined by the experimental geometry, and thus the energy distribution of the Doppler-broadened γ-ray transition is a function only of the correlation coefficients. If $\vec{p}\cdot\vec{k}\approx 0$, then the only correlation for allowed muon capture is $W^0 = 1 + a_2{}^0 P_2(\cos\theta_{\gamma\nu})$. For the $^{28}SiO_2$ data, $\vec{p}\cdot\vec{k}\approx 0$ over the whole target volume, and the transitions were analyzed only in terms of the correlation coefficient $a_2{}^0$. In analyzing the Si (natural) data, we accounted for the variation of $\vec{p}\cdot\vec{k}$ over the target volume using the value for $|\vec{p}|$ measured by Astbury *et al.*[15] The quantities α^0, $a_2{}^0$, $b_2{}^0$, and $c_1{}^0$ were *not* treated as independent variables but as the appropriate functions[9] of C_P/C_A.

Figure 1 shows a nuclear level diagram of the lower levels in ^{28}Al. The transitions indicated by heavy lines have been observed to be Doppler broadened in this experiment. For the observed allowed transitions, the spin sequences, denoted

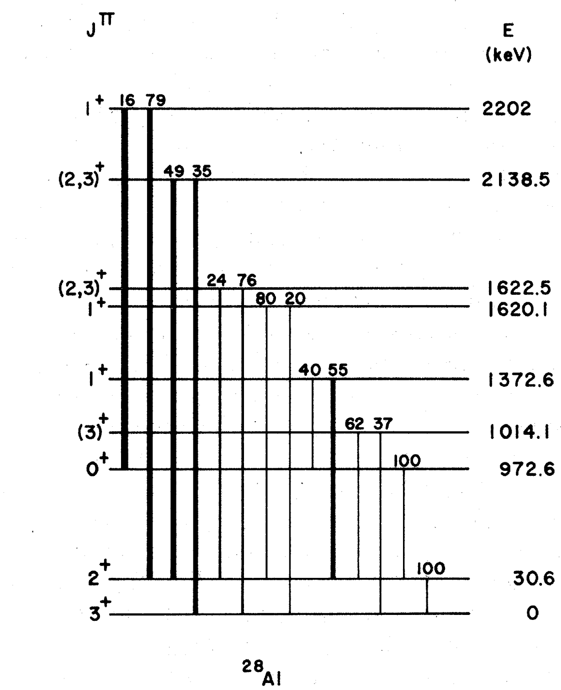

FIG. 1. Nuclear level diagram of ^{28}Al (Ref. 20). The transitions denoted by heavy lines are observed to be Doppler broadened in this experiment. Intensity ratios are also given.

TABLE I. Correlation coefficients from muon capture in Si to states in ^{28}Al. The coefficients are the result of a least-squares fit to Doppler-broadened γ-ray transitions. Energies are in keV.

Transition Energy	Coefficient	Measured Value
2171	$\alpha^0 + \frac{2}{3}c_1^0$	0.54 ± 0.10
	a_2^0	-0.37 ± 0.10
	a_2^0	-0.17 ± 0.08[a]
	b_2^0	0.09 ± 0.01
1229	$\alpha^0 + \frac{2}{3}c_1^0$	1.12 ± 0.10
	a_2^0	0.15 ± 0.25
	a_2^0	0.29 ± 0.30[a]
	b_2^0	0.02 ± 0.03
1343	$\alpha^0 + \frac{2}{3}c_1^0$	0.65 ± 0.20
	a_2^0	-0.29 ± 0.15
	a_2^0	-0.55 ± 0.20[a]
	b_2^0	0.06 ± 0.01
2108	$\alpha^2 + \frac{2}{3}c_1^2$	1.08 ± 0.25
	a_2^2	0.06 ± 0.01
	b_2^2	0.14 ± 0.10
2139	$\alpha^2 + \frac{2}{3}c_1^2$	0.68 ± 0.25
	a_2^2	-0.41 ± 0.20
	b_2^2	0.11 ± 0.10

[a]Values obtained from $^{28}SiO_2$ data; all others obtained from natural-Si data.

by $j_0^\pi \xrightarrow{\mu} j_1^\pi \xrightarrow{\gamma} j_2^\pi$, are $0^+ \xrightarrow{\mu} 1^+ \xrightarrow{\gamma} 2^+$ and $0^+ \xrightarrow{\mu} 1^+ \xrightarrow{\gamma} 0^+$. The respective values of Λ_2^{11} are $1/10\sqrt{6}$ and $1/\sqrt{6}$ for $M1$ γ transitions. In the region near $C_P/C_A = 8$, the $0^+ \xrightarrow{\mu} 1^+ \xrightarrow{M1\gamma} 0^+$ transition would have a correlation a_2^0 of 30%, while the $0^+ \xrightarrow{\mu} 1^+ \xrightarrow{M1\gamma} 2^+$ transition would exhibit only a 3% correlation. These results hold because of the value of Λ_2^{11} only, and similar results are obtained if the nuclear levels de-excite by $E2$ transitions. Similar analyses for the other correlation coefficients indicate that a measurement of the coefficient a_2^0 for the $0^+ \xrightarrow{\mu} 1^+ \xrightarrow{\gamma} 0^+$ transition represents the best possibility for a sensitive determination of C_P/C_A from these allowed transitions.

The results of the least-squares fits are listed in Table I, and the experimental data and fitted line shape for the 1229-keV transition are shown in Fig. 2. Because of the yield[16] of the transitions, the correlation coefficients for the 2202 keV $(1^+) \to 30.6$ keV (2^+) transition are determined with the best precision.

The result of the measurement of the correlation coefficient a_2^0 for the $0^+ \xrightarrow{\mu} 1^+ \xrightarrow{\gamma} 0^+$ transition is in agreement with theoretical predictions; but our corresponding result, $C_P/C_A = 5 \pm 8$, is imprecise primarily as a result of the background level and detector resolution. The measured correlation coefficient a_2^0 for both $0^+ \xrightarrow{\mu} 1^+ \xrightarrow{\gamma} 2^+$ transitions differ by more than 2 standard deviations from theoretical predictions. This fact is perplexing since one of the latter transitions is a result of muon capture to the same level as in the $0^+ \xrightarrow{\mu} 1^+ \xrightarrow{\gamma} 0^+$ transition. The four separate measurements of the coefficient a_2^0 for these $0^+ \xrightarrow{\mu} 1^+ \xrightarrow{M1\gamma} 2^+$ transitions in $^{28}SiO_2$ and natural Si indicate that the experimental data are internally consistent. Because of the uncertainties involved, the value of a_2^0 obtained for the $0^+ \xrightarrow{\mu} 1^+ \xrightarrow{\gamma} 0^+$ transition is also statistically compatible with negative values.

A possible mechanism to explain the disparity between the theoretical and experimental results is substantial population of the levels of interest by cascades from higher-energy levels rather

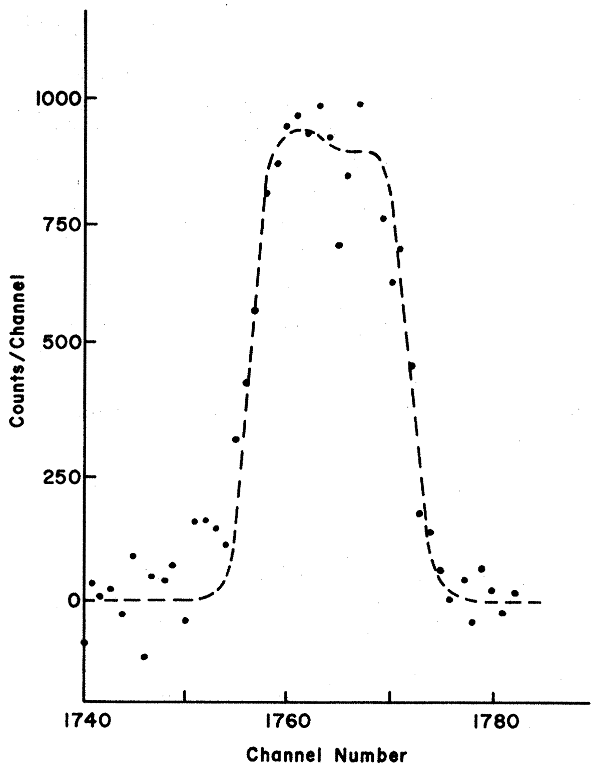

FIG. 2. Energy spectrum of the 1229-keV line in ^{28}Al from μ^- capture in Si (natural). Dots, experimental data points minus background; dashes, least-squares fit to the data.

than by direct capture only. In the energy region below the neutron emission level, no such cascades were observed.[16] It is well known,[17] however, that the level structure of ^{28}Al is very complex and the possibility that the low-lying levels in ^{28}Al are populated by a large number of very weak transitions does exist. Thermal neutron capture in ^{27}Al, however, does not lead to population of the 2202-keV 1^+ state.[18,19] Likewise a comparison[16] of the yield of the 2202-keV state from muon capture with the yield of the analog state in ^{28}Si from 180° electron scattering indicates that there is no substantial population of this level due to cascades. This last statement is not true for the 1373-keV state. Comparison with the electron scattering data for this level indicates that substantial population by cascades may occur.

A complete understanding of the disagreement between the experimental results and the theory of γ-ν correlations as developed by Popov and others[2-9] will require a more precise experimental measurement of a_2^0 for the $0^+ \xrightarrow{\mu} 1^+ \xrightarrow{\gamma} 0^+$ transition. The results of a least-squares analysis of phenomenologically generated line shapes indicate that a factor of 4 increase in data collected, a signal/noise ratio of $\frac{1}{2}$, and an experimental resolution of 1.8-keV FWHM for the 1229-keV transition in ^{28}Al would permit a measurement of a_2^0 to ± 0.03. For $C_P/C_A = 8$ this is a 10% uncertainty.

Some nuclear-structure information can be obtained by analyzing the two γ-ray de-excitations following the second-forbidden transition to the 2139-keV state in ^{28}Al in terms of the γ-ν correlation. Interpretation of the correlation coefficients for these transitions as a function of the weak-interaction form factors is complicated. However, it is possible to deduce[9,13] the spin of the 2139-keV level by merely measuring the *sign* of the correlation coefficient a_2^2. We find the spin of the 2139-keV level to be 2.

In order to employ the γ-ν correlation technique in future experiments with the confidence that an interpretation in terms of weak-interaction coupling constants would be meaningful, one must seek (1) conclusive proof that the population of the low-lying states is the result of direct capture and not cascades, and (2) an understanding of the theoretically contradictory a_2^0 correlation coefficient measured for the $0^+ \xrightarrow{\mu} 1^+ \xrightarrow{\gamma} 2^+$ transition involving the 2202-keV level in ^{28}Al.

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Axial-Vector Anomalies and the Scaling Property of Field Theory*

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A proof of the Adler-Bardeen theorem is given with the aid of the Callan-Symanzik equation.

The recent realization that the processes $\gamma \to 3\pi$ and $2\gamma \to 3\pi$ will provide basic information[1] about the partially conserved-axial-vector-current (PCAC) triangle anomaly[2] has provoked new interest in this subject. If the anomaly is to provide a test of the relevance of the renormalized perturbation series to hadron physics, it is clearly essential that the value of the anomaly remains the same up to any finite order in perturbation theory.[3] Let us elaborate. Consider a perturbative calculation of the amplitude

$$R_{D\mu\nu}(k,q) = i\int d^4x\, d^4y\, e^{i(kx+qy)}\langle 0|\,T\,\partial A(0)\,V_\mu{}^*(x)\,V_\nu{}^*(y)\,|0\rangle = \epsilon_{\mu\nu\lambda\sigma}k^\lambda q^\sigma f(k^2,q^2,kq) \tag{1}$$

in any renormalizable quantum field theory with fermions and a partially conserved axial-vector current. This theory may be, for example, quantum electrodynamics, or the σ model, or a quark-gluon model. [In what follows the discussion is given for quantum electrodynamics (QED). It is straightforward to modify the discussion to a form appropriate for other theories.] The notation $V_\mu{}^*$ indicates that when calculating $R_{D\mu\nu}$, we omit those diagrams in which the vector current $V_\mu = \bar{\psi}_0\gamma_\mu\psi_0$ hooks eventually onto a photon propagator.[4]

The theorem alluded to above then states the following: To any finite order in perturbation theory, $f(0,0,0)$ is given by the basic fermion triangle graph. This is an extraordinary assertion for it tells us that PCAC and gauge invariance imply the existence of a spectacular cancelation among the infinite[5] collection of Feynman diagrams, thus providing a unique opportunity to confront renormalized perturbation theory with data.[6] Moreover, this theorem provides a springboard for several other deductions on the behavior of field theories.[7]

A constructive proof of this important theorem was given by Adler and Bardeen.[8,9] In this paper we propose an alternative proof. This paper also serves to illustrate the Callan-Symanzik[10] equation at work. So let us begin by writing down the version of this equation appropriate for current correlation functions:

$$\left[\lambda(\alpha)m\frac{\partial}{\partial m}+\beta(\alpha)\frac{\partial}{\partial\alpha}\right]R_{D\mu\nu}(k,q) = -\tfrac{1}{2}R_{SD\mu\nu}(0,k,q)+R_{D\mu\nu}(k,q). \tag{2}$$