Higher-Precision Determination of the Fine-Structure Interval in the Ground State of Positronium, and the Fine-Structure Density Shift in Nitrogen*

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The fine-structure interval $\Delta\nu$ in the ground state of positronium has been remeasured with higher precision to obtain $\Delta\nu_{\rm expt} = 203.396 \pm 0.005$ GHz (24 ppm). The current theoretical value is $\Delta\nu_{\rm theor} = 203.4155$ GHz. The difference $\Delta\nu_{\rm expt} - \Delta\nu_{\rm theor} = -19 \pm 5$ MHz is about 4 standard deviations of the experimental error, indicating the need for calculations of higher-order radiative contributions. Values are given for both the linear and quadratic coefficients of the fine-structure density shift in N_2.

The importance of a highly precise measurement of the fine-structure interval $\Delta\nu$ in the ground state of positronium for a test of quantum electrodynamics, and in particular of the Bethe-Salpeter equation for the bound-state lepton-antilepton system, is well known.[1,2] The present Letter reports a value for $\Delta\nu$ with a substantially smaller error than the best previously quoted value.[3] Significant comparison of this new precise value for $\Delta\nu$ with theory clearly requires the calculation of higher-order radiative contributions to $\Delta\nu$.[4]

The method of the present experiment is the measurement of the Zeeman transition frequency between the $M=0$ and $M=\pm 1$ magnetic sublevels of orthopositronium by a magnetic resonance technique, and is basically similar to that used in previous measurements.[3,5,6] The transition frequency f_{01} in a magnetic field H is given by

$$f_{01} = \tfrac{1}{2}\Delta\nu[(1+x^2)^{1/2} - 1] \quad (1)$$

in which $x = 2\mu_{\rm B} g' H / h\Delta\nu$. The quantity g' is given by

$$g' = g(1 - \tfrac{5}{24}\alpha^2) = g(1 - 11.1\times 10^{-6}), \quad (2)$$

where g is the gyromagnetic ratio of the free electron (or minus that of the free positron),[7] $g = 2(1.001\,159\,656\,7 \pm 0.000\,000\,003\,5)$.

The factor involving α^2 is a relativistic bound-state contribution.[8-10] It was obtained using a Hamiltonian for positronium in an external magnetic field consisting of the Pauli Hamiltonian,[11] the Breit interaction,[12] and the virtual annihilation interaction.[13] A transformation developed by Chraplyvy[14] for two-body equations was used to decouple the large and small components. This approach is an extension of the Foldy-Wouthysen transformation[15] for the Dirac equation. The α^2 term arises from the relativistic change of mass with velocity and from the spin-orbit and spin-other-orbit interactions, but does not have a contribution from the virtual annihilation interaction. The α^2 factor was also derived[16] from a two-particle Breit equation by the method of Perl and Hughes.[17] Finally this factor can be obtained by the general derivation of Grotch and Hegstrom.[18]

Important improvements in apparatus and in experimental technique have enabled us to obtain the substantial improvement in accuracy. A schematic diagram of the experimental apparatus is shown in Fig. 1. A new electromagnet[19] with a pole diameter of 38 cm and a pole gap of 10 cm was used to produce a field of about 7800 G homogeneous to ± 4 ppm over a volume of 7 cm^3. The field was regulated to ± 0.5 ppm by an NMR stabilization system. The microwave system at 2.3 GHz was essentially that previously used,[3] and the positron source was 2.5 mCi of Na^{22}. The stopping gas was ultrahigh purity N_2, which allowed the use of sufficiently high input microwave

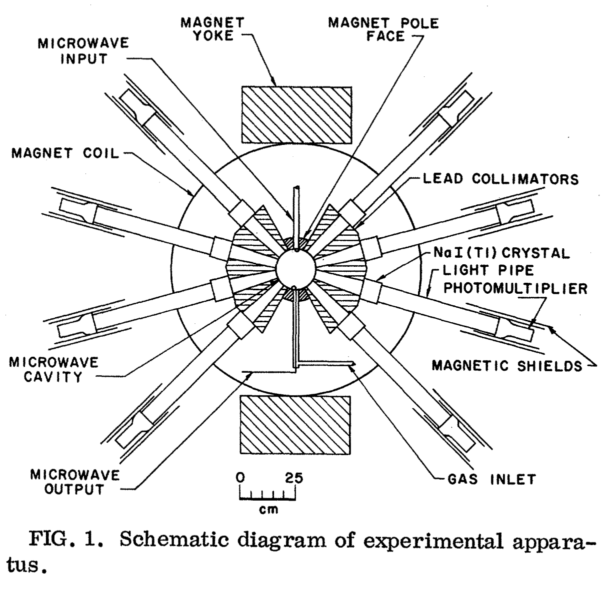

FIG. 1. Schematic diagram of experimental apparatus.

power at the lower pressures without electrical discharge. Eight NaI (Tl) γ-ray detectors were used to provide four coincidence detectors for two 0.51-MeV annihilation γ rays with a 0.2-μsec time resolution and an overall increase in detection solid angle by a factor of 9 compared to an earlier experiment.[3] The active volume for positronium observation of 7 cm^3 is about twice that of the previous experiment. Data were collected with a multichannel pulse-height analyzer.

A resonance curve was obtained by varying the magnetic field H with fixed microwave frequency and power. Signal heights were 3% to 11% and linewidths (full width at half-maximum) were 30 to 47 G. A nonresonant magnetic-field-dependent background, which arises from the magnetic focusing of positrons and from the increase in the $2\gamma/3\gamma$ branching ratio with increase in field and which amounts to about 5% of the resonance signal, was subtracted from the raw data by using a representation of the background linear in H over the resonance region. The remaining signal was fitted by the theoretical line shape as described for a previous experiment.[3] 54 resonance curves were obtained for N_2 pressures from 0.14 to 3.40 atm.

For each resonance curve a resonance microwave frequency f_{01} and central magnetic field H_0 is determined by the fitting procedure. Then with the use of Eq. (1) a value for $\Delta\nu$ is calculated. A plot of the values of $\Delta\nu$ versus N_2 density is shown in Fig. 2. The solid circles are the weighted means of all values of $\Delta\nu$ obtained at a given density, and the associated error bars are 1-standard-deviation errors, due predominantly to statistical counting errors. The data are fitted by the function

$$\Delta\nu(D) = \Delta\nu(0)(1 + aD + bD^2) \tag{3}$$

in which D is the gas density, and a and b are the linear and quadratic coefficients, respectively, of the fine-structure density shift. The results are given in Table I and indicate that not only a linear but also a quadratic density shift is present.

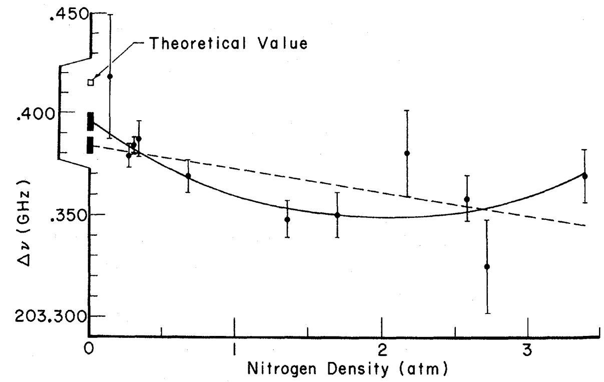

FIG. 2. Plot of $\Delta\nu$ versus N_2 gas density in units of atmospheres at 23°C. The solid and dashed lines are the quadratic and linear fits using Eq. (3), respectively. The solid bars show 1-standard-deviation errors for the fitted values $\Delta\nu(0)$.

The principal result of our experiment is a value for $\Delta\nu$ of free positronium based on the fit of Eq. (3) to our data:

$$\Delta\nu_{\rm expt} = 203.396 \pm 0.005 \text{ GHz (25 ppm)}, \tag{4}$$

in which the error is mainly due to a statistical counting error.[20] This result agrees with, and has 1/2.5 times the quoted error of, the best previously reported result[3]:

$$\Delta\nu_{\rm expt} = 203.403 \pm 0.012 \text{ GHz (60 ppm)}.$$

The current theoretical value is[4]

$$\Delta\nu_{\rm theor} = \tfrac{1}{2}\alpha^2 cR_\infty\left[\frac{7}{3} - \frac{\alpha}{\pi}\left(\frac{32}{9} + 2\ln 2\right) - \frac{3}{2}\alpha^2\ln\alpha\right]$$

$$= 203.4155 \pm 0.0006 \text{ GHz (3.1 ppm)} \tag{5}$$

using[21]

$$\alpha^{-1} = 137.036\,02 \pm 0.000\,21 \text{ (1.5 ppm)},$$

$$R_\infty = (1.097\,373\,1 \pm 0.000\,000\,1)\times 10^5 \text{ cm}^{-1} \text{ (0.1 ppm)},$$

TABLE I. Results of fits of the data $\Delta\nu(D)$ by Eq. (3) for N_2 gas.

	Linear fit	Quadratic fit
$\Delta\nu(0)$ (GHz)	203.384 ± 0.004	203.396 ± 0.005
a (10^{-5} atm^{-1} at 23°C)	−5.7 ± 1.4	−23 ± 6
b (10^{-5} atm^{-2} at 23°C)	0	+5.7 ± 1.8
χ^2 per degree of freedom	17.1/9	6.8/8
$\Delta\nu_{\rm expt} - \Delta\nu_{\rm theor}$ (MHz)	−31 ± 4	−19 ± 5

and[22]

$$c = (2.997\,924\,62 \pm 0.000\,000\,18) \times 10^{10} \text{ cm/sec} \quad (0.06 \text{ ppm}).$$

The indicated uncertainty in $\Delta\nu_{\text{theor}}$ is due to the uncertainty in the constants used in evaluating $\Delta\nu_{\text{theor}}$ from Eq. (5). The next uncalculated radiative correction term is of order $\alpha^4 R_\infty$ which is about 50 ppm.

The difference between the experimental and theoretical values for $\Delta\nu$ is

$$\Delta\nu_{\text{expt}} - \Delta\nu_{\text{theor}} = -0.019 \pm 0.005 \text{ GHz}, \tag{6}$$

where the uncertainty is the 1-standard-deviation experimental error. This difference is about 4 standard deviations of the experimental error. Clearly a more significant comparison of experiment and theory requires the calculation of the $\alpha^4 R_\infty$ radiative correction term.

The experimental values for the linear and quadratic coefficients of the fine-structure density shift are obtained from the quadratic fit of Table I. We note the large difference between the values of a determined with or without the allowance for a quadratic term. In earlier work with Ar gas, not enough data were obtained to test for the quadratic term.[3] No quantitative theory has yet been given for the positronium fine-structure pressure shifts.

We believe that our present apparatus and technique, when supplemented with improved data acquisition equipment and a higher-intensity Na^{22} source, are capable of achieving an accuracy of 10 ppm in the determination of $\Delta\nu$, which would correspond to a choice of the resonance line center of $\frac{1}{1000}$ of its full width at half-maximum.

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[1]S. J. Brodsky and S. D. Drell, Annu. Rev. Nucl. Sci. 20, 147 (1970).

[2]V. W. Hughes, in *Atomic Physics,* edited by V. W. Hughes, B. Bederson, V. W. Cohen, and F. M. J. Pichanick (Plenum, New York, 1969), p. 15.

[3]E. D. Theriot, Jr., R. H. Beers, V. W. Hughes, and K. O. H. Ziock, Phys. Rev. A 2, 707 (1970).

[4]T. Fulton, D. A. Owen, and W. W. Repko, Phys. Rev A 4, 1802 (1971).

[5]M. Deutsch and S. C. Brown, Phys. Rev. 85, 1047 (1952); R. Weinstein, M. Deutsch, and S. Brown, *ibid.* 94, 758 (1954), and 98, 223 (1955).

[6]V. W. Hughes, S. Marder, and C. S. Wu, Phys. Rev. 106, 934 (1957).

[7]J. C. Wesley and A. Rich, Phys. Rev. A 4, 1341 (1971); S. Granger and G. W. Ford, Phys. Rev. Lett. 28, 1479 (1972).

[8]E. R. Carlson, V. W. Hughes, and I. Lindgren, Bull. Amer. Phys. Soc. 17, 454 (1972).

[9]E. R. Carlson, V. W. Hughes, and I. Lindgren, in Abstracts of the Third International Conference on Atomic Physics, Boulder, Colorado, August, 1972 (unpublished), p. 32.

[10]M. L. Lewis and V. W. Hughes, in Abstracts of Third International Conference on Atomic Physics, Boulder, Colorado, August, 1972 (unpublished), p. 149.

[11]W. Pauli, Rev. Mod. Phys. 13, 203 (1941).

[12]V. W. Hughes, in *Facets of Physics*, edited by D. A. Bromley and V. W. Hughes (Academic, New York, 1970), p. 125.

[13]W. A. Barker and F. N. Glover, Phys. Rev. 99, 317 (1955).

[14]Z. V. Chraplyvy, Phys. Rev. 92, 1310 (1953).

[15]L. L. Foldy and S. A. Wouthuysen, Phys. Rev. 78, 29 (1950).

[16]E. R. Carlson, Ph.D. thesis, Yale University, 1972 (unpublished).

[17]W. Perl and V. W. Hughes, Phys. Rev. 91, 842 (1953).

[18]H. Grotch and R. A. Hegstrom, Phys. Rev. A 4, 59 (1971).

[19]Varian 15-in. electromagnet (V-3802).

[20]In Ref. 8 a preliminary value for $\Delta\nu_{\text{expt}}$ was reported. The value in Eq. (4) is based on more extensive data.

[21]B. N. Taylor, W. H. Parker, and D. N. Langenberg, Rev. Mod. Phys. 41, 375 (1969).

[22]Z. Bay, G. G. Luther, and J. A. White, Phys. Rev. Lett. 29, 189 (1972).