Instability of the Whistler Structure of Oblique Hydromagnetic Shocks*

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The characteristic whistler structure of oblique hydromagnetic shocks is observed experimentally to suffer from a rapidly growing short-wavelength instability, which is tentatively identified as the decay instability first predicted by Galeev and Karpman. It is suggested that this instability is responsible for the turbulent whistler shock structure observed when a super-Alfvénic plasma stream encounters a magnetic obstacle.

It has been shown theoretically[1] and demonstrated experimentally[2,3] that the magnetic structure of a shock wave traveling at an oblique angle to the field in a magnetized plasma has the form of a large-amplitude, circularly polarized wave. This structure is a consequence of the limiting of the shock steepening by dispersion; for a steady-state shock, the wavelength λ may be derived by equating the shock velocity to the phase velocity of a wave on the "whistler" branch of the low-frequency plasma dispersion relation. We thus obtain

$$\lambda=\frac{2\pi\cos\theta}{(M_A^2-1)^{1/2}}\frac{c}{\omega_{pi}}, \tag{1}$$

where θ is the angle between the direction of propagation and the initial magnetic field, and M_A is the Alfvén number (shock speed/Alfvén speed). This relationship has been verified in an experiment at the Univesity of Texas[3] in which a curved shock front was propagated through a uniformly magnetized plasma. The radius of curvature of the shock front was sufficiently great compared to the shock thickness that it was possible to treat local regions of the shock as plane and to compare the structure at each point with one-dimensional theory.

Under typical experiment conditions (initial plasma density $n_e=5\times10^{14}$ cm^{-3} in hydrogen, initial axial magnetic field $B_0=500$ G, shock velocity $u=2\times10^7$ cm sec^{-1}, $M_A=4$) a single-mode whistler structure was seen, clearly detached from the driving magnetic piston. The wavelength was given by (1) and the damping of the wave could be ascribed to a combination of classical and anomalous resistivity, as in the case of perpendicular shocks. When the plasma was changed to deuterium at a density of 2.5×10^{14} cm^{-3}, all other conditions being unchanged, evidence was found of a shorter-wavelength component apparently growing on the original whistler. In this Letter we describe some further observations of this perturbation, suggest its probable origin, and consider its significance in relating laboratory and geophysical observations of oblique shocks.

In our experiment, the whistler wavelength at $M_A=4$ and $\theta=45°$ is about 1 cm. To study the short-wavelength perturbations, we used miniature magnetic probes[4] with outside diameter ~0.6 mm and time response ~2 nsec. The probes could be oriented to measure all field components in the shock. We define a local coordinate system in which the shock moves in the x direction, the shock front is in the y-z plane, and the magnetic field in front of and behind the shock is in the x-y plane. For convenience we usually measure the z component of the field, since this is zero on both sides of the shock and only appears within the shock front due to the characteristic rotation of the magnetic field of the whistler.

Figure 1 shows unintegrated signals from a $\dot{B}_z$

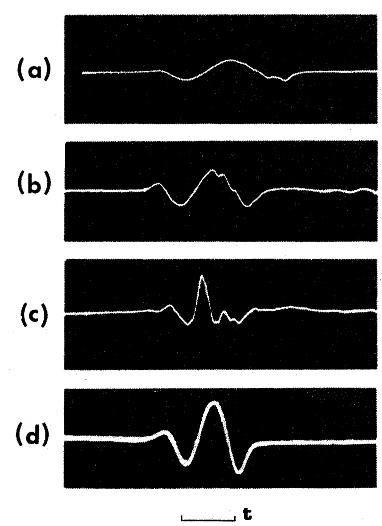

FIG. 1. Appearance of the perturbation on $\dot{B}_z$ as M_A is increased by increasing the shock velocity. Probe at 9.6 cm radius. Conditions: deuterium, $n_e=2.5\times10^{14}$ cm^{-3}, $B_0=500$ G, $\theta=45°$. (a) $M_A=2.7$, (b) $M_A=3.2$, (c) $M_A=4.0$, (d) hydrogen, $n_e=5.0\times10^{14}$ cm^{-3}, $M_A=4.0$. Time marker: (a)–(c) 100 nsec, (d) 50 nsec.

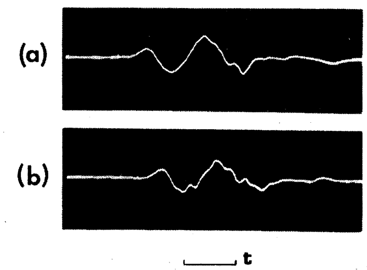

FIG. 2. Development with time of the perturbation on $\dot{B}_z$. Conditions: deuterium, $n_e = 2.5\times10^{14}$ cm^{-3}, $B_0 \doteq 500$ G, $\theta = 45°$, $M_A = 3.2$. (a) Upper probe at 9.6 cm radius; (b) lower probe displaced 7 mm in the direction of shock propagation. Time marker, 100 nsec.

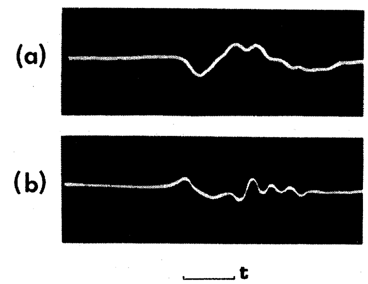

FIG. 3. (a) $\dot{B}_z$, (b) $\dot{B}_x$. Conditions as in Fig. 2(a). Note that the $\dot{B}_z$ signal is *attenuated by a factor of 3* compared to the $\dot{B}_x$ signal. Time marker, 100 nsec.

probe in a deuterium shock as M_A is increased by increasing the shock speed. At low M_A the perturbation appears at the back of the shock while at $M_A = 4.0$ it also appears on the first oscillation of the whistler. An unperturbed hydrogen shock is shown for comparison. Figure 2 shows the signals from two B_z probes separated by a distance of 7 mm in the direction of shock propagation. Here we see the perturbation developing, appearing later in time on the leading oscillation of the main shock structure. From similar measurements at different probe separations we find that the positions of the perturbation peaks with respect to the whistler do not change significantly within the time of observation ($\sim\lambda/u$). We conclude that the perturbation is traveling at about the same velocity as the shock.

Figure 3 shows simultaneous signals from two closely spaced probes measuring $\dot{B}_z$ and $\dot{B}_x$, respectively. The appearance of a component $\dot{B}_x$ having the same frequency as the perturbation on $\dot{B}_z$ indicates that the perturbation is propagating at an angle to the shock front, since the normal component of field does not change through a plane shock. (The small x component at the frequency of the main whistler is attributed to the small but finite curvature of the shock front.) Because of the small scale of the phenomenon it was not possible to obtain sufficiently accurate phase correlation between the two probes to establish the polarization of the perturbation.

The perturbation grows more rapidly as the direction of the shock becomes closer to perpendicular, and is then seen at late time in hydrogen as well as deuterium. Figure 4 shows a strongly perturbed hydrogen shock at $\theta = 65°$. Here the perturbation appears to cover the entire structure, and even extends ahead of it. At this stage the perturbation may have propagated from a different region of the shock front, and we can probably no longer treat local regions of the front as plane.

We can summarize these observations as follows: (1) A perturbation with a wavelength of $\frac{1}{4}$ to $\frac{1}{5}$ of the whistler wavelength grows with time upon the main whistler structure. (2) The perturbation appears earlier at larger Alfvén numbers, and at propagation angles closer to the perpendicular. (3) The perturbation usually appears to be stationary in the frame of the shock, but its k vector is at an angle to the shock front. (4) At late times the perturbation grows to the point where it completely destroys the fundamental single-mode whistler structure.

The stability of the whistler structure of the oblique shock was first considered by Galeev and Karpman,[5] who predicted that the structure would be unstable to a decay instability. The problem has recently been treated in some detail by Sloan[6] who finds that a finite-amplitude, obliquely propagating whistler will decay into whistler waves of shorter wavelengths with a growth rate $\gamma \sim k_0 u\tilde{B}/B_x$, where k_0 is the wave number of the original whistler, u its velocity of propagation, $\tilde{B}$ the amplitude of the magnetic field of the whistler, and B_x the component of the unperturbed magnetic field in the direction of k_0. It is an essential feature of Sloan's theory that the unstable perturba-

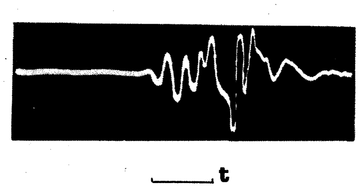

FIG. 4. Destruction of the whistler by the instability. Conditions: hydrogen, $n_e = 5\times10^{14}$ cm^{-3}, $B_0 = 500$ G, $M_A = 4.0$, $\theta = 65°$. Probe at 6 cm radius. Time marker, 100 nsec.

tions propagate obliquely to k_0; the whistler is stable to perturbations propagating parallel to itself, and hence no instability is found in one-dimensional analysis.

Sloan's rather complicated stability condition has been evaluated numerically by Macmahon[7] who finds that for a $\theta = 45°$ shock the fastest growing modes have $k_\parallel \approx k_\perp \approx (4-5)k_0$, and that these modes have their phase velocity close to the shock velocity. (Modes whose group velocity equals the shock velocity are also unstable.) The growth rate increases with increasing M_A (increasing $\tilde{B}$) and increasing θ (decreasing B_x). Thus, although the theory is derived for $\tilde{B}/B_x \ll 1$, while in the experiment this ratio is typically ~ 3, there is generally good agreement between the predictions of the theory and the experimental observations. The theory does not consider either binary collisions or effective collisions due to current-driven microinstabilities; inclusion of these effects might shed some light on the earlier onset of the instability in deuterium, which is otherwise unexplained.

The extremely rapid growth rate of the instability means that the single-mode whistler shock predicted by one-dimensional theory can be observed only for a brief period after the shock has been formed. Experiments in which stationary shock waves are produced by directing a stream of plasma against a magnetic dipole[8] and also Earth's bow shock[9] generally show a turbulent spectrum of whistler waves. In the present experiment we appear to be observing the initial stage of the instability, and the transition from a single-mode laminar shock structure to a turbulent one.

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Anomalous Penetration of a Magnetic Pulse into a Plasma*

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The penetration of a magnetic pulse into a cylindrical, field-free plasma has been studied under conditions where the penetration was determined by turbulent conductivity. The penetration time was found to vary as $(M_i n_e)^{1/2}/B$, where M_i is the ion mass, n_e is electron density, and B_0 is peak amplitude of the applied magnetic field. This corresponds to a mean conductivity $\sigma \propto M_i n_e/B_0^2$. These results can not be accounted for by any present theoretical model.

The development of an enhanced resistivity through the excitation of instabilities in a current-carrying plasma is a well-established phenomenon. Both experiments[1,2] and theory[3-5] have shown that an enhancement by many orders of magnitude over the classical resistivity can be obtained when large current densities are driven through a plasma by high electric fields. A con-