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[12] In Fig. 3(b) we show the intensity I (Weber's histograms $\sim I^{1/2}$) due to such an unpolarized source at the Galactic center, averaged for two antennas spaced 12° apart in longitude. The histogram for 4-h bins is indistinguishable from Weber's, which is in agreement with his conclusions. Note that grouping the data into 3-h bins rather than the usual 4-h bins greatly increases the histogram modulation. This is a sensitive test for unpolarized transverse tensor radiation.

Interpretation of Gravitational-Wave Observations*

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If Weber's gravitational-wave observations are interpreted in terms of a source at the Galactic center, both the intensity and the frequency of the waves are more reasonable if the source is assumed to emit in a synchrotron mode (narrow angles, high harmonics). Although presently studied sources for such modes are astrophysically unsatisfactory —high-energy, nearly circular, scattering orbits—other possible sources are under study.

Weber[1] has estimated that a straightforward interpretation of his observations would involve a source at the center of the Galaxy radiating isotropically an average power of $10^3 M_\odot c^2/\mathrm{yr}$ in the form of gravitational waves. In part, simply because of the fundamental significance of the detection of gravitational waves, but in part also because of the lack of more appealing astrophysical proposals for a source for this radiation, Weber's observations are not yet considered definitive. I am not, however, aware that there remains outstanding any proposal to explain the observations as artifacts arising from any cause other than unconscious observer bias. This last possible source of error is being eliminated by increasing automation of the experiment. Since, further, several independent attempts to verify Weber's observations are underway, the observational data could soon become undebatable. In this paper I presume that the gravitational-wave flux at the earth is that indicated by Weber's experiments, and suggest directions in which more satisfactory theories of the source can be sought.

The essence of my proposal is that one not focus attention on sources which emit primarily gravitational quadrupole radiation, but instead seek sources for synchrotron modes of gravitational radiation, i.e., modes which radiate high harmonics of the source motion frequencies in narrow angular distributions. In the following Letter it is shown that sources can, in principle, be manufactured which emit gravitational synchrotron radiation. The remainder of this note indicates why the observational evidence leads to a presumption that the source is emitting gravitational synchrotron radiation, and then indicates some directions in which one might hope to create a theory of the source mechanism.

I assume that the source is located near the center of our Galaxy for the reasons Weber[1] has suggested, and because assuming a distribution of sources throughout the Galactic disk (with the one closest to the Sun at a typical distance) can only increase the total energy requirements.

The assumption that a source at the Galactic center is radiating synchrotron modes ameliorates two difficulties faced by a quadrupole source: high power and low frequency. Weber's power estimate of $10^3 M_\odot c^2$/yr required for a quadrupole source would consume the entire mass of the Galaxy ($\sim 10^{11} M_\odot$) in only 1% of its age (1% of 10^{10} yr). But if the source were to radiate only within an angle $\Delta\vartheta \sim 10^{-3}$ of the Galactic plane, "only" 10% of the original Galaxy will have been consumed by a source operating since the birth of the Galaxy. It is possible to postulate such a very narrowly beamed source ($\Delta\vartheta \sim 10^{-3}$) since the distance of the Sun from the Galactic plane ($z_\odot = 4 \pm 12$ pc) is so very small[2] compared to its distance from the Galactic center[3] ($\sim 10^4$ pc). A very substantial reduction from the $10^3 M_\odot$/yr for isotropic radiation is required by observations of the mass loss from the center of the Galaxy. High mass-loss rates change the Galaxy's gravitational field and can lead to observable effects on the motion of gas and stars in the Galaxy. Sciama, Field, and Rees[4] show that mass-loss rates of $70 M_\odot$/yr persisting only 10^8 yr would already be at the observationally permissible limit.

Even a "low" radiated power of $1 M_\odot c^2$/yr from the Galactic center would require a very massive source to sustain the radiation for a reasonable time. But a mass M cannot be smaller than its Schwarzschild radius $2GM/c^2$, so a characteristic time scale for motions (at the velocity of light) cannot be shorter than $\omega_0^{-1} = 2GM/c^3 = (M/M_\odot) \times 10^{-5}$ sec. Thus quadrupole radiation at Weber's frequency $\omega \sim 10^4$ sec^{-1} cannot be emitted by sources more massive than $20 M_\odot$. The period for a small mass orbiting or falling into a $10^8 M_\odot$ black hole is about an hour:

$$T_0 \equiv 2\pi/\omega_0 = 4\pi M = (1.8 \text{ h})(M/10^8 M_\odot), \tag{1}$$

in units with $G = c = 1$. Since this is much longer than the millisecond period of Weber's bar, it is plausible to think in terms of a massive source at the center of the Galaxy only if the radiated frequency is a high harmonic of the fundamental frequency of mechanical motions within the source:

$$\omega = m\omega_0, \quad m \gg 1. \tag{2}$$

(Press[5] has also noted that high-frequency harmonics are desireable in connection with Weber's observations.) These two features of radiation —narrowly beamed angular distributions

$$\Delta\vartheta \ll 1 \tag{3}$$

and high-frequency harmonics m—which these observational considerations suggest, are characteristic of synchrotron modes of radiation.

The simplest way for a theoretician to compute the excitation of such modes of the radiation field is to use as a source a highly ralativistic particle in a circular orbit. For geodesic orbits the accompanying paper[6] shows that the beaming angle $\Delta\vartheta$ and the harmonic order m are related by $\Delta\vartheta = m^{-1/2}$. If this relation were valid for a more plausible source, one might have a black hole of mass $M = 10^7 M_\odot$, with $\omega_0 = 10^{-2}$ sec^{-1}, radiating in order $m = 10^6$ to give Weber's frequency $\omega = 10^4$/sec and a radiation pattern with $\Delta\vartheta = 10^{-3}$ rad. However, a more realistic source mechanism could show closer analogies to ordinary electromagnetic synchrotron radiation from accelerated charges where the relation $\Delta\vartheta \simeq m^{-1/3}$ holds. In this case one might propose $m = 10^7$, $M = 10^8 M_\odot$, and $\Delta\vartheta = 5 \times 10^{-3}$ as a possible set of parameters. Lower masses and lower harmonic orders give larger $\Delta\vartheta$ and increased power requirements, with $\Delta\vartheta < 0.05$ the limit for $< 50 M_\odot$/yr radiated.

It is natural to think that a large, rapidly rotating black hole at the center of the Galaxy could play a dominant role in any mechanism to generate gravitational synchrotron radiation (GSR). Only a black hole provides a gravitational potential well deep enough to accelerate masses to the relativistic velocities which GSR is expected to require. A rotating black hole, in addition, can act preferentially in its equatorial plane. This plane can reasonably be assumed to be aligned with the Galactic plane since the black-hole mass will be a significant fraction of the Galactic mass. The existence of a black hole of mass $10^7 M_\odot$ to $10^8 M_\odot$ at the center of our Galaxy has been proposed on other grounds by Lynden-Bell[7] in a theory of galactic nuclei designed to encompass Seyfert galaxies and quasars. Bardeen[8] then pointed out that it is most natural to assume that any such black hole has nearly the maximum angular momentum consistent with collapse.

No satisfactory theory of a source mechanism for GSR exists at present. I will describe below some proposals which have been studied enough to appear unlikely, but serve to illustrate relevent considerations which are as yet little explored. A further proposal which is still under study and has not yet been excluded is also outlined.

In a previous version of this Letter I conjectured that a mass in a bound stable circular orbit near the horizon of a rapidly rotating Kerr[9,10] black hole ($a \simeq M$) would emit GSR. Bardeen[11,12] objected that the particle motion was not adequately relativistic in a reasonable reference frame, and when the wave-equation computations were completed[13] they showed, in fact, no synchrotron radiation. Several other possible source mechanisms were then considered, in all of which the Penrose[14] mechanism for extracting rotational energy from a black hole was kept in mind. First, there will evidently be highly relativistic unbound, unstable circular Kerr geodesic orbits which, as in the Schwarzschild case, will emit GSR. Could a particle be accelerated into such an orbit by radiating negative-energy gravitons down the black hole? Apparently not, since in order to radiate predominantly negative-energy gravitons the particle would itself likely have to be in a negative-energy orbit and any such mechanisim would shut off before large positive energies were accumulated by the radiating particle. Next the question was raised, could a stellar or planetary mass break up under the action of tidal forces in such a way that some fragments had negative energies but others were highly relativistic? A negative answer is implied by the following idealized calculation. A mass μ of energy $\mu\gamma$ and angular momentum zero (referred to infinity) falls inward and then fissions into two equal fragments, each with energy per unit mass γ_f in the rest frame of the fissioning particle, so $\mu = 2\mu_f\gamma_f$. The infinity energies of these fragments are at best $\gamma_\pm = \gamma_f(\gamma \pm \beta_f)$ when the fission occurs transversely near the horizon of an extreme ($a \simeq M$) Kerr black hole. Since $\beta_f^2 = 1 - \gamma_f^{-2} < 1$, no large advantage is gained over the energies γ_f released locally (which would be comparable to the binding energy of the original mass) although for $\gamma < \beta_f$ some energy is extracted from the black hole.

As another attempt to find sources for synchrotron modes, some aspects of the stability of the Kerr metric were considered. Perhaps if an infalling mass spins up a black hole above a critical angular momentum $J = a_{\mathrm{crit}}M$, the black hole becomes unstable and begins throwing out gravitational waves balanced by others of negative energy sent down the hole? But a model calculation for scalar "gravitons" shows that a black hole, when perturbed by an incident scalar wave, will never bounce the wave back out with more than a small amplification, i.e., the reflection coefficient does not exceed unity by more than a small amount for any mode. Thus these simple provocations do not result in a strong burst of gravitational radiation for any $a \leq M$. (The details of this will be published elsewhere.)

"Kerr plunge radiation" remains a plausible speculation as a source for GSR and is being actively studied. This is the radiation emitted by a particle falling into a rapidly rotating black hole. An analogous problem in the Schwarzschild metric has been studied,[15] and the radiation emitted is unspectacular. However, the problem for a nearly maximal Kerr metric ($a \simeq M$) is qualitatively different. One difference is the Penrose effect[14] which allows some of the radiated energy to be extracted from the rotation of the black hole instead of from the energy of the in-falling particle. The second and perhaps more important difference is that in the case $a \simeq M$ an in-falling particle can be accelerated to relativistic local energies before reaching the "radiation belt" near the null circular orbit. In addition, the red-shift formula in the Kerr metric is direction dependent so that radiation (such as the synchrotron modes) emitted with a momentum component along the direct equatorial direction is not significantly shifted to the red. Thus the energy gained by a particle in falling into the gravitational potential need not be proportionately expended by a radiated graviton climbing back out of the potential well if it is emitted in favorable directions. This can be thought of as due to an $\vec{L}\cdot\vec{J}$ term in the potential binding a mass to a black hole of angular momentum J. This term is also responsible for the possibility of negative-energy orbits for gravitons or other particles with negative orbital angular momentum $\Phi \equiv \vec{L}\cdot\vec{J}/|J|$ and sufficiently small r (in the "ergosphere"[16]).

The formula the above words describe is

$$E = (\rho^2\Delta/B)^{1/2}E_{\mathrm{B}} + (2r/B)\vec{J}\cdot\vec{L}, \tag{4}$$

where E is the total energy of the particle in question, while E_{B} is its locally measured energy in the standard ("Bardeen") frame defined by a unique family of locally nonrotating stationary observers.[8,17] The other notations are conventional for the Kerr metric, namely, $\Delta \equiv r^2 + a^2 - 2Mr$, $\rho^2 \equiv r^2 + a^2\cos^2\theta$, and $B \equiv \rho^2(r^2 + a^2) + Mra^2 \times \sin^2\theta$, where (t, r, θ, φ) are the Boyer-Lindquist coordinates, while M and $J = aM$ are the mass and angular momentum of the black hole.[8,10] The horizon (one-way surface) is defined by $\Delta = 0$. The relation of $\vec{J}\cdot\vec{L} \equiv aM\Phi$ to the locally measured transverse linear-momentum component $p_{\hat{\varphi}}$ is given by

$$\Phi = p_{\hat{\varphi}}(B/\rho^2)^{1/2}\sin\theta. \quad (5)$$

In the most interesting case where $\alpha^2 \equiv 1-(a/M)^2 \ll 1$, the direct null circular orbit (which is the region of an effective potential maximum in the wave equation, hence the region in which falling particles might radiate most intensely) is $r = r_\gamma \simeq M[1+(2/\sqrt{3})\alpha]$ and $\theta = \pi/2$ so Eq. (4) reads, at r_γ, $E \simeq (\alpha/2\sqrt{3})E_B + p_{\hat{\varphi}}$. (For the Schwarzschild metric the equivalent formula at $r_\gamma = 3M$ is $E = E_B/\sqrt{3}$.) For particles falling in with nonrelativistic transverse momenta, $|p_{\hat{\varphi}}| \ll E_B$, this shows high binding energy, $E \ll E_B$. For gravitons, or other relativistic particles with $p_{\hat{\varphi}} \sim E_B$ it shows little red shift ($E \sim E_B$) except for limited angles satisfying $|p_{\hat{\varphi}}/E_B| \ll 1$. Retrograde gravitons with $p_{\hat{\varphi}}/E_B < -\alpha/2\sqrt{3}$ have negative energy and are captured by the black hole, thus partially compensating the energy losses due to radiation of positive-energy gravitons, of which some are captured and others escape.

Calculations of radiation based on a wave equation[18] are needed not only to define the radiation pattern and intensities for which "intuitive" estimates are not yet reliable, but also to compute (from electromagnetic[19] or tensor[20] wave equations) the polarizations. It is not clear that plunge orbits would produce the high polarizations which characterize synchrotron radiation from circular orbits.

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