The intensity problem could be solved by combining a high-flux reactor with a heavy-ion accelerator.

[5]L. Westgaard, private communication, and in the Sixth European Conference on the Interactions of High Energy Particles and Complex Nuclei, Kitzbuhel, Austria, 26 September to 2 October 1971 (unpublished). He estimated a production rate of about 5000 nuclei per month for the reaction $^{238}U(^{56}Ca,xn)^{294-x}112$ when using CERN's 25-GeV 10^{12}-protons/sec beam. A. Ghiorso and W. J. Swiatecki recently proposed a "low-energy" reaction in order to increase the N/Z ratio in heavy ions. At energies of at least 8-MeV per particle, projectiles such as Ca or Ge can pick up a few neutrons (or be stripped of few protons) in grazing collisions with a Th nucleus prior to compound system formation in the same thick Th target.

[6]W. J. Swiatecki, in Proceedings of the Nordic-Dutch Accelerator Symposium, Ebeltoft, Denmark, 19 May 1971 (unpublished), and private communication.

[7]M. Lefort, private communication, and in the Sixth European Conference on the Interactions of High Energy Particles and Complex Nuclei, Kitzbuhel, Austria, 26 September to 2 October 1971 (unpublished).

[8]By this we mean process characterized by millisecond time constants rather than microseconds (prompt) or seconds (rapid), as discussed in the text.

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[12]See, e.g., Fig. 4 of J. R. Nix, LASL Report No. LA-DC-12488, 1971 (unpublished).

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[14]G. I. Bell, Rev. Mod. Phys. 39, 59 (1967).

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[16]D. W. Dorn and R. W. Hoff, Phys. Rev. Lett. 14, 440 (1965). See also G. I. Bell, Phys. Rev. 158, 1127 (1967); and J. S. Ingley, Nucl. Phys. A124, 130 (1969). Eccles (Ref. 11) was, in fact, able to reporduce all yield curves by using *measured* (n,p) and (n,α) cross sections together with extrapolated capture rates.

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Nuclear Surface Energy and Neutron-Star Matter*

D. G. Ravenhall,† C. D. Bennett,‡ and C. J. Pethick†§
University of Illinois, Urbana, Illinois 61801
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Direct calculations of the nuclear surface energy are made for a Hamiltonian containing the Skyrme nucleon-nucleon interaction. A plane surface separating nuclear matter and a neutron gas or a vacuum is considered in Hartree-Fock and Thomas-Fermi approximations. These surface energies are incorporated in the compressible liquid-drop model to obtain properties of neutron-star matter. The Hartree-Fock results lead to Z values for the nuclei roughly constant at around $Z \sim 36$–38.

Neutron-star matter at densities between 4×10^{11} gm/cm^3 and approximately nuclear densities consists of neutron-rich nuclei immersed in a gas of pure neutrons. The size of the nuclei is determined by competition between the nuclear surface energy and the electrostatic Coulomb energy, and the atomic number Z is directly proportional to the energy per unit area of the nuclear surface. In the calculation of Baym, Bethe, and Pethick[1] (BBP) the surface energy inserted into their compressible liquid-drop model was estimated on the basis of dimensional arguments. The resulting

Z increases continuously with increasing neutron-gas density. On the other hand, Buchler and Barkat[2] (BB) performed differential Thomas-Fermi calculations of a whole unit cell of neutron-star matter, in which the surface energy is contained only implicitly. They found that *Z decreases* continuously, once the neutron-gas density is above a rather small value.

In this paper we make a direct calculation of the nuclear surface energy. We examine the properties of a plane interface between two nuclear-matter phases, in the Hartree-Fock (HF) and the Thomas-Fermi (TF) approximations. We are thereby able to check the values of the surface energy used by BBP. By constructing neutron-star matter using these surface energies, we can also make a comparison with the work of BB. Furthermore, we can compare the semiclassical TF approximation with the quantum-mechanical HF method. Our results are as follows: The HF surface energies are about 25% bigger than the TF values in the region of interest for neutron-star matter, although they are about a factor of 4 below the BBP values.[1] The Z values we obtain are comparable to those obtained by BB.[2]

The plane-surface approach enables us to study properties of the nuclear surface itself, without any contamination from shell effects. It is therefore particularly useful for comparing different methods of calculating properties of the surface. Also we can investigate readily the gross features of nuclear energies for a range of nuclei, using the compressible liquid-drop model, without having to perform a detailed microscopic calculation for each nucleus.[3]

The nuclear model we consider is that of nucleons interacting through a contact pseudopotential, as suggested by Skyrme.[4] The particular Hamiltonian $H(R)$ we use follows closely the form given by Vautherin and Brink,[5] with a slight modification in the isospin dependence of the three-body interaction[6] to improve the agreement with the neutron-gas results of Siemens and Pandharipande,[7] obtained using nuclear-matter theory.

The physical system consists of a planar-surface region separating a mixture of neutrons and protons to the left of the surface, and a pure neutron gas (or a vacuum) to the right of the surface. Far to the left of the surface the neutron, proton, and total nucleon densities approach the values $n_{0,n}$, $n_{0,p}$, and n_0 ($\equiv n_{0,n}+n_{0,p}$), and far to the right of the surface the neutron density approaches the value n_d. At the start of the calculation these asymptotic densities are chosen to make pressures and neutron chemical potentials equal on the two sides of the surface; this ensures that the two phases can coexist in thermodynamic equilibrium. The enumeration of states necessary to satisfy the bulk-equilibrium boundary condition is taken from Bennett and Duke's calculation of properties of a metallic interface.[8] The rest of the procedures are intricate but fairly standard. We start with appropriately chosen interpolating functions for the particle densities and the effective masses between two positions R_1 and R_2 to the left and to the right of the surface, and sufficiently distant from it. In about five HF iterations the surface stabilizes in shape, and we can examine its properties.

Surface profiles obtained from the HF calculation are shown in Fig. 1, for a typical proton concentration. For comparison purposes we show also the results of TF calculations with exactly the same Hamiltonian and boundary conditions. We use two versions of the TF approximation: The semiclassical neutron and proton fluids are either independent (two-fluid version, TF-II), or are proportional according to the relationship $n_p(R) \propto n_n(R) - n_d$ (single-fluid version, TF-I). The latter version formed the basis for BBP's consideration of surface energy, whereas the former is close in spirit to BB's calculations. Differences between the two TF profiles give information about the role of neutron-proton separation and symmetry energy, thereby calibrating the contribution of this effect in the HF calculation, in which the neutrons and protons are neces-

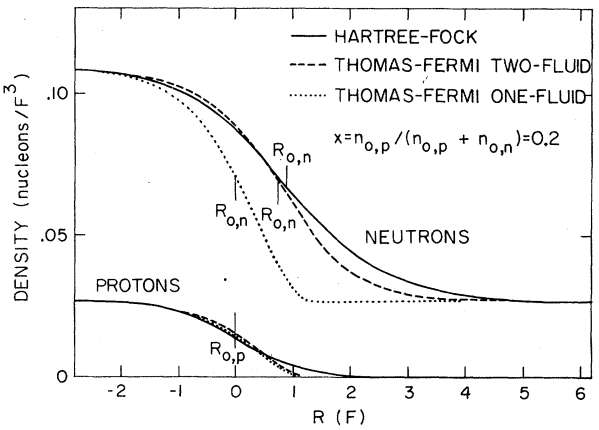

FIG. 1. Dependence of particle densities on distance in the region of the surface. To the left is a neutron-proton mixture with $x=0.2$, to the right a neutron gas. The origin R is taken arbitrarily as the proton radius R_p. The boundaries R_1 and R_2 are typically -15 F and $+15$ F, respectively.

sarily independent.

One has to be rather careful when defining the surface energy for the two-component system, since the positions R_n and R_p of the surfaces for the neutrons and protons differ. By studying the way in which the surface energy is used in mass formulas for ordinary nuclei and in the neutron-matter calculations of BBP, it may be shown that the appropriate definition is

$$E_{\text{surf}} = \int_{R_1}^{R_2} H(R)\,dR - (R_p - R_1)n_0 W_0 - (R_2 - R_p)n_d W_d - \mu_n \Delta N_n, \qquad (1)$$

where $\Delta N_n = (R_n - R_p)(n_{0,n} - n_d)$ is the number of excess neutrons in the surface region, μ_n is the neutron chemical potential, and W_0, W_d are the asymptotic energies per nucleon. R_1 and R_2 are the distant points, to the left and to the right of the surface, respectively, which define our system. The proton and neutron radii R_p and R_n are defined by the equations

$$\int_{R_1}^{R_2} n_n(R)\,dR = n_{0,n}(R_n - R_1) + n_d(R_2 - R_n), \qquad (2a)$$

$$\int_{R_1}^{R_2} n_p(R)\,dR = n_{0,p}(R_p - R_1). \qquad (2b)$$

E_{surf} is the energy per unit area of our plane surface. The customary surface energy coefficient in the nuclear mass formula is $W_{\text{surf}} = 4\pi r_0^2 E_{\text{surf}}$, where r_0 is obtained from the saturation density at the particular value of x.

Plots of the surface energy as a function of $x = n_{0,p}/n_0$, the fraction of protons in the matter to the left of the boundary, are shown in Fig. 2. E_{surf} tends rapidly to zero as $x \to 0$, and yet it must be obtained, according to Eq. (1), as the difference of two quantities necessarily of order 10 MeV/F^2. The TF results shown in Fig. 2 present no problem on this score, but the much more complex HF calculations may be accurate only down to about $E_{\text{surf}} \sim 0.01$ MeV/F^2. This is adequate for present purposes. Points to be noticed are as follows:

(i) For symmetrical nuclear matter ($x = 0.5$) corresponding to physical nuclei with $N = Z$, the HF surface energy is about 25% above the TF value; the HF value for $W_{\text{surf}}(0.5)$ is 19.2 MeV, compared with the empirical value of 18.6 MeV.

(ii) As one would expect, the TF two-fluid calculation, which has an extra degree of freedom associated with neutron-proton separation, gives a surface energy considerably lower than the TF one-fluid calculation.

(iii) The HF value remains larger than the TF two-fluid value for all x; if, however, the TF values are "renormalized" to give the same value at $x = 0.5$, a reasonable practical procedure, then HF values are different from TF-II by at most 2% for $x > 0.16$ (which corresponds to densities less than $\sim 10^{14}$ g/cm^3).

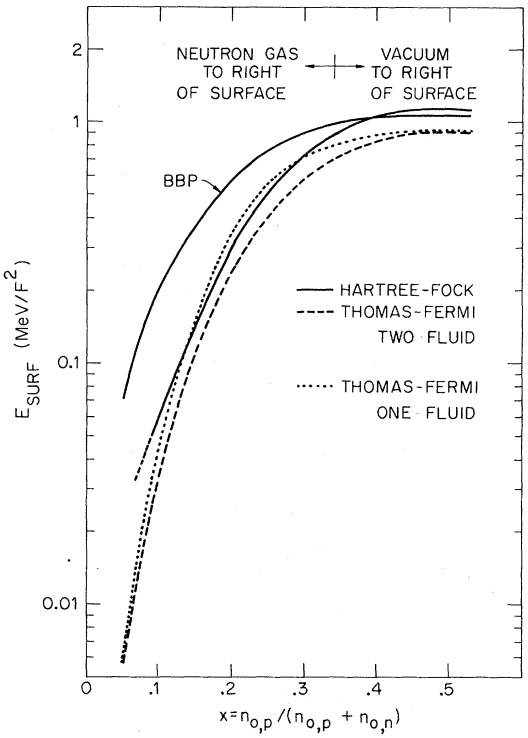

FIG. 2. The surface energy per unit area as a function of x, the proton concentration to the left. The curve labeled BBP comes from Ref. 1.

(iv) The symmetry coefficient C_{sym} of the customary surface energy W_{surf} is defined by $W_{\text{surf}}(x) \simeq W_{\text{surf}}(0.5)[1 - (1-2x)^2 C_{\text{sym}}]$, for $x \to 0.5$. For our HF results, $C_{\text{sym}} \simeq 2.1 \pm 0.2$. The corresponding symmetry coefficient B_{sym} of the bulk energy W has the value 1.82. This indicates that the assumption made by Myers and Swiatecki[9] in their semiempirical mass formula, that $B_{\text{sym}} = C_{\text{sym}}$, is a reasonable approximation.

(v) The results of the TF and HF calculations are considerably below those of BBP, the ratio being a factor ~ 0.25 at $x = 0.15$. BBP's result for the surface energy varied as $\sim x^2$, whereas our TF results appear to vary as $\sim x^3$. The reason for this difference can be understood physically, and will be discussed elsewhere.

(vi) As far as we can tell from the numerical calculations, the slope of the surface energy, $\partial E_{\text{surf}}/\partial x$, is continuous at $x = 0.34$, the value below which free neutrons appear outside the nucle-

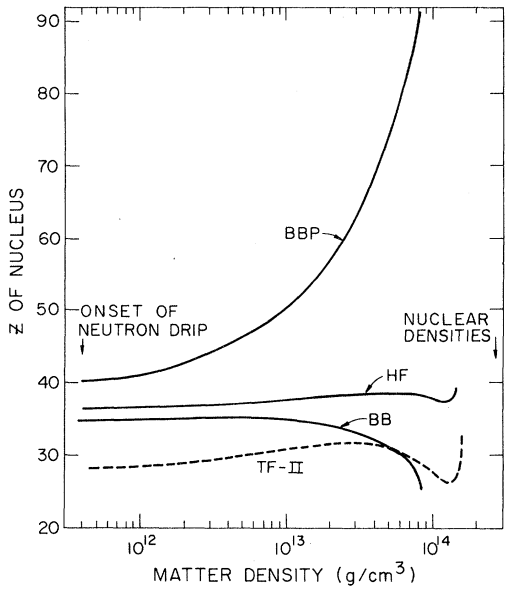

FIG. 3. Number of protons, Z, per nucleus in neutron-star matter, as a function of the matter density. The curve labeled BBP comes from Ref. 1, and the curve labeled BB from Ref. 2.

us.

Our results for the surface energy may be incorporated into the compressible liquid-drop model, along with bulk-matter properties obtained from the original Hamiltonian. We thereby have a "mass formula" in which no coefficients are directly empirical, but all come consistently from one nuclear Hamiltonian. With this model we have calculated the properties of neutron-star matter, and in Fig. 3 we show plots of Z, the quantity most sensitive to the surface energy, as a function of the matter density ρ. The (HF) calculations give Z values typically some 25% larger than the TF-II calculations, which is consistent with the relation $Z \sim E_{\text{surf}}$. At low densities the TF-II Z values are about 20% below those of BB, a difference which is completely taken care of by the renormalization described in (iii). This is as we expect, since BBP's liquid-drop model should be an adequate description of nuclei as large as the ones we find ($A \sim 100$–200). At densities between 10^{13} and 10^{14} g/cm^3, our results have a slightly different trend from those of BB, and this is probably due to the differences in the Hamiltonians, since both our TF-II and the HF results display it. For densities above 10^{14} gm/cm^3, the intriguing behavior of Z displayed by our model is probably dependent on unverified details of the Hamiltonian. The rather small values of the surface energy needed there are obtainable from our calculation, however, and the liquid-drop approach provides a good method for exploring the different phases of matter that occur at high densities.

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†Physics Department, University of Illinois, Urbana, Ill. 61801.

‡Computer-based Education Research Laboratory, University of Illinois, Urbana, Ill. 61801.

§Alfred P. Sloan Research Fellow.

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