the walls. The local values of conductivity are nearly the same as the average conductivity measured by penetration time.

According to the experimental results the conductivity varies as $\sigma \propto M_i n_e / B_0^2$ or the effective collision frequency varies as $\nu_{\rm eff} \sim B_0^2/M_i$. For our typical [Fig. 2(c)] conditions $\nu \simeq 3\times 10^9$ sec^{-1}, i.e., near the ω_{ce} at the peak of the magnetic field and roughly 2 orders of magnitude higher than either the electron-ion or electron-neutral collision frequency.

During the last few years, several theories have been developed concerning instabilities that can be excited in a plasma when an electric current flows perpendicular to the magnetic field. Based on the quasilinear equation, these theories give approximate expressions for the effective collision frequency. However, the work[5] apparently most relevant to this experiment does not give the scaling we observe. The fact that the conductivity is somewhat independent of r and τ probably means that saturation of the instability has occurred and therefore comparison with predictions from quasilinear theory is not valid. No existing theory describes this situation of considerable practical importance in the magnetic compression of a field-free plasma.

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Electron Fluctuations and Transport in Toroidal Plasmas

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Negative-energy modes driven by a normal gradient of the electron temperature are found in two-dimensional equilibrium configurations such as the toroidal diffuse pinch. These modes tend to grow by transferring (positive) energy to the resonating electrons; they have properties that make them suitable to alter considerably the orbits of the deeply trapped electrons by proper resonant interactions, and make them lead to electron thermal energy transport across the magnetic field without corresponding particle transport.

To understand the macroscopic transport properties of two-dimensional confined plasmas, a detailed knowledge of the modes which can be excited in them[1] is necessary. In particular, an important question is whether the orbit of deeply trapped electrons in a toroidal confinement configuration can be significantly altered by the collective modes[2] to which the plasma is subject. An analysis of the needed characteristics of such modes leads to requiring that (i) they exist for frequencies $\omega \sim \hat{\omega}_{be}$, where $\hat{\omega}_{be}$ is the average bounce frequency of trapped electrons; (ii) the profile of the resulting electric field fluctuations is correlated with the periodic variation of the magnetic field and is nonzero and even around the point of minimum magnetic field; (iii) they should not be damped by the process of resonant interaction with trapped electrons. This last requirement can be met, for instance, if the relevant modes are of negative energy,[3] in the sense that they tend to grow when positive energy is transferred to the resonant particles from the wave.

Now we recall that in a symmetric torus in which the magnetic field can be represented as $B \approx B_0/[1+(r/R_0)\cos\theta]$, r and R_0 being the minor and major radii of a magnetic surface, respectively, $\hat{\omega}_{be}$ is of order $(r/R_0)^{1/2}v_{\text{th}e}/2qR_0$, where $v_{\text{th}e}$ is the electron thermal velocity, and $q \approx rB_T/R_0B_p$, B_T and B_p being the toroidal and poloidal magnetic field components, respectively. An analysis of the possible modes that can be excited with frequency $\omega \sim \hat{\omega}_{be}$ on the basis of the microinstabilities known to occur in one-dimensional equilibrium configurations leads to unrealistic results. For instance, if we consider drift modes with electric potential $\tilde{\Phi}=\tilde{\varphi}(r)\exp(-i\omega t - im^0\theta + ik_\zeta\zeta)$, where ζ is the direction of the magnetic field, we obtain

$$\omega \approx \omega_{*e}\frac{F(b_i)}{1+(T_e/T_i)[1+F(b_i)]}$$

$$\lesssim \frac{1}{2\sqrt{\pi}}\left(\frac{T_e}{T_e+T_i}\right)\frac{v_{\text{th}i}}{r_n}. \tag{1}$$

Here $\omega_{*e}=(m^0/r)(cT_e/eBn)\,dn/dr = m^0(\rho_e/r)v_{\text{th}e}/2r_n$, with ρ_e being the electron Larmor radius; r_n indicates the density gradient scale distance; $b_i=\frac{1}{2}(m^0\rho_i/r)^2$, with ρ_i the ion Larmor radius; $v_{\text{th}i}$ is the ion thermal velocity; $F(b_i)\equiv I_0(b_i)\times\exp(-b_i)$, with I_0 the known modified Bessel function. Therefore, the condition $\omega \gtrsim \hat{\omega}_{be}$ would require

$$\frac{r_n}{R_0 q}\left(\frac{r}{R}\right)^{1/2} < \left(\frac{m_e T_i}{m_i \pi T_e}\right)^{1/2}\frac{T_e}{T_e+T_i},$$

which is not satisfied in realistic diffuse-pinch configurations.

We could also consider ion-sound waves, but these are not likely to be excited in experiments where T_e is not much larger than T_i and the electron drift velocity is much less than $v_{\text{th}e}$, as is usually the case.

The modes we shall find here satisfy all three requirements indicated previously, and unlike the known drift modes they can be made unstable by a gradient of the electron temperature in the direction of the density gradient. For a two-dimensional toroidal configuration which is inhomogeneous and periodic in θ, the appropriate normal modes are of the form $\tilde{\Phi}\equiv\tilde{\varphi}_{m^0,n^0}(\theta,r)\exp(-i\omega t - im^0\theta + in^0\zeta)$, where we assume that $\partial\tilde{\varphi}_{m^0,n^0}(\theta,r)/\partial\theta \ll m^0\tilde{\varphi}_{m^0,n^0}(\theta,r)$. We consider in particular those modes which are radially localized around a rational surface $r=r_0$ such that $q(r_0)=m^0/n^0$. The longitudinal electric field, that is important for the resonant mode-particle interaction, is $\tilde{E}_\parallel = \vec{\mathrm{E}}_1\cdot\vec{\mathrm{B}}/B = -(B_p/r_0B)(\partial/\partial\theta)\tilde{\varphi}_m(\theta)$, where $\vec{\mathrm{E}}_1 = -\nabla\tilde{\Phi}$ and $\tilde{\varphi}_m(\theta)\equiv\tilde{\varphi}_{m^0,n^0}(r_0,\theta)$. Since we are interested in the interactions with deeply trapped electrons, we shall analyze modes with $\tilde{\varphi}_m(\theta)$ *odd* (in θ) around $\theta=0$, so that $\tilde{E}_\parallel$ is even and nonzero around the same point.

We consider the frequency range $\hat{\omega}_{bi}<\omega\lesssim\omega_{be}$ and short transverse wavelengths so that $\omega_{*e}\gtrsim\hat{\omega}_{be}$. This implies $b_i \gg 1$ and the perturbed ion distribution function is then[1]

$$\tilde{n}_i = -(en_i/T_i)\tilde{\varphi}_m. \tag{2}$$

To determine the perturbed electron density $\tilde{n}_e$ we derive the perturbed electron distribution, in the guiding center approximation, by integrating the linearized Vlasov equation along particle orbits.[1] For this we define $\epsilon = m(v_\parallel^2+v_\perp^2)/2$ as the particle energy, $\mu = mv_\perp^2/2B$ as its magnetic moment, and $\Lambda=\mu B_0/\epsilon$ as representing the pitch angle so that $v_\parallel = (2\epsilon/m)^{1/2}[1-\Lambda B(\theta)/B_0]^{1/2}$. The quantity ωb defines the bounce frequency for trapped particles ($\frac{1}{2}R_0q_0\oint d\theta/|v_\parallel|$ for $1-r_0/R_0<\Lambda<1+r_0/R_0$) and the transit frequency for circulating particles ($R_0q_0\int_0^{2\pi}d\theta/v_\parallel$ for $0<\Lambda<1+r_0/R_0$). Then we decompose $\tilde{\varphi}_m(\theta)$ in harmonics of the orbit periodicity, so that $\tilde{\varphi}_m(\theta)=\sum_p\Phi_m^{(p)}(\Lambda)\exp(ip\omega_b\hat{t})$, where $\hat{t}=\int^\theta d\theta'/v_\parallel$, and $\omega_b\hat{t}$ is a function of Λ only.

The unperturbed electron distribution is assumed to be of the form $f_e = f_{\text{M}e}(1+\hat{f}_e)$, where $f_{\text{M}e}$ is the Maxwellian with temperature T_e,

$$\hat{f}_e = \frac{v_\parallel}{|\Omega_{pe}|}\left(\frac{dn_e/dr}{n_e} - \frac{dT_e}{dr}\frac{\frac{3}{2}-\epsilon/T_e}{T_e}\right)$$

and $|\Omega_{pe}| = eB_p/m_ec \approx |\Omega_e|B_p/B$. This form of $\hat{f}_e$ is appropriate for trapped electrons in regimes[4] where their average collision frequency $\langle\nu_e\rangle$ is smaller than $\hat{\omega}_{be}$. In the following we shall consider the limit of sufficiently large values of R_0/r_0 for which, as we can see, the contribution of $\hat{f}_e$ that is appropriate for circulating particles can be neglected to order $(r_0/R_0)^{1/2}$. We have also extended[5] our results to include the contributions of circulating particles which are numerically significant for realistic values of r_0/R_0 by adopting, for circulating particles

$$\hat{f}_e = -\frac{v_\parallel}{|\Omega_{pe}|}\left(\frac{dn}{dr}\frac{T_i}{nT_e} + \sigma\frac{dT_i/dr}{T_e}\right) + \frac{v_\theta}{|\Omega_e|}\left(\frac{dn}{dr}\frac{1+T_i/T_e}{n} + \sigma\frac{dT_i/dr}{T_e} - \frac{dT_e}{dr}\frac{\frac{3}{2}-\epsilon/T_e}{T_e}\right).$$

For $\sigma \approx 0.172$ this expression is consistent, to order r_0/R_0, with the results of Ref. 4. Then, following Refs. 1 or 2, we are led to

$$\tilde{n}_e = (en/T_e)\{\tilde{\varphi}_m - n^{-1}\int d^3v\, f_{\mathrm{M}e}[\omega - \omega_{*e} + \omega_{Te}(\tfrac{3}{2} - \epsilon/T_i)]\textstyle\sum_p[\tilde{\Phi}_m^{(p)}(\Lambda)\exp(ip\omega_b\hat{t})]/(\omega - p\omega_b)\}, \tag{3}$$

where $\omega_{Te} = \omega_{*e}(d\ln T_e/dr)/(d\ln n/dr)$. Recalling that $m^0\tilde{\varphi}_m \gg \partial\tilde{\varphi}_m/\partial\theta$, we consider Poisson's equation $(m^0/r_0)^2\tilde{\varphi}_m = 4\pi e(\tilde{n}_i - \tilde{n}_e)$ and take the quadratic form

$$(m^0/r_0)^2\oint dl\,|\tilde{\varphi}_m|^2/B - 4\pi e\oint dl\,\tilde{\varphi}_m^*(\tilde{n}_i - \tilde{n}_e)/B = 0, \tag{4}$$

where $dl = R_0 q_0\, d\theta$. Then we obtain,[1] for $(m^0\lambda_\mathrm{D}/r_0)^2 < 1$,

$$\left(1+\frac{T_e}{T_i}\right)n\oint dl\,\frac{|\tilde{\varphi}_m|^2}{B} = \frac{1}{2}\pi\left(\frac{2}{m_e}\right)^2\iint d\epsilon\, d\mu f_{\mathrm{M}e}(\epsilon)|\tau|\left[\omega - \omega_{*e} + \omega_{Te}\left(\frac{3}{2} - \frac{\epsilon}{T_e}\right)\right]\sum_{p\neq 0}\frac{|\tilde{\Phi}_m^{(p)}(\Lambda)|^2}{\omega - p\omega_b} = 0, \tag{5}$$

where λ_D is the Debye length, and $\omega_b = 2\pi/\tau$. We also have expressed $\int d^3v$ as $(2\pi/m_e^2)\int d\mu\, d\epsilon\, B/|v_\parallel|$ with the convention that contributions from positive and negative values of $v_\parallel$ are to be added, and have made use of the fact that $\int dl/B \iint d\mu\, d\epsilon\, B/|v_\parallel| = \iint d\mu\, d\epsilon\, |\tau|$.

For simplicity, we consider the limits $\omega/\hat{\omega}_{be} < 1$ and $R_0/r_0 > 1$. We expand Eq. (5) in these two parameters, and carry out the integration over ϵ to obtain

$$\left(1+\frac{T_e}{T_i}\right)\int_0^{2\pi} d\theta\,|\tilde{\varphi}_m|^2 - \left(\frac{R_0}{r_0}\right)^{1/2}\frac{(\omega_{*e} - \omega_{Te})\omega}{2\pi^2\hat{\omega}_{ce}^2}\int_0^1 d\chi^2 L^3(\chi^2)\sum_{p>0}\frac{|\tilde{\Phi}_m^{(p)}(\chi^2)|^2}{p^2}$$
$$+ i\sqrt{\pi}\,\frac{R_0}{r_0}\,\frac{(\frac{3}{2}\omega_{Te} - \omega_{*e})\omega^2}{4\pi^3\hat{\omega}_{ce}^3}\int_0^1 d\chi^2 L^4(\chi^2)\sum_{p>0}\frac{|\tilde{\Phi}^{(p)}(\chi^2)|^2}{p^3} = 0, \tag{6}$$

where as indicated earlier we have neglected the contribution of circulating particles within the last two terms.

In order to derive Eq. (6) from Eq. (5) we have observed that

$$\sum_p \frac{|\tilde{\Phi}_m^{(p)}|^2}{\omega - p\omega_b} = \sum_{p>0}|\tilde{\Phi}_m^{(p)}|^2\frac{2\omega}{\omega^2 - p^2\omega_b^2}, \quad (\omega - p\omega_b)^{-1} = P(\omega - p\omega_b)^{-1} + i\pi\delta(\omega - p\omega_b),$$

where $\delta(\omega - p\omega_b) = (\epsilon/|\omega|)\delta(\epsilon - p\epsilon(\chi))$, $\epsilon(\chi) = (\omega R_0 q_0)^2 L^2(\chi) R_0/r_0$, and that $P[2\omega/(\omega^2 - p^2\omega_b^2)] \approx -2\omega/(p\omega_b)^2$ in the considered limit $\omega/\hat{\omega}_{be} < 1$. We also have defined $\hat{\omega}_{ce} = v_{\mathrm{th}e}/R_0 q_0$ as the average transit frequency, chosen m^0 such that $\omega > 0$, and taken $\chi^2 \equiv \frac{1}{2}[1 + (1-\Lambda)R_0/r_0]$ so that $0 \leq \chi \leq 1$ is equal to half of the excursion amplitude in θ of deeply trapped particles. In addition, $L(\chi^2) \approx \frac{1}{2}\oint d\theta/(2\chi^2 - 1 + \cos\theta)^{1/2}$.

The last term in Eq. (6) results from resonances of the considered wave with trapped particles having bounce frequency $\omega_b = \omega/p$. The second term, which is larger by a factor of order $\hat{\omega}_{be}/\omega$, has no correspondence[1] in the stability theory of one-dimensional plasmas involving resonances of the form $\omega = k_\zeta v_\parallel$, so that, in the limit $\omega < k_\zeta v_{\mathrm{th}}$, the resonant contribution to the dispersion relation is of order $\omega/k_\zeta v_{\mathrm{th}}$ instead of $\omega^3/\hat{\omega}_{be}^3$ as is in the present case.

If we neglect the last (resonant) term in Eq. (6), we can use the remaining quadratic form as a variational form in order to evaluate ω. Thus, the imaginary part of ω is obtained as a perturbation, and in this sense we can estimate the stability of the modes under consideration by the equation

$$\left(1+\frac{T_e}{T_i}\right) - \left(\frac{R_0}{r_0}\right)^{1/2}\frac{(\omega_{*e} - \omega_{Te})\omega}{\hat{\omega}_{ce}^2}\,\mathcal{T}_1 - i\,\frac{R_0}{r}\,\frac{(\frac{3}{2}\omega_{Te} - \omega_{*e})\omega^2}{\hat{\omega}_{ce}^3}\,\mathcal{T}_2 = 0. \tag{7}$$

Here $\mathcal{T}_1$ and $\mathcal{T}_2$ are positive numbers resulting from the evaluation of the integrals appearing in Eq. (6) when $\tilde{\varphi}$ is replaced by a trial function $\tilde{\varphi}'$ which is found by applying the variational principle indicated earlier.

Now we see that instability is found for $\frac{2}{3}d\ln n/dr < d\ln T_e/dr \lesssim d\ln n/dr$, and this is compatible with the assumption that $\omega < \hat{\omega}_{be}$. However, the general quadratic form Eq. (5) shows no evidence that these modes disappear for $\omega \sim \hat{\omega}_{be}$. In particular, when $d \ln T_e/dr = d \ln n/dr$, the relevant instability ceases to be of resonant type, as $\mathrm{Re}\,\omega \sim \mathrm{Im}\,\omega$, and an additional term has to be included in Eq. (7).

We recall that the known electron drift modes[1] are damped by the contribution of ω_{Te} when $(d \ln T_e/dr)/(d \ln n/dr) > 0$ and are such that $\tilde{n}$ and $\tilde{\varphi}$ are out of phase. Instead, for the modes present here, $\tilde{n}$ and $\tilde{\varphi}$ are in phase, as indicated by Eq. (2), and we expect, on the basis of quasilinear theory, that they lead to electron thermal energy transport across the magnetic field without a corresponding particle transport. We also notice that *odd* modes, of the type considered here, are less susceptible to the effects of collisions than the *even* modes treated in Refs. 6 and 7.

We refer to the quadratic form (4) and define an effective dielectric constant ϵ in terms of the integrals $\mathcal{T}_1$ and $\mathcal{T}_2$ in Eq. (7). Then we may argue that the wave energy is proportional to $\omega\, \partial\epsilon/\partial\omega = -(r_0/m^0\lambda_D)^2(1+T_e/T_i)$ which is evidently negative. From Eq. (6) we can also see that most of the electrons (linearly) resonating with this wave are barely trapped.

The influence of well-developed modes of the type considered here on the orbit of deeply trapped ions[8] has been studied analytically and numerically.[9] It has been found[9] that, depending on the value of the parameter $\alpha^{1/3}$, where $\alpha = e\tilde{\varphi}_c R_0/\mu B_0 r \sim (R_0/r)e\tilde{\varphi}_c/T$ and $\tilde{\varphi}_c$ is the characteristic mode amplitude, the trapped-particle oscillations can be amplified up to $2(16\alpha)^{1/3}$, but still remain trapped if α is sufficiently small. The resulting orbits are composed of a sequence of open banana-like excursions in the r,θ plane, have a superperiod about equal to $(16\alpha)^{2/3}2/\omega_b$. When α is larger than a reasonable value [such as $\alpha \approx 0.15$ which has been obtained in Ref. 6 for $\tilde{\varphi}_m(\theta) = \tilde{\varphi}_c \sin\theta$], the resonating particles can be untrapped and circulate for a considerable part of their superperiod. The radial particle excursions can be considerably larger than the known banana width of trapped particles, and the average magnetic curvature drift can be more favorable to the stability of interchange[8] modes than in the case where fluctuations are absent.

So in the presence of modes which are odd in $\tilde{\varphi}_m$ and have frequency close to the average bounce frequency of the ions and of the electrons, we can have "quasibanana" orbits with relatively large amplitudes for the particles which remain trapped, or quasicirculating orbits for the particles which become periodically untrapped. We can infer that the stability of lower-frequency[8,9] modes, such as the trapped-particle interchange modes, and the particle and energy transport across the magnetic field will have to be re-evaluated by taking these effects into proper account.

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