TABLE 1. $B(E1)$, $B(E2)$, and M_{fi}, and energy-weighted sum-rule (EWSR) limits.

E_x (MeV)	J^π	P^{a}	$E_x P^{b}$	(EWSR)^{b}	$\frac{E_x P}{\text{EWSR}}$
16.65	1^-	17.0 ± 5.0	283 ± 86	264	1.07 ± 0.32
14.0	2^+	990 ± 300	13900 ± 4200	24900	0.56 ± 0.17
14.0	0^+	2050 ± 610	28700 ± 8500	28000	1.03 ± 0.3

^{a}P is $B(E1)$ in units e^2 F^2 for $J^\pi = 1^-$, $B(E2)$ in e^2 F^4 for $J^\pi = 2^+$, and $|M_{fi}|^2$ in F^4 for $J^\pi = 0^+$.

^{b}Units are MeV times units of P.

ter. The resonance around 28 MeV also shows a collective nature. We have found the same kind of resonances for the relatively spherical nuclei ^{54}Fe, ^{116}Sn, and ^{208}Pb as well as for deformed ^{152}Sm. In contrast to the general relation $80A^{1/3}$ MeV for the peak energies of the giant dipole resonances, the newly discovered giant resonances are described by $65A^{1/3}$ MeV and $\sim 120A^{1/3}$ MeV, respectively.

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[1]R. Pitthan and Th. Walcher, Phys. Lett. 36B, 563 (1971).

[2]M. Kimura *et al.*, Nucl. Instrum. Methods 95, 403 (1971).

[3]A. Yamaguchi, T. Terasawa, K. Nakahara, and Y. Torizuka, Phys. Rev. C 3, 1750 (1971).

[4]B. L. Berman *et al.*, Phys. Rev. 162, 1098 (1967); A. Lepretre *et al.*, Nucl. Phys. A175, 609 (1971).

[5]D. G. Ravenhall, quoted in R. Hofstadter, Rev. Mod. Phys. 28, 214 (1956).

[6]M. Danos and H. Steinwedel, Z. Naturforsch. 69, 217 (1951).

[7]V. G. Shevchenko and B. A. Yuryev, Nucl. Phys. 37, 495 (1962).

[8]L. J. Tassie, Aust. J. Phys. 9, 407 (1956).

[9]H. Uberall, *Electron Scattering From Complex Nuclei* (Academic, New York, 1971), Part B, Chap. 6.

[10]O. Nathan and S. G. Nilsson, in *Alpha, Beta, and Gamma-ray Spectroscopy,* edited by K. Siegbahn (North-Holland, Amsterdam, 1965), Chap. X.

[11]R. A. Ferrell, Phys. Rev. 107, 1631 (1957).

Limits on the Magnetic Moments of Doubly Odd $4N+2$ Nuclei with $T=0, J^\pi=1^+$

Pham Tri Nang
Centre de Physique Theorique de l'Ecole Polytechnique, Paris V, France*
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Assuming that the low-lying states of doubly odd nuclei contain only the two lowest SU(4) supermultiplets, I have derived the following lower and upper limits on the magnetic moments of ^{6}Li, ^{10}B*, and ^{18}F, using only experimental data on superallowed Gamow-Teller transitions: $(0.800 \pm 0.016)\mu_N \leq \mu(^6\text{Li}) \leq 0.88\mu_N$, $0.65\mu_N \leq \mu(^{10}\text{B}^*) \leq 0.88\mu_N$, $0.66\mu_N \leq \mu(^{18}\text{F}) \leq 0.88\mu_N$. This shows that relativistic corrections to the magnetic moment of ^{6}Li cannot exceed 7.3%.

In a previous paper[1] I showed that the ground state of ^{6}Li is an almost pure ($T=0$, $S=1$) state of the lowest (100) SU(4) supermultiplet,[2] consistent with good SU(4) symmetry.

The purpose of this note is to present rigorous lower and upper limits on the magnetic moments of doubly odd nuclei under the assumption that SU(4) is a good symmetry and that the ground states do not contain higher supermultiplets other than the (111) supermultiplet. Thus, the ground states of ^{6}Li and

^{6}He can be expressed as follows:

$$\psi(^6\mathrm{Li}) = x\varphi_0(S=1, L=0) + y'\varphi_0(S=1, L=1) + z\varphi_0(S=1, L=2) + \beta\varphi_1(S=0, L=1),$$

$$\psi(^6\mathrm{He}) = \alpha'\varphi_0(S=0, L=0) + \beta'\varphi_1(S=1, L=1),$$

in obvious notation. The parameters are normalized such that

$$|x|^2 + |y|^2 + |z|^2 = 1, \quad |\alpha|^2 + |\beta|^2 = 1, \quad |\alpha'|^2 + |\beta'|^2 = 1,$$

and they satisfy

$$|\alpha|^2 = |x|^2 + |y'|^2 + |z|^2, \quad |y|^2 = |y'|^2 + |\beta|^2.$$

Consider

$$\langle \psi(^6\mathrm{He}) | Y_z^3 | \psi(^6\mathrm{Li}) \rangle = \alpha'^* x M_0 + \beta'^* \beta M_1,$$

where

$$M_0 = \langle \varphi_0(S=0, L=0) | Y_z^3 | \varphi_0(S=1, L=0) \rangle$$

and

$$M_1 = \langle \varphi_1(S=1, L=1) | Y_z^3 | \varphi_1(S=0, L=1) \rangle .$$

The simplest way to compute M_0 and M_1 is to proceed through the following sum rules:

$$(2T+1)^{-1}[\sum_n |\langle 0T \| Y_z \| nT-1\rangle|^2 + (T+1)^{-1}\sum_n |\langle 0T \| Y_z \| nT\rangle|^2 - T(T+1)^{-1}\sum_n |\langle 0T \| Y_z \| nT+1\rangle|^2] = |T_3|, \quad (1)$$

which is derived from the commutation relation

$$[Y_z^+, Y_z^-] = 2T^3;$$

and

$$(2T+1)^{-1}[\sum_n |\langle 0T \| Y_z \| nT-1\rangle|^2 + \sum_n |\langle 0T \| Y_z \| nT\rangle|^2 + \sum_n |\langle 0T \| Y_z \| nT+1\rangle|^2] = \tfrac{1}{3}\langle 0T | Y^2 | 0T\rangle, \quad (2)$$

where T_3 is the value of T^3, the three-component of the isospin operator T^a,

$$Y_z^{\pm} = Y_z^1 \pm i Y_z^2, \quad Y^2 = \sum_{a,\lambda} Y_\lambda^a Y_\lambda^a, \quad Y_\lambda^a = \tfrac{1}{2}\sum_i^A \tau_a^i \sigma_\lambda^i$$

($a = 1, 2, 3$; $\lambda = x, y, z$), and $\langle 0T \| Y_z \| nT'\rangle$ is the reduced matrix element of Y_z^a in isospin space.

For the ground state of ^{6}He, $\langle 0T | Y^2 | 0T\rangle = 3 + 2|\beta'|^2$, which is easily obtained from the expectation values of $T^2 = \sum_a T^a T^a$, $S^2 = \sum_\lambda S_\lambda S_\lambda$ (S_λ are the spin operators), and $C_2^4 = T^2 + S^2 + Y^2$, the second-order Casimir operator of SU(4). When $\beta, \beta' = 0$, it is easy to see that the sum rules (1) and (2) are saturated by the only $(T=1, S=0, L=0) \rightarrow (T=0, S=1, L=0)$ Gamow-Teller (GT) transition; hence $|M_0|^2 = 1$. Similarly, by setting $\beta, \beta' = 1$, we obtain $|M_1|^2 = \frac{1}{3}$.

By definition

$$\tfrac{1}{6} \left| \int \sigma \right|^2_{^6\mathrm{He} \rightarrow {^6\mathrm{Li}}} = |\alpha'^* x M_0 + \beta'^* \beta M_1|^2 \equiv 1 - \epsilon^2.$$

This implies

$$|\alpha'||x| + 3^{-1/2}|\beta'||\beta| \geq (1-\epsilon^2)^{1/2} \quad (3)$$

or

$$|\alpha'||x \geq (1-\epsilon^2)^{1/2} - 3^{-1/2}|\beta'||\beta|.$$

Good SU(4) symmetry means that β' and β are small. Thus, $(1-\epsilon^2)^{1/2} - 3^{-1/2}|\beta'||\beta|$ is positive in most of the observed superallowed GT transitions.

By squaring both sides of (3), one gets

$$(1 - \rho^2 + \tfrac{1}{3}|\beta|^2)|\beta'|^2 - \tfrac{2}{3}\sqrt{3}(1-\epsilon^2)^{1/2}|\beta||\beta'| - (\epsilon^2 - \rho^2) \leq 0 \quad (4)$$

(where $\rho^2 \equiv |y|^2 + |z|^2$; $|x|^2 = 1 - \rho^2$). Inequality (4) can be satisfied if and only if

$$\rho^4 - (1 + \epsilon^2 + \tfrac{1}{3}|\beta|^2)\rho^2 + (\epsilon^2 + \tfrac{1}{3}|\beta|^2) \geq 0. \quad (5)$$

Equation (5) in turn is satisfied when $\rho^2 \leq \epsilon^2 + \frac{1}{3}|\beta|^2$. Since $|\beta|^2 \leq |y|^2$, we must have

$$\tfrac{2}{3}|y|^2 + |z|^2 \leq \epsilon^2. \quad (6)$$

The magnetic moment of ^{6}Li is given by

$$\mu(^6\mathrm{Li}) = 0.88 - 0.38 \times \tfrac{1}{2}(|\beta|^2 + |y|^2 + 3|z|^2).$$

From the fact that $|\beta|^2 \leq |y|^2$ and from (6), we have

$$\mu(^6\mathrm{Li}) \geq 0.88 - 0.57\epsilon^2. \quad (7)$$

The recent experimental value of $|G_A/G_v|^2$ reduces[3] $|\int\sigma|^2_{^6\mathrm{He}\to{}^6\mathrm{Li}}$ from the old value of 5.6 ± 0.2 to 5.16 ± 0.2. This gives $\epsilon^2 = 1 - \frac{1}{6}|\int\sigma|^2 = 0.14 \pm 0.03$, which leads to $\mu(^6\mathrm{Li}) \geq (0.800 \pm 0.016)\mu_N$ as compared with the experimental value of $0.822\mu_N$.

Other examples of superallowed GT transitions can be found in the $1p$ and $2s$-$1d$ shell nuclei; for example, the superallowed transition

$$^{10}\mathrm{C}(T=1, J^\pi = 0^+) \to {}^{10}\mathrm{B}^*(T=0, J^\pi = 1^+)$$

has $ft = 1.0 \times 10^3$, corresponding to $|\int\sigma|^2 = 3.69 \pm 0.2$ as reported by Kavanagh[3] and Bahcall,[4] but not yet firmly established. From this value, one gets $\epsilon^2 = 0.40$, which implies that $\mu(^{10}\mathrm{B}^*)$ is greater than $0.652\mu_N$ as compared with the recently measured value[5] of $(0.63 \pm 0.12)\mu_N$. The superallowed transition $^{18}\mathrm{Ne} \to {}^{18}\mathrm{F}$ has $|\int\sigma|^2 = 3.81$, and hence $\epsilon^2 = 0.375$, and $\mu(^{18}\mathrm{F})$ is thus predicted to be greater than $0.666\mu_N$.

We note that since $|\beta| \leq |y|$, $|x| \leq |\alpha|$, inequality (3) also implies that

$$|\beta|^2 \leq \tfrac{3}{2}\epsilon^2, \quad |\beta'|^2 \leq \tfrac{3}{2}\epsilon^2, \quad |y|^2 \leq \tfrac{3}{2}\epsilon^2. \quad (8)$$

So far we have only made use of the information extracted from the superallowed transition and the limits obtained are interesting only when ϵ^2 is small. However, it is possible to improve the upper limit of $|\beta|^2$ by making use of (1) and (2); one gets

$$|\beta'|^2 = \tfrac{3}{2}(\epsilon^2 - \textstyle\sum_n |\langle g1|Y_z{}^3|n0\rangle|^2), \quad (9)$$

where $\sum_n |\langle g1|Y_z{}^3|n0\rangle|^2$ is the total $|\Delta T| = 1$ GT transition strength except the strongest superallowed transition strength defined by $\frac{1}{6}|\int\sigma|^2 = 1 - \epsilon^2$. Therefore a very accurate measurement of a large number of $(1, 0^+) \to (0, 1^+)$ GT transitions can give a good estimate of the upper bound on $|\beta'|^2$.

Similarly, the upper limit on $|z|^2$ for the ground state of ^{6}Li can be derived with the help of Eqs. (9) and (11) of Ref. 1:

$$|z|^2 \leq \epsilon^2 - \tfrac{1}{3}(\textstyle\sum_n |\langle g\|Y^3\|0^+n\rangle|^2 + \tfrac{2}{3}\sum_n |\langle g\|Y^3\|1^+n\rangle|^2), \quad (10)$$

where $\langle g\|Y^3\|J'n\rangle$ is the reduced matrix element of $Y_\lambda{}^3$ in ordinary space.

Thus the existence of GT transitions to all excited states with $T=1$, $J^\pi = 0^+, 1^+$ further reduces the upper bound on $|z|^2$. The upper limit on the magnetic moment of ^{6}Li is derived by means of Eq. (7) of Ref. 1 and the sum rules for the ground-state expectation value of S^2. We have

$$2|\alpha|^2 - \langle S_z\rangle - \langle S_z\rangle^2 = \tfrac{1}{3}(\tfrac{1}{2}\sum_{n\neq g} |\langle g\|S\|1^+n\rangle|^2 + \tfrac{3}{2}\sum_n |\langle g\|S\|2^+n\rangle|^2), \quad (11)$$

where $\langle g\|S\|J'n\rangle$ is the reduced matrix element of S_λ. Because of our lack of experimental data on $\Delta T = 0$, isoscalar $M1$ transitions, all we can tell from (11) is that

$$\langle S_z\rangle \leq \tfrac{1}{2}[-1 + (1 + 8|\alpha|^2)^{1/2}] \leq 1. \quad (12)$$

This gives an upper limit on $\mu(^6\mathrm{Li})$ and we arrive at

$$(0.800 \pm 0.016)\mu_N \leq \mu(^6\mathrm{Li}) \leq 0.88\mu_N.$$

Since SU(4) is a good symmetry for the ground states of $A = 6$ nuclei, the bounds on $\mu(^6\mathrm{Li})$ derived above are essentially model independent.

A 10% reduction of the GT transition strength due to meson exchange currents (as in the case of the threshold neutron capture $n + p \to d + \gamma$) can reduce the lower limit to 0.76. In any case, we must conclude that relativistic corrections to the magnetic moment of ^{6}Li cannot exceed 7.3%.

At present very few data are available on $M1$ transitions in $4N+2$ nuclei, so that an upper limit on the supermultiplet impurity cannot be determined in the same way as for ^{6}Li. However, there is good reason to believe that the impurities for ^{10}B* and ^{18}F are small since these two nuclei can be described as two nucleons in a triplet state coupled to ^{8}Be and ^{16}O cores, respectively, and it is known[1] that the ground states of ^{8}Be and ^{16}O are nearly pure scalar supermultiplets. Thus, inequalities (6), (7), and (12) are still valid and one gets

$$0.65\mu_N \leq \mu(^{10}\mathrm{B}^*) \leq 0.88\mu_N,$$

$$0.66\mu_N \leq \mu(^{18}\mathrm{F}) \leq 0.88\mu_N.$$

We note that a recent theoretical calculation[6] gives $\mu(^{18}\mathrm{F}) = 1.14\mu_N$, in disagreement with the above upper limit.

In concluding, I would like to point out that more accurate experimental data on $\Delta T = 1$ GT and $M1$ transitions from the ground to all excited states in doubly odd $4N+2$ nuclei will give a good estimate of the supermultiplet impurities of the ground state, thus enabling us to establish the validity of SU(4) in these nuclei.

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*Equipe de Recherche associée au Centre National de la Recherche Scientifique.

[1]Pham T. N., Nucl. Phys. A185, 413 (1972).

[2]For a discussion of supermultiplet theory, see, for example, L. A. Radicati, in *Application of Mathematics to Problems in Theoretical Physics,* edited by F. Lurçat (Gordon and Breach, New York, 1967), and other references cited therein.

[3]R. W. Kavanagh, Nucl. Phys. A129, 172 (1969).

[4]J. N. Bahcall, Nucl. Phys. 75, 10 (1966).

[5]R. Avida, I. Ben-Zvi, G. Goldring, S. S. Hanna, P. N. Tandon, and Y. Wolfson, Nucl. Phys. A182, 359 (1972).

[6]M. R. Gunye and S. B. Khadkikar, Phys. Rev. C 4, 1073 (1971).

Rotating Black Holes: Separable Wave Equations for Gravitational and Electromagnetic Perturbations*

Saul A. Teukolsky
California Institute of Technology, Pasadena, California 91109
(Received 10 July 1972)

Separable wave equations with source terms are presented for electromagnetic and gravitational perturbations of an uncharged, rotating black hole. These equations describe the radiative field completely, and also part of the nonradiative field. Nontrivial, source-free, stationary perturbations are shown not to exist. The barrier integral governing synchrotron radiation from particles in circular orbits is shown to be the same as for scalar radiation. Future applications (stability of rotating black holes, "spin-down," superradiant scattering, floating orbits) are outlined.

It is generally accepted that the gravitational collapse of a massive rotating star can produce a rotating black hole. Moreover, black holes may play important roles in a number of astrophysical phenomena: (i) One or more black holes near the center of the Galaxy might be the origin of Weber's[1] gravitational-wave events; (ii) a massive black hole at the center of the Galaxy has been postulated[2] to explain radio and infrared phenomena there; (iii) the x-ray source Cyg-XI —and also 2U-0900-40—is likely to be a black hole in close orbit around a B-type supergiant star, with the x rays emitted by gas flowing from star to hole.[3]

These developments create an urgent need for two types of black-hole calculations: first, cal-