hole, the particle would be in a "floating orbit." Preliminary calculations[26] with the scalar equation suggest that the details of super-radiant scattering and floating orbits may depend strongly on the spin of the field. The calculations for electromagnetic and gravitational radiation are now underway.

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Evidence for Direct Emission in the Decay $K^{\pm}\to\pi^{\pm}\pi^{0}\gamma$

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We have obtained about 4000 events above background which satisfy the two-constraint fit with the hypothesis $K^{\pm}\to\pi^{\pm}\pi^{0}\gamma$. Their Dalitz-plot population shows the existence of direct radiation in addition to that of the expected inner bremsstrahlung. The best fit to the sum of K^+ and K^- decays in the charged-pion kinetic energy interval of 55 to 90 MeV gives a branching ratio for direct emission of $(1.56\pm0.35)\times10^{-5}$, with a systematic uncertainty of $\pm0.5\times10^{-5}$. No interference is required for a good fit.

It has been apparent for some time that the $\pi^{\pm}\pi^{0}\gamma$ decay of the $K^{\pm}$ meson could have a direct emission (DE) contribution of the same order as the inner bremsstrahlung part (IB).[1,2] Such DE would be a new type of decay of the K meson since the IB part necessarily follows from the $K_{\pi 2}$ decay. Several authors[3] have attempted to calculate this direct amplitude, and others[4] have looked for such a contribution to the $\pi^{\pm}\pi^{0}\gamma$ decay. A direct decay into $\pi^{\pm}\pi^{0}\gamma$ could proceed without violation of the $\Delta I=\frac{1}{2}$ rule, which suppresses by a factor of ~500 the $K_{\pi 2}$ rate and hence the IB rate. For this reason the $K^{\pm}\to\pi^{\pm}\pi^{0}\gamma$ decay mode has been considered to have excellent prospects of exhibiting direct emission. This decay rate has also been suggested as one in which CP nonconservation could possibly be observed.[5]

We have studied the $\pi^{\pm}\pi^{0}\gamma$ decay mode from kaon decays in flight in an experiment carried out in a 1.8-GeV/c partially separated kaon beam at the Brookhaven National Laboratory (BNL) alternating gradient synchrotron. A schematic of the setup is shown in Fig. 1. The incident kaon and its charged decay pion were recorded with a

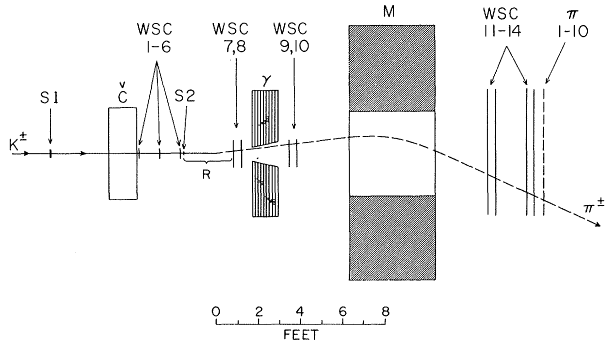

FIG. 1. Plan view of experimental layout, with a superimposed event. S_1, S_2, π_{1-10} are scintillation counters; WSC_{1-14} are wire spark chambers; $\check{C}$ is a differential Čerenkov counter to identify incident kaons; R is the kaon decay region; γ is the γ detector; M is a magnet (aperture 4 ft wide by 4 ft long by 3 ft high, with a field of 5 kG).

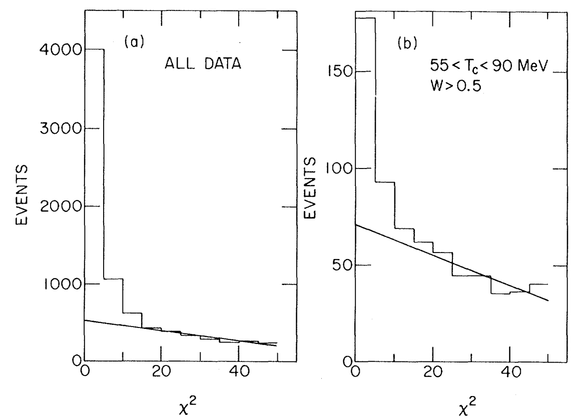

FIG. 2. χ^2 distribution for the fitted events. The solid lines indicate the background subtractions used in the data analysis. (a) All events (4100 events above background); (b) events in a region of the Dalitz plot in which 41% of the signal is DE (W and T_c are defined in the text).

core read-out wire-spark-chamber spectrometer. The three γ's were recorded in the γ detector[6] which consisted of eight layers of lead, an optical spark chamber, and a 32-element scintillator hodoscope. The trigger required an incident kaon, three or more γ's in the γ detector, and a charged particle in the hodoscope at the end of the spectrometer. For each trigger, the optical chambers were photographed and the wire-chamber information was recorded on magnetic tape. The apparatus was monitored on line with the BNL PDP 6 computer. Of the 2.4×10^6 events recorded (from 17×10^9 incident kaons), approximately equal numbers were K^+ and K^- events.

From the wire-chamber information, the c.m. momentum of the charged secondary was calculated for those events with a decay vertex in the fiducial volume (10^6 events). Since most triggers were due to $K_{\pi2}$ and τ' decays, the optical-chamber pictures were scanned only for events with pion c.m. kinetic energy from 51 to 100 MeV (10^5 events). At least three γ's were required within the hodoscope elements that triggered the event, and the conversion point of each γ was measured (3×10^4 events).

These measurements were used with the wire-chamber information to make a two-constraint fit with the $\pi^\pm\pi^0\gamma$ hypothesis. This fit could be threefold ambiguous because of the three possible pairings of the γ's to form the π^0. For events with smallest $\chi^2<5$, 51% had a second pairing with $\chi^2<15$. IB events tend to populate the region of large T_{π^0} on the Dalitz plot while DE events tend to populate the region of small T_{π^0}. Because the incorrect pairing of γ's from IB events can simulate DE, i.e., move events to smaller T_{π^0} values, it is essential to determine the errors on the various input variables so that this effect can be correctly taken into account. The errors were independently measured in studies of the wire spark chambers and $K_{\pi2}$ decays. The χ^2 distribution of the fitted events is shown in Fig. 2 where the smallest χ^2 has been chosen. A cut of $\chi^2<5$ and a linear background subtraction using the events from $15<\chi^2<50$ were made to obtain data distributions. The resulting χ^2 distribution for fitted $\pi\pi\gamma$ events above background agrees with that expected for two degrees of freedom.

To obtain the final data distributions, a method of analysis that takes into account the ambiguous solutions must be adopted. Additional information is contained in the π^0 opening angle in the kaon cm system. The γ's from a real π^0 predominantly occur near their minimum opening angle whereas an incorrect pairing of the γ's which kinematically fits a π^0 does not. A method was chosen which assigned weights to the two best solutions. These weights were a product of their χ^2 probability, their $\pi^0\to2\gamma$ opening angle probability, and their IB decay probability. These weights were normalized to 1 and the data bins populated by these weights. This method essentially eliminated the effect of incorrect pairings according to Monte Carlo studies.

In presenting the data, two projections are used to display the dynamics. The charged-pion c.m. kinetic energy T_c is used because it is insensitive to the γ pairing ambiguity. The other variable, W, almost orthogonal to T_c on the Dalitz plot, is used because it carries the maximum sensitivity to DE; furthermore, the detection efficiency as a function of W is the same for IB and DE. In terms of the kaon c.m. variables, W is defined as $[E_\gamma(\frac{1}{2}m - E_0)/\mu^2]^{1/2}$, where m is the K mass, μ is the average π mass, E_γ is the γ energy, and E_0 is the π^0 energy.

Following the analysis of Good[2] and others,[5] the differential decay probability into $\pi^\pm\pi^0\gamma$ per incident kaon can be written in the lowest multipole approximation as

$$d(R^+ + R^-)/dW \approx 2f\epsilon(W)I(W)[1 + 2\mathcal{E}\cos(\delta_1{}^1 - \delta_0{}^2)\cos(\varphi)\mu^2W^2/m^2 + (\mathcal{E}^2 + \mathfrak{M}^2)\mu^4W^4/m^4], \qquad (1)$$

where $I(W)$ is the IB branching ratio as a function of W, $\delta_1{}^1 - \delta_0{}^2$ is the $\pi\pi$ phase-shift difference which is of the order of 10 deg,[7] φ is the CP-nonconserving phase which would have a value of 90° for maximum CP-invariance violation, $\epsilon(W)$ is the detection efficiency (a function of W determined by Monte Carlo means), and f is the fraction of incident kaons decaying in our fiducial volume (0.053). The electric dipole amplitude $\mathcal{E}$ can interfere with the IB amplitude to produce the second term in the brackets. The magnetic dipole amplitude $\mathfrak{M}$ does not interfere with IB or $\mathcal{E}$ when the photon polarization is summed over.[2,5]

The three terms in Eq. (1) used in fitting the data were generated by Monte Carlo means and take into account the apparatus acceptance, the resolution, and the incorrect γ pairings for the measured events. The Monte Carlo events were given the measured errors, fitted with the same fitting program, and ambiguous γ pairings treated by the identical procedure that was used for the data. The average apparatus acceptance of 2.8% is comprised of the geometrical acceptance of 4.2%, 3γ trigger acceptance of 74%, and the loss of 6% of the γ's because they had too low an energy to be recorded in the γ detector. The trigger acceptance factor arises from the requirement that each γ shower must be separated from the others by at least two counters in the γ hodoscope to be counted as a separate γ.[6]

Our results, with statistical error, are shown in Fig. 3(a) for the W projection. We have used only those events with $55 < T_c < 90$ MeV (2100 events) to avoid possible boundary effects. The dashed curve is a best fit to IB alone and gives a χ^2 of 40 for 17 degrees of freedom. The solid curve is a best fit to the sum of IB and DE only (i.e., no interference term). It gives a χ^2 of 17 for 16 degrees of freedom. A fit including the interference term did not improve the χ^2 significantly. However, the interference term is strongly correlated with the direct term and cannot be ruled out, as shown in Fig. 4.

To obtain the absolute IB rate, the result of the

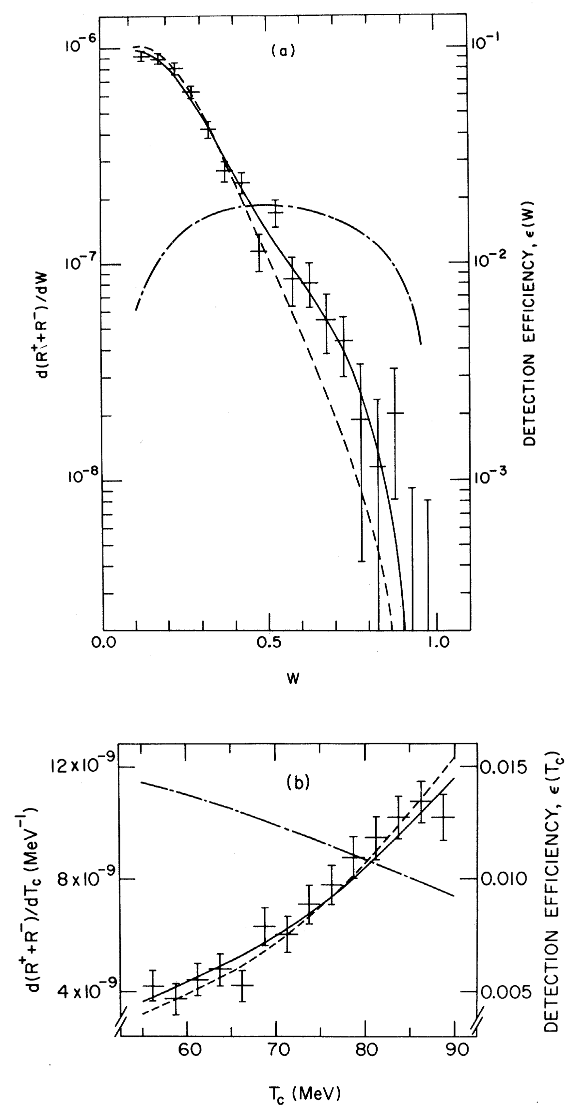

FIG. 3. (a) Results for $d(R^+ + R^-)/dW$ [defined in Eq. (1)] as a function of W. (b) Results for $d(R^+ + R^-)/dT_c$ as a function of T_c. The solid curves are the best fits to the sum of IB and DE; the dashed curves are the best fits to IB alone. Also shown as dot-dashed curves are the detection efficiencies $\epsilon(W)$, used in Eq. (1), and $\epsilon(T_c)$.

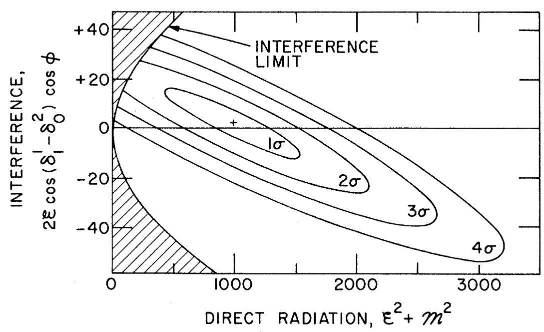

FIG. 4. Results for fitting our data to the sum of IB, DE, and interference term; the axes are defined in Eq. (1). Our best fit is the cross, and contours are shown for 1 to 4 standard deviations. The interference-limit curve is obtained by setting $\mathfrak{M}=0$ and $\cos(\delta_1^1 - \delta_0^2)\cos\varphi = 1$.

fit has been corrected for the scanning efficiency (79 ± 3%), the wire-plane spark efficiency (94%), the wire-plane track reconstruction efficiency (89 ± 3%), the measured $K_{\pi 2}$ rate (80 ± 5%), and a loss of 22% from a combination of other minor effects. The branching ratio for IB is $(2.55 \pm 0.18) \times 10^{-4}$ compared with a theoretical value of 2.50×10^{-4} in our T_c region. When normalized to the theoretical IB rate, the best fit to the W projection with no interference gives a kaon branching ratio of $(1.56 \pm 0.35) \times 10^{-5}$ for the direct $\pi^\pm\pi^0\gamma$ mode in our T_c region. Previous experimental upper limits[4] are consistent with this result. It is in agreement with the expectation that the DE could be large,[5] although the observed rate is larger than all model-dependent calculations.[3]

The charged pion kinetic energy spectrum [Fig. 3(b)] was fitted for IB and DE in the region $55 < T_c < 90$ MeV. A branching ratio of $(1.36 \pm 0.72) \times 10^{-5}$ was obtained for DE. This projection is appreciably less sensitive to DE but it gives a value consistent with that obtained from the W projection.

The data were also analyzed by several alternative methods as a check on proper handling of the ambiguities. A second method involved using only the γ pairing with the smallest χ^2 for the fit. It was determined from Monte Carlo studies that this method selected an incorrect pairing about 15% of the time. A third method reduced the incorrect pairing to 5% by selecting the γ pairing which had the best combined probability of χ^2 and $\pi^0 \to 2\gamma$ opening angle. The results from these analyses were consistent with the weighting method.

In a fourth method of analysis, events were selected using the invariant mass squared of all four decay products and the invariant mass squared of each pair of γ's computed from the unconstrained measurements. For plausible choices of background subtraction, the DE varied by less than the statistical error and was consistent with the other methods of analysis.

Several different procedures were followed to check the stability of the DE term and detect any systematics if present. These included varying the interval of the χ^2 curve used for background subtraction, the type of extrapolation used, the acceptance χ^2 cut, and the charged-pion kinetic energy interval. A partial rescan of the data shows no evidence for bias that would affect the conclusions concerning the direct emission. The lowest-energy γ spectrum was compared with the Monte Carlo results and agreed very well. This indicated no unaccounted loss of low-energy γ's in the detection or scanning.

From the alternate methods of analysis and stability checks we find that the limits of the systematic uncertainty in the DE branching ratio are $\pm 0.5 \times 10^{-5}$. When we have completed a careful check on the small systematics affecting the difference between the K^+ and K^- decays, results on this difference will be presented.

We conclude that we have observed direct emission in $\pi^\pm\pi^0\gamma$ decay which is an essentially new type of kaon decay. From Fig. 4 it can be seen that if CP invariance is not violated (i.e., $\cos\varphi = 1$), then the best fit favors pure magnetic dipole emission; however, the correlations are such that pure electric dipole emission that interferes constructively with IB cannot be ruled out.

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ERRATA

HIGHER-ORDER VACUUM POLARIZATION CORRECTIONS IN MUONIC ATOMS. M. K. Sundaresan and P. J. S. Watson [Phys. Rev. Lett. 29, 15 (1972)].

It has been found on closer examination that the Källén-Sabry form of the vacuum-polarization function used for calculating the contribution due to Figs. 1(b), 1(c), and 1(d) also includes that due to the diagram Fig. 1(a). The contribution from Fig. 1(a) should not have been added separately. The essential conclusion of the paper remains unchanged, however, since the error is only about 5–6 eV for the Ba and Pb transitions. We thank Professor N. M. Kroll for bringing this error to our attention.

SOME COMMENTS ON THE CROSS SECTION OF ^{37}Cl FOR SOLAR NEUTRINO ABSORPTION. W. A. Lanford and B. H. Wildenthal [Phys. Rev. Lett. 29, 606 (1972)].

The cross section quoted on page 607, column 2, line 11 should read $\langle\sigma(^8\mathrm{B})\rangle = 1.15\times10^{-42}$ cm^2.