Turbulent Heating in Computer Simulations of the Modified Plasma Two-Stream Instability

Edward Ott,* John B. McBride, Joseph H. Orens, and Jay P. Boris
Naval Research Laboratory, Washington, D. C. 20390
(Received 1 November 1971)

Computer simulations of the fluidlike modified two-stream instability, driven by relative electron-ion drifts across a magnetic field, show strong anomalous plasma heating. The electron and ion heating rates are comparable and result in ion thermal speeds which approach the relative drift speed within a few ion plasma periods.

We report preliminary results of one- and two-dimensional computer simulations of the "modified two-stream" instability.[1-5] These simulations show anomalously large plasma heating and electron and ion heating rates which are approximately equal. The final ion thermal speed is $v_i \sim U \sim c_s$, where U is the relative drift speed between electrons and ions and $c_s = (T_e/m_i)^{1/2}$. The results give strong indication that this instability can be a very important turbulent heating mechanism in plasmas in which there is a relative drift of electrons and ions across a magnetic field. This instability can be operative even in regimes where no other electrostatic instabilities are present. Moreover, as discussed by Krall and Liewer,[1] the fluidlike character of the instability and its insensitivity to the electron-ion temperature ratio make it attractive for explaining a number of experiments. The strong interaction of the instability with ions is of considerable interest from a fusion standpoint. Situations in which the modified two-stream instability may be significant include anomalous resistance and ion heating in turbulent heating experiments.

Consider a current driven across a magnetic field $\vec{B}_0$. In the rest frame of the electrons the ions have a drift velocity $\vec{U}$. We assume $k \gg k_\parallel$ (i.e., propagation vectors nearly perpendicular to $\vec{B}_0$), $\Omega_i \ll \omega \ll \Omega_e$, $k\rho_i \gg 1 \gg k\rho_e$, $kv_i < |\omega - \vec{k}\cdot\vec{U}|$, and $k_\parallel v_e < \omega$, where $\Omega_{e,i}$, $\rho_{e,i}$, and $v_{e,i}$ denote the electron and ion gyrofrequencies, gyroradii, and thermal velocities, respectively. Thus the electrons and ions are relatively cold and the ions are effectively unmagnetized. In this case the electrostatic dispersion relation[6] takes on a particularly simple form,

$$1 + \frac{\omega_{pe}^2}{\Omega_e^2} - \frac{\omega_{pi}^2}{(\omega - \vec{k}\cdot\vec{U})^2} - \frac{k_\parallel^2}{k^2}\frac{\omega_{pe}^2}{\omega^2} = 0, \qquad (1)$$

which is analogous in form to the well-known two-stream instability dispersion relation, but is valid even for drifts U which are considerably smaller than v_e. The unstable mode associated with (1), which we call the modified two-stream instability, has $\mathrm{Re}\omega \sim \mathrm{Im}\omega \sim \omega_{pi}(1+\omega_{pe}^2/\Omega_e^2)^{-1/2}$ for $k_\parallel/k \sim (m_e/m_i)^{1/2}$. Since $\omega \sim kU$, the condition to avoid electron Landau resonance, $k_\parallel v_e \lesssim \omega$, becomes a condition on the angle of wave propagation, that is $k_\parallel/k \lesssim U/v_e$. For larger wave-vector angles the dispersion relation is no longer of the modified two-stream type but is more properly an ion acoustic wave.

The factor $(k_\parallel/k)^2$ in Eq. (1) arises because electrons are free to accelerate under an applied force only along the magnetic field. That is, the electrons behave as if they had an effective mass $\bar{m}_e = m_e(k/k_\parallel)^2$. Since $\vec{k}$ is nearly perpendicular to $\vec{B}_0$, we expect ion heating primarily in the direction perpendicular to $\vec{B}_0$. Electrons will be heated in the parallel direction, however. This suggests that waves with $k_\parallel/k \sim (m_e/m_i)^{1/2}$ (i.e., $\bar{m}_e \sim m_i$) should cause a parallel electron heating rate roughly the same as the perpendicular ion heating rate.

Comparisons between the computer experiments discussed below and the nonresonant quasilinear rate equations show that the initial nonlinear development of the modified two-stream instability can be described by quasilinear theory. However, for later times the electron and ion thermal energies increase much faster than the quasilinear equations would indicate, leading to our final thermal energy being much larger than the final electric field energy. The inapplicability of nonresonant quasilinear arguments to the final heating phase is expected because of the presence of strongly nonlinear processes which lead to irreversible heating,[4] e.g., particle trapping. At the equal effective-mass angle, $k_\parallel/k = (m_e/m_i)^{1/2}$, the electrons and ions are trapped and heated simultaneously, and stabilization is ultimately expected when $v_i = c_s \sim U$. This is confirmed by the computer simulations discussed below.

The one- and two-dimensional plasma simulations reported here were performed on the IBM 360/91 computer using the PPOWER simulation code developed at the Naval Research Laboratory.[7] This program uses the distribution-in-cell parti-

cle interpolation technique[7] and the finite-sized particle approximation of Dawson, Hsi, and Shanny[8] to reduce numerical fluctuations in the simulations.

The one-dimensional (1D) calculation was performed on a system 256 cells long with 2^{14} ions and 2^{14} electrons. The two-dimensional simulation was performed on a 256×64 system with 2^{15} ions and 2^{15} electrons. In both calculations we set $\omega_{pe}\delta t = 0.2$. The mass ratio m_i/m_e for the 1D calculation was set to 512 for computational convenience while the 2D simulation had $m_i/m_e = 128$. In both calculations $\Omega_e/\omega_{pe} = 3$, $T_i/T_e = 1.5$, $U/v_e = 0.5$, the electron Debye length was set to one cell, the ions were unmagnetized, and the simulations were performed in the initial rest frame of the electrons. Previous tests with this plasma simultaion program[7] allow us to conclude that numerically enhanced binary collisions have negligible effect on these calculations.

The parameters in these simulations were chosen so as to exclude other known electrostatic instabilities; in particular, $U<v_e$ eliminates Buneman-type instabilities while $T_e < T_i$ prevents ion acoustic waves from developing. The condition $\Omega_e > \omega_{pe}$ eliminates the beam-cyclotron instability[9] which requires $\Omega_e v_e/\omega_{pe}U<1$ for an appreciable growth rate. Thus, when a spatially homogeneous description is adequate, the modified two-stream instability should be the strongest electrostatic instability operative. The main results of the computer experiments are contained in Figs. 1 and 2 and in Table I.

In the 1D case the magnetic field was chosen such that $B_z = 0$, $B_x/B_y = (m_e/m_i)^{1/2}$, implying that the effective mass of the magnetized electrons equals the mass of the unmagnetized ions. The observed growth rate for the electrostatic fields was $\gamma/\omega_{pi} = 0.4$, in close agreement with the maximum growth rate predicted by linear theory, $\gamma/\omega_{pi} = 0.5$. We see also that the electrons heat primarily parallel to $\vec{B}_0$, that the ions heat primarily perpendicular to $\vec{B}_0$ (cf. Table I), and that these two heating rates are approximately the same, as predicted since $\bar{m}_e = m_i$ (cf. Fig. 1). From Table I we note the very important result that about half of the original free energy (ion drift energy) was converted into plasma thermal energy and partitioned almost equally between the electrons and ions. The remainder of

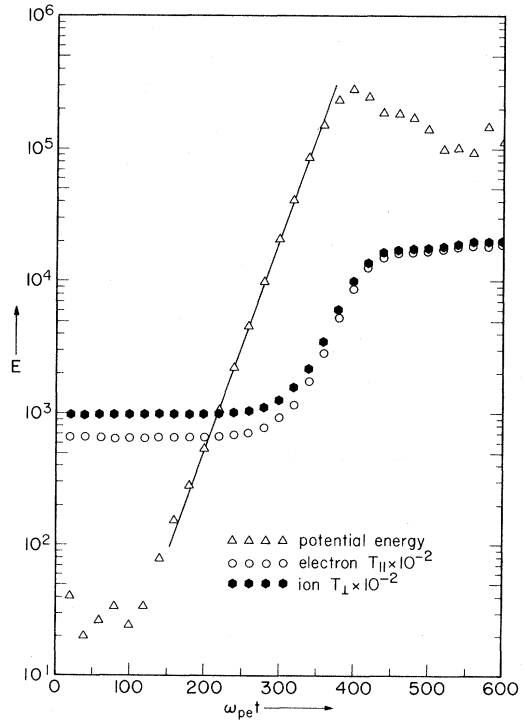

FIG. 1. Plots of electrostatic field energy ($\sum|E_k|^2/8\pi$), the parallel electron thermal energy, and the perpendicular ion thermal energy, in arbitrary units for the 1D case. The straight line emphasizes the exponential growth stage.

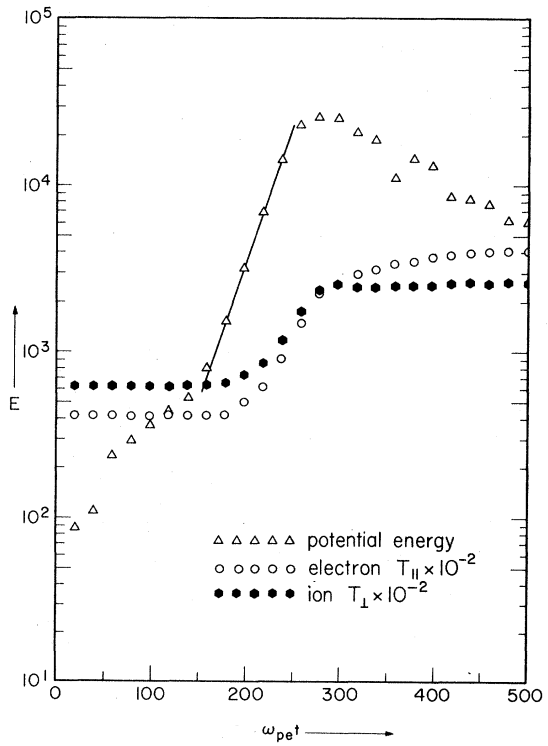

FIG. 2. Plots of electrostatic field energy ($\sum|E_k|^2/8\pi$), the parallel electron thermal energy, and the perpendicular ion thermal energy in arbitrary units for the 2D case.

TABLE I. The first four columns refer to the changes in the x, y, z components of thermal energy E_{th}, and the drift energy E_d (electron drift is along $\vec{B}$) as fractions of the initial drift energy of the ions, $\frac{1}{2}m_iU_0^2$ (which is nearly equal to the enitre system energy). The change in the field energy is always 0.01 ($\frac{1}{2}m_iU_0^2$). All changes Δ refer to the values of the indicated quantities at the end of the experiment ($t = 600\omega_{pe}^{-1}$ in the 1D simulation and $t = 500\omega_{pe}^{-1}$ in the 2D simulation) minus the initial value. The last column gives the ratio of the final total thermal energy E_{th}(total) to the final ion drift energy E_d(ion).

Species	$\Delta E_{th\,x}$	$\Delta E_{th\,y}$	$\Delta E_{th\,z}$	ΔE_d	Final E_{th}(total) / Final E_d(ion)
One-dimensional simulation					
Electrons	0.00	0.21	0.00	0.11	0.56
Ions	0.23	0.00	0.00	−0.56	0.63
Two-dimensional simulation					
Electrons	0.28	0.00	0.00	0.00	0.70
Ions	0.02	0.15	0.00	−0.46	0.48

the transferred energy is converted mainly to electron drift motion along $\vec{B}_0$ while only a very small fraction appears finally as electrostatic field energy. In the more realistic 2D calculation discussed below, symmetry of the k vectors about the direction perpendicular to $\vec{B}_0$ essentially eliminates a systematic electron drift along $\vec{B}_0$. From the last column of Table I we see that the final plasma thermal energy is comparable to the ion streaming energy, corresponding to final thermal spreads $v_i \sim c_s \sim U$. That this value of v_i corresponds to random ion thermal motion is evidenced in the printouts of the ion phase space. At the end of the exponential-growth stage a regular vortex structure characteristic of strong ion trapping is observed. At the end of the run these vortices have essentially been filled in. The presence of strong ion trapping can account for the small final ratio of electrostatic to thermal energy and is consistent with the observed slowing down of the beam since the trapping width in velocity space is approximately $U/2$.

Figure 2 shows the time development of the field and thermal energies for the two-dimensional simulation. The magnetic field is taken to be in the x direction and the initial drift in the y direction. Waves are allowed to develop in the plane of $\vec{U}$ and $\vec{B}_0$ with $\vec{k}$ at all angles subject only to the quantization imposed by the periodic boundary conditions of the finite computer grid. The observed growth rate of the electrostatic fields in the exponential phase is now $\gamma/\omega_{pi} \approx 0.25$ which agrees somewhat less well with the maximum growth rate predicted from linear theory than in the one-dimensional run. This discrepancy is explained by the finite $\vec{k}$-space grid in the 2D simulation which severely limits the number of unstable modes and does not necessarily admit the most rapidly growing modes as calculated from a theory assuming a continuous spectrum.[10] As in the 1D case, we see that the electron and ion heating rates are comparable and that nearly half of the ion streaming energy is converted quickly into plasma thermal energy. Once again Table I shows that the electrons are heated along the magnetic field and the ions are heated mainly in the direction of $\vec{U}$. From the last column of Table I we see that the final thermal energy is comparable to the ion drift energy. Therefore, the final thermal spread is $v_i \sim c_s \sim U$ as in the 1D run. The fact that the temperature increase relative to the initial temperature is less in the 2D run (see Figs. 1 and 2) is a result of the larger value of m_e/m_i chosen for this case and the correspondingly lower saturation level. The energy partitioned to the field fluctuations is again small compared to the thermal energy. Only a negligible fraction of the energy is converted to electron drift energy along $\vec{B}_0$, in contrast to the 1D run.

The results of the 2D simulation confirm that in real plasmas the modified two-stream instability can be an important turbulent heating mechanism. In particular this mechanism efficiently heats ions and can operate in parameter regimes where other electrostatic instabilities cannot. A detailed theoretical and numerical investigation of the modified two-stream instability, including simulations for all regions of parameter space of practical importance, will be reported in a future full-length article.

*On leave of absence from Cornell University, Ithaca, N. Y.

[1]N. A. Krall and P. C. Liewer, Phys. Rev. A 4, 2094 (1971), and private conversations.

[2]A. B. Mikhailovskii and V. S. Tsypin, Zh. Eksp. Teor. Fiz., Pis'ma Red. 3, 247 (1966) [JETP Lett. 3, 158 (1966)].

[3]K. N. Stepanov, Zh. Tekh. Fiz. 34, 2146 (1964) [Sov. Phys. Tech. Phys. 9, 1653 (1965)].

[4]V. V. Demchenko, V. L. Sizonenko, and K. N. Stepanov, Nucl. Fusion 11, 245 (1971).

[5]J. B. McBride and E. Ott, to be published.

[6]References 1–4 are based on the electrostatic approximation. Reference 5 considers electromagnetic

and finite-β effects. These are found to have a stabilizing effect. The electrostatic approximation is good provided that U is much less than the Alfvén speed.

[7]J. H. Orens, J. P. Boris, and I. Haber, in *Proceedings of the Fourth Annual Conference on the Numerical Simulation of Plasmas*, Naval Research Laboratory, Washington, D. C., 1970 (U. S. GPO, Washington, D. C., 1971).

[8]J. M. Dawson, C. G. Hsi, and R. Shanny, in *Plasma Physics and Controlled Nuclear Fusion Research* (International Atomic Energy Agency, Vienna, 1969), Vol. I, p. 735.

[9]M. Lampe, W. Manheimer, J. B. McBride, J. H. Orens, R. Shanny, and R. N. Sudan, Phys. Rev. Lett. 26, 1221 (1971).

[10]Future runs are being planned to minimize this quantization problem in $\vec{k}$ space by substantially increasing the number of available grid points.

Possible Inverted Band Structure in PbPo

Richard Dalven

Department of Physics, University of California, Berkeley, California 94720

(Received 15 November 1971)

Based on empirical evidence, it is suggested that the energy band structure in PbPo may be inverted from that in PbS, PbSe, and PbTe. This inversion would result in PbPo having an L_6^+ conduction-band edge and an L_6^- valence-band edge, implying a negative value of the energy gap $E_G = [E(L_6^-) - E(L_6^+)]$, and a *negative* value of dE_G/dT. We propose an experiment to test this hypothesis.

The band structure of PbS, PbSe, and PbTe is currently believed to be inverted[1] from that of SnTe. By this is meant that in these lead compounds, the conduction-band edge is an L_6^- state and the valence-band edge is an L_6^+ state; in SnTe the inverse is the case. The purpose of this Letter is to suggest, based on empirical evidence, that PbPo, the fourth member of the series, may have a band structure inverted from those of PbS, PbSe, and PbTe. This possibility is amenable to a direct experimental test by relatively simple means.

A knowledge of the energy gap E_G of PbPo would be useful in the study of the well-known anomaly[2] $[E_G(\text{PbTe}) > E_G(\text{PbSe})]$ in the series of the energy gap values of PbS, PbSe, and PbTe. However, the only experimental data[3] available for PbPo are that it has the face-centered cubic crystal structure with lattice constant $a_0 = 6.590$ Å at room temperature. The fact that PbPo has the same rock-salt crystal structure as do PbS, PbSe, and PbTe suggests that PbPo has the same general band structure (a semiconductor with a direct energy gap at the L point) as do the other three lead salts. To obtain an estimate of the energy gap of PbPo, the well-known empirical relation[4] between energy gap E_G and lattice constant a_0 was considered. This relation, which states that E_G is directly proportional to $1/a_0^2$ for a series of related semiconductors, was known[4] to hold reasonably well for the group IV elemental semiconductors, diamond, Ge, Si, and α-Sn.

The validity of this relation has been investigated for a number of semiconductors with the rock-salt structure. Values of energy gap were plotted as a function of $1/a_0^2$ for (1) BaS, BaSe, BaTe,[5,6] (2) CaS, CaSe, CaTe,[5,6] (3) SrS, SrSe, SrTe,[5,6] (4) solid solutions of SnSe in PbSe,[7,8] (5) solid solutions of SnTe in PbTe.[9,10] The results were that, within the uncertainty of the experimental data, E_G was a linear function of $1/a_0^2$ for each series of semiconductors. A representative result is given in Fig. 1, which shows a plot[11] of E_G as a function of $1/a_0^2$ for several solid solutions of SnSe in PbSe. It should be noted that the data cover both positive and negative values of $E_G = E(L_6^-) - E(L_6^+)$. Since the empirical relation $E_G \propto 1/a_0^2$ holds for a large number of semiconductors with the NaCl fcc structure, it appeared reasonable to use this relation to calculate a value of the energy gap of lead polonide.

Using values of the energy gap[12] at 77 and 300 K, and lattice constants a_0 at ~300 K,[13] E_G was plotted as a function of $1/a_0^2$ for PbS, PbSe, and PbTe, as shown in Fig. 2. It is seen that the PbS-PbSe line has been extrapolated; the point for PbTe has not been used. The justification for this procedure is as follows. In proposing[2] that PbTe is the irregular member of this series, it was pointed out that the outer s-electron energies of the S, Se, and Po atoms are a linear function of their atomic numbers Z, whereas the Te 5s electron energy does *not* lie on the S-Se-Po