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†Now at University of Toronto, Toronto, Canada.

‡Now at University of Washington, Seattle, Wash.

§Now at National Accelerator Laboratory, Batavia, Ill.

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[8]The data for $|t'|$ less than 0.02 GeV^2 are not shown in Fig. 1 since, for part of the momentum region studied, the recoil protons in the t' interval are unobservable in the hydrogen bubble chamber.

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[11]One may interpret $\langle \mathrm{Im}R\rangle$ as being approximately the helicity nonflip t-channel Regge amplitude since t-channel helicity is nearly conserved and there is no evidence for a forward turnover in the differential cross section for diffraction dissociation processes.

[12]Over the t' range studied, we note that an almost indistinguishable curve is obtained using the parametrization form of Ref. 2: $\langle \mathrm{Im}R\rangle = 0.3\exp(0.7t')J_0(6.5\times\sqrt{-t'})$ mb$^{1/2}$ GeV^{-1}. However, the factor of 6.5 GeV^{-1} should not be taken literally as an interaction radius since several helicity amplitudes probably contribute to $\langle \mathrm{Im}R\rangle$ as discussed in the text.

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Rapidity Correlations between the Negative Pions Produced in the K^+p Interaction*

Winston Ko

Department of Physics, University of California, Davis, California 95616

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Dramatic behavior is observed in the correlation between π^-'s as a function of their rapidity separation Δy. At small Δy, the π's are strongly positively correlated. At large Δy the correlation length is measured to be 0.52 ± 0.03. Particle-production and particle-particle correlations are distinguished and studied.

In the attempts to understand the fundamental nature of strong interactions by studying inclusive reactions, the correlation between particles along the longitudinal direction is one of the most important[1-7] measurements. The "correlation length," which estimates the rapidity separation beyond which particle productions become independent of one another, could be of great value. The shape of the correlation curve as a function of rapidity separation is a quantity that a correct inclusive theory must be able to derive in addition to agreeing with single-particle spectra.

A simple longitudinal correlation function was proposed by Wilson[1] and thereafter has been used in many models and theories, among which is Mueller's generalized optical theorem.[8] This correlation function between particles c and d is defined by

$$g(y_1, y_2) = \frac{d^2\sigma}{dy_1^{\,c}\,dy_2^{\,d}} - \frac{1}{\sigma_T}\frac{d\sigma}{dy_1^{\,c}}\frac{d\sigma}{dy_2^{\,d}}. \quad (1)$$

It is the difference between the coincidence of c and d at rapidity values y_1 and y_2 and the product of their singles counts at y_1 and y_2 divided by the total cross section. At finite energies, this measure of correlations has its difficulties; among others, the kinematic limits of the two terms in Eq. (1) are different.

It is the purpose of this Letter to study critically this longitudinal correlation using a high-statistics sample of π^-'s from a 31-events/μb K^+p interaction[9] experiment at 12 GeV/c. A total of 67 463 π^- pairs were studied. No events with charge multiplicity of 10 or higher are considered.

First we compare the two terms of Eq. (1). In Fig. 1(a), the solid histogram is the coincidence of two π^-'s when their rapidities are Δy from each other. The dotted histogram is the product of the single-particle rapidity-distribution histograms, when two bins are Δy apart, divided by the total cross section (17 mb). This dotted histogram includes the four-prong events although no π^--π^- coincidence is possible for those events. To obtain the correlation functions of Eq. (1) we will take the difference of the solid and the dotted histograms. Now, in addition to the statistical error, the errors in knowing the absolute normalization will play an important role. Unlike the single-particle distribution where the normalization only affects the absolute value, the *shape* of the curve can be affected here.

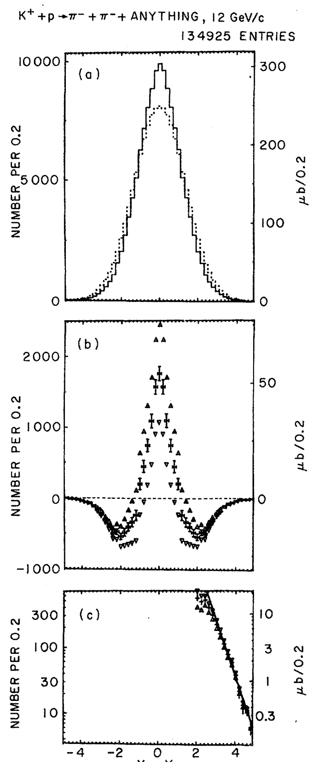

FIG. 1. (a) Solid histogram, coincidence $d^2\sigma/dy_1dy_2$ of two π^-'s with rapidity separation Δy. Dotted histogram, $(1/\sigma_T)(d\sigma/dy_1)(d\sigma/dy_2)$, the product of inclusive one-π^- distributions divided by total cross section. (b) The difference of the two histograms of (a). Upright and inverted trianges, values when absolute normalization is $\pm 7\%$ off. (c) Absolute value of the quantities in (b) for $\Delta y \geqslant 2$. Dotted curve, error of the straight-line fit to points for $\Delta y > 3$.

Figure 1(b) shows the correlation as a function of Δy. The error bars are for the statistical error only, and the upright and inverted triangles indicate the value of the correlation when the path length is $\pm 7\%$ off. One observes in the plot a striking correlation at $\Delta y = 0$. This correlation drops rapidly and reaches zero correlation at $\Delta y = 1$. At $\Delta y = 2$ the two π^-'s negative correlation reaches the maximum. This negative correlation gradually tapers down at 0.

As in the single-particle spectrum, we can divide the particle production into fragmentation and central regions; we are interested to know the correlations within one region and between different regions. Figure 2 shows the correlation function as a function of Δy when we fix one of the particles involved in the target fragmentation region ($-3.3 < y_1 < -1$), the central regions ($-1 < y_1 < 0$, $0 < y_1 < 1$), and the projectile fragmentation region ($1 < y_1 < 3.3$).

The strong correlation at $\Delta y \simeq 0$ occurs only when one (and therefore both) particle is in the central region. So far we know of no models that predict this effect. It could be the manifestation of the Bose-statistical behavior we have observed earlier.[10]

At large Δy, the rate of decrease of the negative correlation as we increase Δy determines the correlation length L defined by

$$g(\Delta y; \Delta y \gg L) \sim e^{-\Delta y/L}. \qquad (2)$$

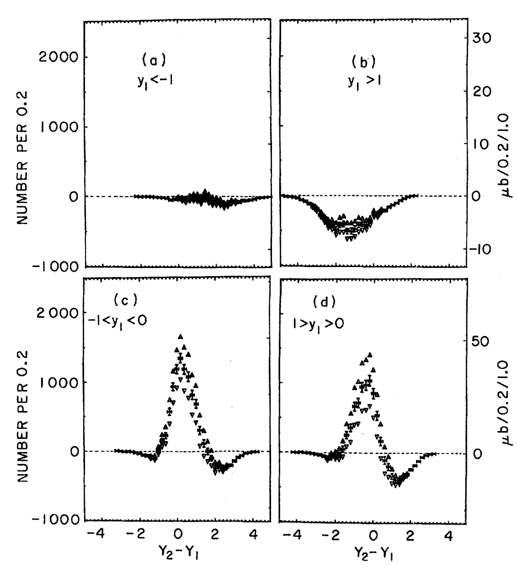

FIG. 2. The quantities of Fig. 1(b) with cuts (a) $-3.3 < y_1 < -1$, (b) $1 < y_1 < 3.3$, (c) $-1 < y_1 < 0$, (d) $0 < y_1 < 1$.

This exponential behavior is apparently observed in a semilog plot of $|g|$ versus Δy in Fig. 1(c). A correlation length of 0.52 ± 0.03 is measured for $\Delta y > 3$. The contribution to the negative correlation is largely from the π^-'s in the projectile fragmentation region (Fig. 2). This correlation length we have measured is, therefore, the correlation between the projectile fragments with both the central and target fragments.[11]

The behavior at large $|\Delta y|$ is, however, dominated by the four-prong contribution to the second term of Eq. (1). In four-prong events, no coincidence is possible; yet the width of its rapidity distribution is wider than that of six- or eight-prong events because of kinematics. When the correlation is affected largely by the contributions where no correlation is possible, it seems to us that the interpretation of the correlation is in doubt.

The question of correlation may become clearer if one separates two kinds of correlations. The previously discussed correlation is a mixture of the two. The first kind is the particle-production correlation, where one asks the question, "If one finds a π^- with some rapidity y, how many other π^-'s can one expect to see with any value of y?" The second kind is the particle-particle correlation in which, given a definite number of particles, one observes the relation of pairs in the rapidity space.

The particle-production correlation is, of course, related to the average multiplicity of the π^- distribution.[12] In Fig. 3 the average number of *other* π^-'s produced in conjunction with the chosen π^- is plotted as a function of the rapidity of the chosen π^-. One sees that a π^- in the central region has a relatively good chance of having another π^- produced with it.

In studying the particle-particle correlation, we first construct an uncorrelated expression that should be the coincidence rate when there is no correlation between any pairs of a given set of n π^-'s. For the events with n π^-'s, we have

$$\int (d\sigma/dy)_n \, dy = n\sigma_n, \tag{3}$$

where σ_n is the particle partial cross section of n-π^- production and $(d\sigma/dy)_n$ is the single-particle spectrum when there are n π^-'s. The probability to find a π^- at y is therefore $(1/n\sigma_n)(d\sigma/dy)_n$. The joint probability for a pair of π^-'s to occur at y_1 and y_2 is

$$\frac{1}{n\sigma_n}\left(\frac{d\sigma}{dy_1}\right)_n \frac{1}{n\sigma_n}\left(\frac{d\sigma}{dy_2}\right)_n; \tag{4}$$

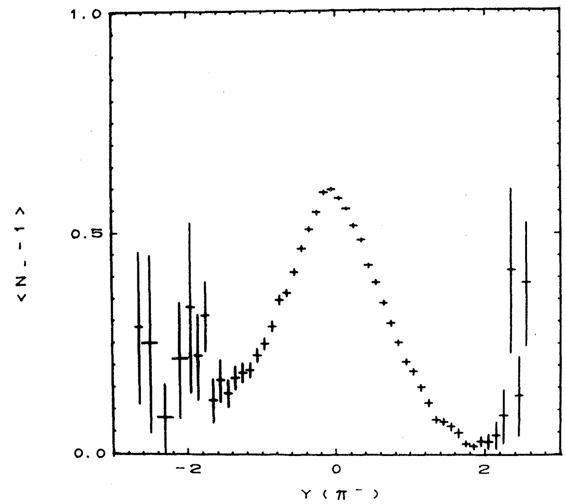

FIG. 3. $\langle N_- - 1\rangle$ as a function of the rapidity of a π^-.

but, for n π^-'s we can construct $n(n-1)$ pairs of π^-'s with values y_1 and y_2, so we should multiply by this factor. Normalized to the particle cross section, the coincidence rate (in millibarns) of n uncorrelated π^-'s is

$$\frac{n-1}{n\sigma_n}\left(\frac{d\sigma}{dy_1}\right)_n\left(\frac{d\sigma}{dy_2}\right)_n. \tag{5}$$

We will investigate the difference of measured coincidence rate to the uncorrelated coincidence rate of Eq. (5). We define

$$G_n(y_1, y_2) = \left(\frac{d^2\sigma}{dy_1 dy_2}\right)_n - \frac{n-1}{n\sigma_n}\left(\frac{d\sigma}{dy_1}\right)_n\left(\frac{d\sigma}{dy_2}\right)_n. \tag{6}$$

Here, since the number of π^-'s are fixed, the integration of G_n over y_1 and y_2 vanishes. In other words, a positive correlation at some point in the rapidity space will cause a negative correlation somewhere else. Another point: The error of G_n should only be statistical; the uncertainty in absolute normalization now only affects the absolute value of G_n, rather than changing the shape as we discussed in the case of the correlation function g of Eq. (1).

Figures 4(a) and 4(b) shows G_2 and G_3 for six and eight prongs, respectively. For the six-prong data we see that the two π^-'s still show strong correlation near $\Delta y \simeq 0$ but the peaking is much narrower so that zero correlation is reached near $\Delta y \sim 0.4$. After reaching maximum negative correlation at $\Delta y \sim 1$, the curve turns again and crosses 0 at $\Delta y \sim 2$. Unlike the correlation function g, we now start to have positive correlation *again;* it reaches its maximum value near $\Delta y \sim 3$.

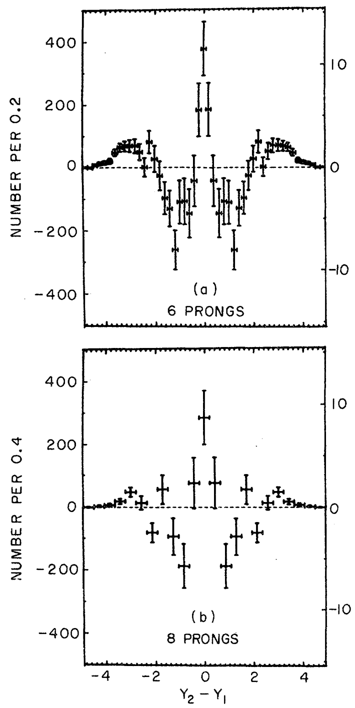

FIG. 4. (a) G_2 and (b) G_3 as defined in the text.

The statistics on the eight-prong data are not good but show characteristics similar to those for six-prong.

We present this dramatic behavior of the two π^-'s as functions of their rapidity separation as a challenge to any theory of inclusive reactions.

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Experimental Test of Local Hidden-Variable Theories*

Stuart J. Freedman and John F. Clauser
Department of Physics and Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
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We have measured the linear polarization correlation of the photons emitted in an atomic cascade of calcium. It has been shown by a generalization of Bell's inequality that the existence of local hidden variables imposes restrictions on this correlation in conflict with the predictions of quantum mechanics. Our data, in agreement with quantum mechanics, violate these restrictions to high statistical accuracy, thus providing strong evidence against local hidden-variable theories.

Since quantum mechanics was first developed, there have been repeated suggestions that its statistical features possibly might be described by an underlying deterministic substructure. Such features, then, arise because a quantum state represents a statistical ensemble of "hidden-variable states." Proofs by von Neumann and others, demonstrating the impossibility of a hid-