Bronzan and Jones[16] have shown that elastic unitarity in the t channel requires that the discontinuity at the tip of the cut vanishes for general t. If the zero of $g_{PPP}(t, q_1^2, q_2^2)$ is a nonsense wrong-signature zero, then their result is still valid at $t=0$, where the pole and cut collide, and implies that $N(0,0)=0$. For the more general type of zero, Eq. (3), it does not follow that $N(0,0)$ is zero. This can be so because $N(\vec{q}_\perp, \vec{\Delta}_\perp)$ is not analytic everywhere in the neighborhood of $t=0$ and $q^2=0$, allowing different limits to be obtained as q^2 and t approach zero along different paths. This is important because it means the application by Abarbanel and Green[9] of the Bronzan-Jones condition is valid at $t=0$ only if g_{PPP} has a nonsense wrong signature zero. A related point is that the bound on dg_{PPP}/dq^2 derived by Ellis, Finkelstein, and Peccei[17] is valid only if the zero is a nonsense wrong-signature zero.[18]

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New Measurement of the Solar Gravitational Red Shift

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The red shift of the solar potassium absorption line at 7699 Å has been measured by means of an atomic-beam resonance-scattering technique. The shift at the center of the solar disk, corrected for Earth's motion, is $(\Delta\lambda)_{\rm exp}=(1.01\pm0.06)(\Delta\lambda)_{\rm theor}$, where the theoretical value, based on the principle of equivalence, is 16.3 mÅ. Further data show negligible center-to-limb wavelength variation for this line.

This Letter reports measurements made during the past year at Oberlin of the red shift of the solar potassium absorption line at 7699 Å relative to the corresponding wavelength of potassium atoms in the laboratory. After correction for Earth's motion, the remaining red shift at the center of the solar disk is found to be 1.01 ± 0.06 times that predicted by Einstein[1] on the basis of the principle of equivalence. Data taken at Kitt Peak show that there is negligible variation of wavelength from center to limb. This is the second clear-cut experiment on the solar red shift, the only previous one being the sodium-$D1$ experiment of Brault[2] which used a different method and gave agreement with the prediction within 5%. The sodium and potassium lines are believed to originate above that region of the photosphere where upward and downward mass motions can produce a net Doppler shift, and they are therefore particularly suited to a test of the gravitational effect. The 4607-Å strontium line examined by Blamont and Roddier,[3,4] using the same method as in the present experiment, was weak and blended with another line; it showed a center-to-limb variation in wavelength as well. The best current result on the red shift is from the terrestrial experiment of Pound and co-workers[5,6] which gave agreement within 1%.

In the present experiment a Zeeman-scanning resonance-scattering technique was used to measure the wavelength shift of the solar line relative to the absorption wavelength of essentially isolated atoms in an atomic beam. A general discussion of the method and a description of an earlier version of the apparatus has been given.[7] A solar image of $2\frac{1}{4}$ in. diam is formed by a $12\frac{1}{2}$-in. $f/6$ reflecting telescope and an auxiliary lens. The light to be studied passes through a circular polarizer made of a sheet of HN7 linear polarizer and a sheet of quarter-wave retarder, then through a $\frac{1}{8}$-in.-diam aperture centered on the image followed by an interference filter which transmits a 15-Å-wide band around the 7699-Å, $4P_{1/2}$-$4S_{1/2}$ line, and finally through one pole of an electromagnet along the direction of the magnetic field. Lenses within the magnet pole focus the light onto a beam of potassium atoms traveling between the magnet poles in a direction accurately perpendicular to the incident light beam. Light which is reasonantly scattered by the beam is guided by a light pipe perpendicular to both the incident light and atomic-beam directions to a cooled EMI 9558 photomultiplier tube connected to a conventional pulse-counting system. The unscattered light passes through the second pole piece and is absorbed in a blackened tube.

The theory of the Zeeman effect shows that a magnetic field applied to the atoms in the beam shifts their resonant wavelength by ±36.9 mÅ/kG, corresponding to right and left circular polarization, respectively, of the input light. This Zeeman slope is valid only for fields sufficiently large that the beam atoms are in the strong-field region of their hyperfine-structure diagram, where the ground-state energy level is split into two separate groups each consisting of four levels whose spacing is uniform and essentially independent of field. The width of the beam-atom absorption profile due to these four separated hyperfine-structure levels is about 7 mÅ. The field was always kept greater than 500 G to satisfy this condition. Line profiles were obtained by measuring the scattered light intensity as a function of the magnetic field strength with the use of opposite senses of input-light circular polarization for the two sides of the line. The full width at half-maximum of the line was found to be about 160 mÅ, and its minimum intensity was roughly 15% of the continuum.

The method adopted for measuring the line shift was a "three-point" series of measurements. The ratio of the scattered-signal count rate at a given field to that at roughly 8000 G, corresponding to the level of the continuum, was measured for two separate values of the field with right circular input polarization and for a third value of the field with left circular polarization. Let us denote these respective normalized signals and corresponding fields by S_{1R}, S_{2R}, S_L and H_{1R}, H_{2R}, H_L. The field for the left circular polarization point was chosen so that $S_{1R} > S_L > S_{2R}$. In this way, the normalized signal at a single location on the short-wavelength side of the line was compared with that at two slightly different locations on the long-wavelength side, so that by linear interpolation the shift of the line center, at a depth in the line determined by the chosen value of H_L, could be obtained:

$$H_s = \tfrac{1}{2}\left[H_{2R} + \frac{S_L - S_{2R}}{S_{1R} - S_{2R}}(H_{1R} - H_{2R}) - H_L\right] \text{ G},$$

where positive H_s means that the line is shifted to the red. If the line is asymmetric this shift will depend upon the depth in the line at which it is measured. No significant variation of the measured shifts was found for data taken over a range from about 20 to 95 mÅ from the line minimum. This method assumes linearity of the line over the range of field values from H_{1R} down to H_{2R}. For the field differences used this was a valid approximation. Typical count rates were in the range from 1000 to 10 000 counts/sec, and the measured shifts were independent of count rate.

Figure 1 shows the measured values of H_s plotted against the calculated values of the Doppler shift in milliangstroms produced by Earth's rotation and orbital revolution, based on data from the *American Ephemeris and Nautical Almanac.*[8] With only one or two exceptions, each point represents the mean of five "three-point" measurements made over a period of roughly 20 min. It is difficult to estimate the uncertainty to be associated with each point. The statistical counting error is of the order of ±10 G; on the basis of the observed fluctuations within each group of five points an error of ±20 G seems reasonable. Also drawn on the figure is a straight line with the calculated Zeeman slope which cuts the horizontal axis at the theoretical red-shift value of $(2.12\times10^{-6})\lambda = 16.3$ mÅ.

An obvious feature of the data presented in Fig. 1 is that the majority of points lie below the straight line. A plausible reason for this is that there are some photons incident on the beam atoms which have the unwanted sense of circular polarization. These photons can induce the transition having a Zeeman slope of opposite sign, corresponding to the other side of the line. The result is an apparent reduction in the line shift. The line drawn through the data points was calculated for an assumed relative intensity of light of the wrong polarization of 12%. Such photons could be present because the polarizers are not

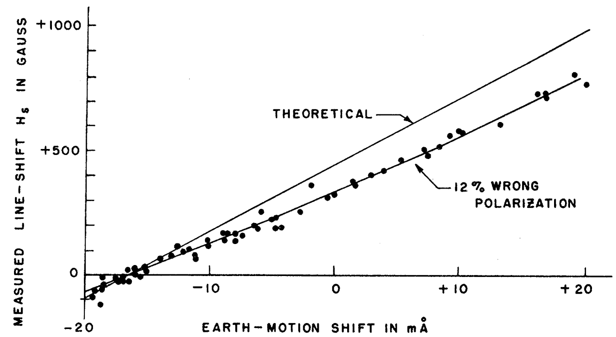

FIG. 1. The experimental data plotted versus the calculated Earth-motion shift.

100% efficient; they could also have had their polarization sense reversed by reflection from the walls of the scattering chamber. The crucial point for our purposes is this: As the net shift of the line approaches zero, this instrumental ef-

TABLE I. Corrected red shifts for data having an Earth-motion shift more negative than −12 mÅ.

H_s in gauss	H_s in mÅ	Earth-motion shift in mÅ	Corrected red-shift in mÅ
+113 ± 20	+4.6 ± 0.8	-12.7 ± 0.1	17.3 ± 1
- 28	-1.1	-17.4	16.3
- 5	-0.2	-16.9	16.7
+ 20	+0.8	-16.1	16.9
+ 26	+1.1	-15.3	16.4
- 12	-0.5	-17.6	17.1
- 21	-0.8	-16.9	16.1
- 2	-0.1	-15.8	15.7
- 94	-3.8	-19.4	15.6
- 5	-0.2	-18.6	18.4
- 71	-2.9	-19.2	16.3
-119	-4.8	-18.9	14.1
- 36	-1.5	-18.1	16.6
- 25	-1.0	-17.3	16.3
- 26	-1.1	-16.5	15.4
+ 10	+0.4	-15.2	15.6
+ 65	+2.6	-14.1	16.7
+ 77	+3.1	-13.2	16.4
+ 90	+3.6	-12.3	15.9
- 63	-2.5	-18.8	16.3
- 27	-1.1	-18.1	17.0
- 38	-1.5	-18.5	17.0
- 14	-0.6	-17.7	17.1
+ 21	+0.8	-16.6	17.4
- 3	-0.1	-15.7	15.6
		Mean	16.4 ± 0.2 mÅ

fect also approaches zero and leaves the location of the intercept on the horizontal axis unaffected. A least-squares fit to the 25 points for which the Earth-motion shift is more negative than −12 mÅ, obtained between 24 August and 28 October 1971, gives a slope of 40.4±3.3 mÅ/kG as compared with the theoretical Zeeman slope of 36.9 mÅ/kG. The value of χ^2 is 26. These points refer to the part of the line which is 25–50 mÅ from the line minimum. Table I lists the measured values of H_s (in gauss), the measured values (in milliangstroms) using the experimental slope, the associated Earth-motion corrections, and the final corrected red shifts, obtained by reversing the sign of each entry in column three and adding it to the corresponding entry in column two. The mean value of these 25 points is 16.4 mÅ, with an error of only a few tenths of a milliangstrom. Doing the calculations using the theoretical Zeeman slope gives an identical result. A reasonable estimate of the overall uncertainty to allow for possible systematic error is ±1 mÅ. Our final result is therefore

$$(\Delta\lambda)_{\rm exp} = (1.01 \pm 0.06)(\Delta\lambda)_{\rm theor}.$$

We have also made measurements of the line shift at various other locations on the solar disk, first of the absolute shifts using the Oberlin apparatus and then of the center-to-limb relative shift with the McMath solar telescope at the Kitt Peak National Observatory. Figure 2 shows the Kitt Peak data. Here, $\mu \equiv \cos\theta$, with θ the angle between the line of sight and the normal to the solar surface at the point observed. Each plotted shift is the mean value of the line shift measured 25 mÅ from the line minimum and that measured 50 mÅ from the minimum, averaged over several line profiles taken on different days. There is no significant center-to-limb shift of the 7699-Å potassium line. The Oberlin results lead to the same conclusion. Most solar lines show a red shift considerably smaller than the predicted gravitational shift at the center of the disk, rising toward the predicted value at the limb.[9] The reason for this is presumably that radial mass motions in the photosphere produce a net Doppler blue shift. The absence of such an effect in sodium and potassium confirms the belief that these lines originate in a region free of such motions.

Various tests of the apparatus have been performed. The rotational speed of the sun has been determined by measuring the Doppler shifts at

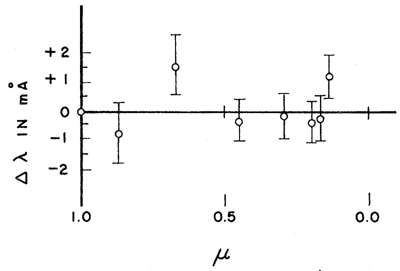

FIG. 2. Center-to-limb wavelength variation of the 7699-Å line as a function of $\mu = \cos\theta$ (Kitt Peak data).

opposite edges of the disk. The result is in good agreement with previous determinations of this quantity. The profiles of the solar sodium $D1$ line and the emission line from a laboratory potassium light source have been measured as a check. The calibration of the rotating-coil gaussmeter has been repeatedly checked with an NMR apparatus. On the basis of these and other tests, we believe that our estimate of error quoted above is reasonable.

Hopefully this experiment will stimulate interest in further theoretical and experimental work on the solar potassium line. A complete discussion of sources of error together with details of the apparatus will be published elsewhere. I thank the undergraduates who have worked with me on this experiment over the past two years: Larry Abbott, Bob Cope, Keith Louden, Dave Muchmore, and Larry Wilson. Thanks are also due to Hank Annable, Ruth Edwards, Harlan Hurd, and Tom Smolka. Jim Brault and Larry Testerman were of great help in our work at Kitt Peak.

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