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Isospin Splitting of the Giant Resonance, and Neutron and Proton rms Radii

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A method for resolving the giant resonance excitation into its isospin components is applied to several nuclei. The relative strengths of the different isospin channels and their energy splittings are calculated. Consistency is found with the existence of the neutron "halo" in heavy nuclei.

The aim of this Letter is to illustrate carefully a useful method for resolving the giant resonance into its isospin components and to evaluate their relative strengths.

The essence of the method was proposed in a previous work[1] in the framework of an isospin analysis of photoreactions, but a careful estimate of the proposed equations has not been done. Further, we exploit the method in a formalism more directly connected with the experimental quantities.

More precisely, in the following we shall prove (a) the isospin splitting (ΔE) of any nucleus may be expressed, with the help of the dipole sum rules given in the isospin channels, in terms of a few physical parameters like giant resonance energy, isospin, and rms radii of neutrons ($\langle r_n^2\rangle^{1/2}$) and protons ($\langle r_p^2\rangle^{1/2}$). (b) The splitting in heavy nuclei is very sensitive to the difference (δr) of $\langle r_n^2\rangle^{1/2}$ and $\langle r_p^2\rangle^{1/2}$. The present experimental data[2,3] on ΔE of heavy nuclei are easily understood if one accepts the idea that in these nuclei $\langle r_n^2\rangle$ is greater than $\langle r_p^2\rangle$. (c) There is an excellent agreement between the direct measurements[4,5] of δr as deduced from pion inelastic and proton elastic scattering, and the ΔE data. So, these last data may be considered independent and suggestive evidence of the neutron "halo" in heavy nuclei.

We start with the following definitions: $\sigma(E)$ is the dipole physical photo cross section, and $\sigma(E,T')$ is the dipole cross section in the channel T'. For a nuclear target with $T_3=T$ (as we assume in the following) T' may be T or $T+1$, and $\sigma=\sigma(T)+\sigma(T+1)$. We define the giant resonance energy and giant resonance energy in the channel T' by the expressions

$$\frac{\int\sigma(E)\,dE}{\int\sigma(E)\,dE/E}=\bar{E},\quad \frac{\int\sigma(E,T')\,dE}{\int\sigma(E,T')\,dE/E}=\bar{E}_{T'}.$$

We define isospin splitting by the expression

$$\Delta E=\bar{E}_{T+1}-\bar{E}_{T}. \tag{1}$$

Furthermore, the following parameters are interesting:

$$\frac{\int\sigma(E,T+1)\,dE}{\int\sigma(E,T)\,dE}=R, \tag{2}$$

$$\frac{\int\sigma(E,T+1)\,dE/E}{\int\sigma(E,T)\,dE/E}=B. \tag{3}$$

Utilizing the analysis of Ref. (1) we have

$$137\int\sigma(E,T)\,dE=\frac{4\pi^2}{3(T+1)}\left[\frac{T}{2}3A_s+\frac{T}{2}3A_v+\frac{T(2T-1)(2T+3)}{2}A_t\right], \tag{4}$$

$$137\int\sigma(E,T+1)\,dE=\frac{4\pi^2}{3(T+1)}\left[\frac{3}{2}A_s-\frac{T}{2}3A_v-\frac{T(2T-1)}{2}A_t\right], \tag{5}$$

$$137\int\sigma(E,T)\frac{dE}{E}=\frac{4\pi^2}{3(T+1)}\left[\frac{T}{2}r_s^{\,2}+\frac{T}{2}r_v^{\,2}+\frac{T(2T-1)(2T+3)}{6}r_t^{\,2}\right],\tag{6}$$

$$137\int\sigma(E,T+1)\frac{dE}{E}=\frac{4\pi^2}{3(T+1)}\left[\frac{r_s^{\,2}}{2}-\frac{T}{2}r_v^{\,2}-\frac{T(2T-1)}{6}r_t^{\,2}\right].\tag{7}$$

So, we relate (1), (2), and (3) via (4), (5), (6), and (7) to the quantities r_s^2, r_v^2, r_t^2, A_s, A_v, and A_t, defined by Eqs. (3) and (5) of Ref 1.[6] Finally we have ($\hbar=c=1$)

$$\frac{\int\sigma(E)\,dE}{\int\sigma(E)\,dE/E}=\bar{E}=\frac{3(A_s+A_t)}{r_s^{\,2}+r_t^{\,2}}=\frac{3\beta}{2M\langle r^2\rangle\alpha},\tag{8}$$

where $\langle r\rangle^2$ is the mean square distance of the nucleon's c.m. from the nucleus's c.m. We have introduced a parameter α (<1), to take into account the effect of correlations on the energy-weighted sum rule, and a parameter β (>1), indicating by how much the integrated cross section exceeds the classical value $0.06NZ/A$ MeV b.[7] β may be determined empirically by measuring

$$\int_0^\Lambda\frac{\sigma(E)\,dE}{0.06NZ/A},$$

where Λ is the π-production threshold. The problems connected with these measurements are discussed, for example, by Danos and Fuller.[8] Remembering that above 30 MeV retardation effects are essentially compensated by higher-order multipoles, we can use the *total* photoabsorption cross section to estimate β. For energy regions where the total cross section (below π-production threshold) is not known experimentally, it may be estimated from the Gell-Mann–Goldberger–Thirring sum rule.[9] We obtain $\beta\simeq1.5$ or slightly smaller for very light nuclei. Similarly, α may be deduced from (8) if $\langle r^2\rangle$ and $\bar{E}$ are experimentally known. The rms nuclear charge radius $\langle r_{\rm ch}^{\,2}\rangle$ (after subtraction of the nucleon finite size effect) may be a reasonable value for $\langle r^2\rangle$. We have $\langle r_{\rm ch}^{\,2}\rangle-0.65=\langle r_p^{\,2}\rangle\simeq\langle r^2\rangle$.

We note that α may also be evaluated consistently in a pure harmonic-oscillator model. In this case $\beta=1$ and $\alpha=\frac{3}{2}(\langle r^2\rangle\omega_0M)^{-1}$, where (from Bohr and Mottelson,[10] page 222) we have $\langle r^2\rangle\simeq(3\hbar/4M\omega_0)(N_{\rm max}+2)$. Calculating α for $A=16$ ($N_{\rm max}=1$), $A=40$ ($N_{\rm max}=2$), $A=56$ ($N_{\rm max}=3$), $A=100$ ($N_{\rm max}=4$), we obtain $\alpha=0.66$, 0.5, 0.4, and 0.33, respectively, in excellent agreement with our corresponding values from Eq. (8).

Yet another way to calculate α directly is the relation

$$\int\sigma(E)\,dE/E=\tfrac{4}{3}\pi^2\tfrac{1}{137}\,\alpha\langle r^2\rangle NZ/A.\tag{9}$$

The measurements available[11] indicate

$$A^{-4/3}\int_0^{30\ \rm MeV}\sigma(E)\,dE/E\simeq0.2\ \rm mb,$$

from which $\alpha>(4A^{1/3}/\langle r^2\rangle)0.2\simeq1.3A^{-1/3}$ in agreement with our lower bound ($\beta=1$) from Eq. (8). In terms of $\langle r_n^{\,2}\rangle$, $\langle r_p^{\,2}\rangle$, $\epsilon=\langle r_n^{\,2}\rangle-\langle r_p^{\,2}\rangle$, A, and M we obtain

$$r_s^{\,2}=\tfrac{1}{2}\alpha(Z\langle r_p^{\,2}\rangle+N\langle r_n^{\,2}\rangle)=\tfrac{1}{2}A\alpha(\langle r_p^{\,2}\rangle+N\epsilon/A)\simeq\tfrac{1}{2}A\alpha\langle r_p^{\,2}\rangle,\tag{10}$$

$$r_v^{\,2}=\tfrac{1}{2}(Z\langle r_p^{\,2}\rangle-N\langle r_n^{\,2}\rangle)/T_3=\langle r_p^{\,2}\rangle-\tfrac{1}{2}N\epsilon/T,\tag{11}$$

$$\frac{r_t^{\,2}}{r_s^{\,2}}\simeq0,\quad\frac{A_t}{A_s}\simeq0,\quad A_s=\beta\frac{A}{4M},\quad A_v\simeq\beta\frac{1}{2M},\tag{12}$$

Disregarding the terms $r_t^{\,2}/r_s^{\,2}$ we have[12]

$$\Delta E=(T+1)\frac{3A_s}{r_s^{\,2}}\left(\frac{r_v^{\,2}}{r_s^{\,2}}-\frac{A_v}{A_s}\right)\left(1+\frac{r_v^{\,2}}{r_s^{\,2}}\right)^{-1}\left(1-T\frac{r_v^{\,2}}{r_s^{\,2}}\right)^{-1},\tag{13}$$

and in terms of our results (8), (10), (11), and (12) we obtain

$$\Delta E=\frac{T+1}{A}2\bar{E}\left(\frac{1-\alpha'}{\alpha'}\right)\left[1+\frac{2}{\alpha'A}(1-T)-\frac{4T}{\alpha'^2A^2}\right]^{-1}=\frac{T+1}{A}U,\tag{14}$$

$$R=\frac{1}{T}\frac{\frac{1}{2}A-T}{\frac{1}{2}A+1},\quad B=\frac{1}{T}\frac{(\frac{1}{2}\alpha'A-T)}{(\frac{1}{2}\alpha'A+1)},\quad\text{with }\alpha'=\frac{3}{2}\frac{\beta}{ME}\left(\langle r_p^{\,2}\rangle-\frac{N\epsilon}{2T}\right)^{-1}.\tag{15}$$

TABLE I. The values of the parameters U [from Eq. (14)] and α' and B [from Eq. (15)] for the assumed giant resonance energies rms charge radii, and the assumed differences between neutron and proton rms radii (δr); β is 1.4 for A =26 and is 1.5 for all the other nuclei.

Nuc.(A)	$\bar{E}$	$r_{ch}(\delta r)$	U	α'	B	Nuc.(A)	$\bar{E}$	$r_{ch}(\delta r)$	U	α'	B
C(13)	23	2.30(.00)	8	.840	1.53	Cd(112)	15	4.53(.10)	88	.397	.077
N(15)	22	2.48(.00)	14	.739	1.54	Sn(116)	15	4.55(.10)	52	.448	.083
Ne(19)	22	2.80(.00)	31	.565	1.43	Sn(117)	15	4.60(.15)	57	.426	.074
Mg(26)	21	2.90(.00)	34	.549	.755	Sn(118)	15	4.60(.15)	60	.421	.068
Ti(48)	18	3.60(.00)	52	.433	.368	Sn(120)	15	4.64(.15)	60	.43	.059
Cr(52)	18	3.66(.00)	55	.418	.373	Sn(122)	15	4.65(.20)	63	.433	.051
Ni(58)	18	3.81(.00)	58	.384	.835	Sn(124)	15	4.67(.20)	72	.418	.043
Ni(60)	18	3.83(.00)	65	.380	.379	Cs(138)	15	4.80(.25)	73	.423	.036
Ni(62)	18	3.87(.00)	74	.372	.227	Ba(138)	15	4.80(.25)	64	.437	.043
Ni(64)	18	3.90(.05)	65	.417	.163	Le(140)	15	4.82(.20)	85	.397	.039
Zn(64)	18	3.90(.00)	68	.366	.382	Ce(140)	15	4.82(.20)	70	.406	.046
Ge(70)	17	3.96(.00)	67	.376	.239	Pr(142)	15	4.85(.20)	73	.400	.046
Ge(72)	17	4.00(.05)	57	.424	.173	Pm(144)	15	4.88(.15)	83	.366	.051
Ge(74)	17	4.02(.05)	67	.408	.125	Sm(144)	15	4.88(.15)	75	.376	.06
Ge(76)	17	4.05(.10)	61	.441	.100	Nd(142)	15	4.85(.20)	59	.43	.056
Sr(88)	16	4.10(.10)	48	.467	.112	Nd(144)	15	4.88(.20)	74	.397	.047
Y (89)	16	4.12(.05)	60	.414	.121	Nd(146)	15	4.90(.20)	85	.381	.040
Zr(90)	16	4.15(.05)	59	.411	.138	Nd(148)	15	4.92(.25)	77	.407	.037
Mo(92)	16	4.20(.00)	73	.351	.177	Nd(150)	15	4.93(.25)	86	.397	.032
Pd(108)	15	4.50(.10)	67	.397	.075	Pb(208)	13	5.45(.25)	113	.351	.018

α' differs from α by the correction term $N\epsilon/2T$ which takes into account the difference between $\langle r_n^2\rangle$ and $\langle r_p^2\rangle$. The interesting analyses of Greenlees, Mokofske, and Pyle[4] and Auerbach, Qureshi, and Sternham[5] permit us to estimate ϵ. From these analyses we assume $\delta r = 0$, $0-0.05$, $0.05-0.1$, $0.1-0.15$, $0.15-0.2$, and $0.2-0.25$ fm for $N/Z<1.2$, $N/Z=1.2-1.3$, $1.3-1.35$, $1.35-1.40$, $1.40-1.45$, and $N/Z>1.45$, respectively. The numerical results calculated with these assumptions are given in Table I.

The agreement with some recent experimental data[2,3,13-16] is good. We point out that U [defined by Eq. (14)], in spite of the complicated dependence on T, α', and A, lies between 55 and 70 MeV,[17] except for very light nuclei in which it is smaller[18] and for nuclei with high N/Z in which it is greater (for Pb^{208} we have $U \gtrsim 100$ MeV). We observe, for example, that for Sn isotopes and Nd isotopes, U increases with increasing A from about 60 to $75-80$ MeV.

Our result (14) is sensitive to the giant resonance assumed. A variation of $\bar{E}$ of 3–4% means a variation in U of 8–10% (however, $\bar{E}$ may be determined experimentally).

Our assumptions on A_v deserve some remarks. If we use a pure harmonic-oscillator model to compute this term we obtain (using $\omega = 3\beta/2M\langle r^2\rangle\alpha$) $A_v = \beta/2\alpha M$ and the splitting ΔE disappears as it should. (In this model the T and $T+1$ dipole states are forced to be degenerate!). Instead we have separated out in A_v a model-independent (one-body) contribution, which, including a proper two-body exchange term, gives $\beta/2M$ and a correlation ($\geqslant 0$) which is strongly model dependent.

Pauli effects and the spatial exchange potential, which give rise to the so-called symmetry potential responsible for the isospin splitting (Ref. 10, page 258), both operate to reduce drastically this correlation with respect to the naive harmonic-oscillator model prediction. We have simply dropped this correlation by assuming $A_v \simeq \beta/2M$. A Fermi-gas model calculation confirms these features. Finally our results on ΔE are not sensitive to a small variation of the ratio A_v/A_s: A 20% increase of A_v gives a 10% reduction of U. This further justifies our approximations.

By far the most interesting and spectacular effect is the sensitivity of ΔE to δr. In principle, the experimental determination of the isospin splitting would be a model for estimating δr. For example, the assumption $\langle r_n^2\rangle = \langle r_p^2\rangle$ in the heavy region would give for U the value $U \simeq 130-140$ MeV for Sn and $U \simeq 350$ MeV for Pb^{208}, in complete disagreement with experimental results. Conversely, a difference $\delta r = 0.5$ fm would give a negligible splitting. So the compatibility of $\delta r \simeq 0.1 - 0.2$ fm with the results on ΔE is very satisfactory and may be considered as further evidence of the existence of the "halo" of the neutrons in the heavy region.

In this respect it would be very interesting to measure the isospin splitting in Ca isotopes in order to analyze the effect on ΔE of the anomalous behavior of δr in this region.[19]

Finally we remember that ΔE also depends on β. Our assumption on β is firmly justified,[7,9] since on very general grounds we have[9] $1.2 \leq \beta \leq 1.6$. Any value smaller than the one we assume would increase ΔE and would suggest an even more pronounced neutron "halo." Conversely, even the extreme value $\beta = 1.9$ is not sufficient to explain the experimental data on ΔE for heavy nuclei without the introduction of $\delta r > 0$. (We have for ^{208}Pb, $U = 150$ with $\delta r = 0$ and $\beta = 1.9$.)

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Ratio of the ^{4}He(γ,p) and ^{4}He(γ,n) Cross Sections

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The ^{4}He$(\gamma,p)^3$H and ^{4}He$(\gamma,n)^3$He cross sections have been determined with a magnetic spectrometer in the energy interval of 30.0 to 51.8 MeV. We find that the average value of $\sigma(\gamma,p)/\sigma(\gamma,n)$ is 1.03 ± 0.04 in the above energy interval. The ^{4}He(γ,p) cross section decreases from 1.52 ± 0.13 mb at 31.7 MeV to 0.36 ± 0.03 mb at 51.8 MeV.

The equality of the ^{4}He(γ,p) and ^{4}He(γ,n) total cross sections was first established by Gorbunov and Spiridonov[1] using a cloud chamber and synchroton bremsstrahlung radiation. Fuller's[2] ^{4}He$(\gamma,p)^3$He data and Livesey and Main's[3] ^{4}He$(\gamma,{}^3$He$)n$ data, and later measurements of Gorbunov[4] and Main,[5] corroborated Gorbunov's[1] original measurements. However, two recent measurements of the ^{4}He(γ,n) cross section have raised serious doubts about the equality of the ^{4}He(γ,p) and ^{4}He(γ,n) cross sections. A measurement of the ^{4}He(γ,n) cross section by Berman, Fultz, and Kelly[6] (BFK), when compared with the ^{4}He(γ,p) cross section measured by Meyerhof, Suffert, and Feldman,[7] indicates that the average ^{4}He(γ,p) cross section is 1.806 ± 0.025 times as large as the average ^{4}He(γ,n) cross section in the energy interval between threshold and 31 MeV. Busso