*Permanent address: University of California, Santa Cruz, Calif. 95000.

†Alfred P. Sloan Foundation Fellow.

¹W. L. McMillan, Phys. Rev. 167, 331 (1968).

³P. B. Allen and M. L. Cohen, Phys. Rev. 187, 525 (1970).

⁴J. J. Hopfield, Phys. Rev. 186, 443 (1969). ⁵The integral

$$\int_0^R d\mathbf{r} \, \mathbf{r}^2 R_{\mathbf{l}} \frac{dv}{d\mathbf{r}} R_{\mathbf{l}+1} \equiv \int_0^R d\mathbf{r} \, u_{\mathbf{l}} \frac{dv}{d\mathbf{r}} u_{\mathbf{l}+1}$$

can be simply evaluated by differentiating the Schrödinger equation for u_{l} , multiplying it by u_{l+1} , and integrating the result. The operation is repeated for u_{l+1} and the two results are added. After integration by parts the remaining integrals are evaluated again using the Schrödinger equation, yielding

$$\int_{0}^{R} dr r^{2}R_{l}(dv/dr)R_{l+1}$$

$$= \frac{1}{2} [u_{l}u_{l+1}'' - 2u_{l}'u_{l+1}' + u_{l}''u_{l+1}]_{0}^{R}$$

$$+ [(l+1)/R] [u_{l}u_{l+1}' - u_{l}'u_{l+1}]_{0}^{R}.$$

For R greater than the range of the potential this gives $\sin(\delta_{l+1} - \delta_l)$ which was quoted in the text.

⁶P. W. Anderson and W. L. McMillan, in *Theory of* Magnetism in Transition Metals, Proceedings of the International School of Physics "Enrico Fermi," Course 37. edited by W. Marshall (Academic, New York, 1967).

⁷R. Evans, D. A. Greenwood, and P. Lloyd, Phys. Lett. 35A, 57 (1971).

⁸The authors are grateful to Dr. R. Evans for providing the phase shifts used in the Fe, W, and Cu calculations. The muffin-tin potentials and band parameters used were those of J. H. Wood, Phys. Rev. 126, 517 (1962), L. F. Mattheiss, Phys. Rev. 139, A1893 (1965), and O. Dreirach, J. Phys. F: Metal Phys. 1, L4 (1971), respectively.

⁹S. Barisic, J. Labbe, and J. Friedel, Phys. Rev. Lett. 25, 919 (1970).

¹⁰Mattheiss, Ref. 8.

¹¹Wood, Ref. 8.

¹²J. S. Faulkner, H. L. Davis, and H. W. Joy, Phys. Rev. 161, 656 (1967).

¹³E. C. Svensson, B. N. Brockhouse, and J. M. Rowe, Phys. Rev. 155, 616 (1967).

Measurement of the Spin-Dependent Part of the Scattering Amplitude of Slow Neutrons on ¹⁹F Using a Polarized Beam and a Polarized Target

A. Abragam, G. L. Bacchella, C. Long,* P. Meriel, J. Peisvaux, and M. Pinot Service de Physique du Solide et de Résonance Magnétique, Centre d'Etudes Nucléaires de Saclay, 91 Gif-sur-Yvette, France (Received 28 December 1971)

Using a polarized target of CaF_2 , we have measured the spin-dependent part of the scattering amplitude of slow neutrons on ¹⁹F. A value $\beta = a_{+} - a_{-} = -0.135 \pm 0.002$ F was found, 10 times smaller than a recent theoretical estimate. A control experiment measuring $\beta = a_+ - a_-$ for the proton by Bragg scattering on a single crystal of LiH yielded the correct value within experimental error.

The scattering amplitude of a slow neutron on a nucleus of spin I can be written in operator form:

$$a = \overline{a} + \beta \overline{\mathbf{I}} \cdot \overline{\mathbf{s}}. \tag{1}$$

For ¹⁹F, \bar{a} is well known,¹

$$\bar{a} = 5.74 \pm 0.03 \, \mathrm{F}.$$
 (2)

The only existing information on β , obtained from the measurement of the total scattering cross section, is that $|\beta|$ is small:

$$\sigma_{\text{tot}}^{sc} = 4\pi (\bar{a}^2 + \frac{3}{16}\beta^2) = 4.0 \pm 0.3 \text{ b}, \qquad (3)$$

whereas $4\pi \overline{a}^2 = 4.14 \pm 0.04$ b. It follows that $|\beta|$ \leq 3 F and its sign is unknown.

A recent theoretical estimate is $\beta = -1.4$ F.² The Bragg scattering of a polarized neutron beam on a polarized target provides a method for obtaining the sign of β and also a much better accuracy for its magnitude when $|\beta|$ is small. With the assumption of a single nuclear species, the intensity of a Bragg-scattered beam is given by

$$g = \overline{a}^2 + \beta I \overline{a} P p + \frac{1}{4} \beta^2 I^2 P^2, \qquad (4)$$

where P and p are the respective nuclear and neutron polarizations counted positively along the applied magnetic field.

If $|\beta I P p| \ll |\overline{\alpha}|$, the ratio of the intensities of scattered neutrons with spins up or down (with the above sign convention) is given with good accuracy by

$$\boldsymbol{g}_{+}/\boldsymbol{g}_{-} = 1 + 2\beta I \boldsymbol{P} |\boldsymbol{p}| / \boldsymbol{\bar{a}}.$$
(5)

It is seen that in contrast to (3), (5) gives an ef-

fect linear with β .

Determinations of β by Bragg scattering of polarized neutrons on polarized nuclear targets have been performed before with $^{59}\mathrm{Co}\,^3$ and $^{51}\mathrm{V}.^4$ In either case the nuclear polarization P was that of thermal equilibrium and thus rather small (lower than 1%) and the experiment was successful thanks to the large value of $|\beta/\overline{a}|$. In the present case $\beta^{(19F)}$ was expected to be small, but the nuclear polarization $P(^{19}F)$ may be increased dynamically to, say, 40% in a single crystal of CaF₂.⁵ Besides, the second term in (5) may be enhanced even further. since we have a crystal with two nuclear species (neglecting ⁴³Ca with an isotopic abundance of 0.145%), by a suitable choice of the Bragg reflection. A straightforward generalization of (5) shows that the best one is (200), (5) being then replaced by

$$g_{+}/g_{-} = 1 - 4\beta I P |p| e / (\bar{a}_{Ca} - 2\bar{a}_{F}),$$
 (6)

where *e* is the efficiency of the device that reverses the neutron polarization (the neutron flipper). With \bar{a}_{Ca} = 4.9 F⁶ and \bar{a}_{F} = 5.74 F,¹ (6) predicts that

$$g_{+}/g_{-} = 1 + 0.3\beta P |p|e_{+}$$
 (7)

 β being expressed in fermis.

Pump

The experimental setup is given in Fig. 1. A "white" unpolarized neutron beam coming from the H₂ beam port of reactor EL₃ impinges on a single crystal of $\text{Co}_{98}\text{Fe}_8$ saturated with a magnetic field H=8 kOe. This crystal reflects, from (200) planes, monoenergetic neutrons of wave length $\lambda = 1.071$ Å and polarization $p = (99.5 \pm 0.4)\%$ parallel to \vec{H} . The polarized beam, guided by the guide field H_{g1} (~ 100 Oe) passes through a flipping coil which, when powered, can flip the neutron spins with an efficiency $e = (99.1 \pm 0.7)\%$.

H Co_{ge}Fe_{ace} flipping coil

FIG. 1. Experimental setup.

The neutron spins are then turned through 90° by means of a guide field H_{g2} , in the same manner as in experiment of Moon, Riste, and Koehler.⁷ The CaF_2 single crystal, a small sphere 2 mm in diameter at the center of a copper resonant cavity fed by a 4-mm wave guide, is dipped in liquid helium at the bottom of the cryostat in the gap of an electromagnet producing a field H_{h} = 13.6 kOe. By pumping on the liquid-helium bath, the temperature may be lowered down to 1.2°K. The counter, which may turn in a vertical plane around an axis parallel to H_{p} , is fixed at the $2\theta_{200}$ position. Small adjustments of the cryostat allowed us to set the crystal at the position giving maximum reflected intensity into the counter. I^+ (flipper off) and I^- (flipper on) were measured alternatively every 30 sec, counts being cumulated over 10 min. The nuclear polarization was given various values between -35% and +40%and was measured from the shape of the derivative of the NMR absorption curve.⁷

The first measurements showed at once that β was much smaller than expected, and a large number of counts were necessary to obtain β with an acceptable accuracy. A least-squares fit of 264 counts (with positive, negative, and zero polarization) by a formula of the type

$$g^+/g^- = A + BP \tag{8}$$

gave

$$A = 1.002 \pm 0.002$$
,

$$B = (-4.4 \pm 0.7) \times 10^{-2}.$$

whence

$$\beta = -0.15 \pm 0.02 \, \mathrm{F}, \tag{9}$$

a value 10 times smaller than the theoretical prediction!

Such a discrepancy could have been due to (1) a depolarization of the neutron beam which falls on the crystal. A careful measurement of the polarization of the neutron beam transmitted by the cryostat and sample, with another $\text{Co}_{92}\text{Fe}_8$ crystal, has shown that such is not the case; depolarization is, in the limits of statistical error, less than $\frac{1}{2}\%$. (2) Extinction effects. We first notice that the crystal is very small (4 mm³) and that the (200) reflection is weak. However, in absence of information about the mosaic spread, intensity measurements of (111), (200), and (220) reflections have been performed on this crystal, which gave relative values in accordance with

VOLUME 28, NUMBER 13

calculated ones, assuming no extinction.

Nevertheless, to be sure that no other source of error was present, an overall test experiment was performed, with the same experimental conditions, on a crystal in which nuclear polarization effects on Bragg scattering of polarized neutrons could be predicted. Since hydrogen has a well-known and large β value, we used a single crystal of LiH where the nuclear polarizations were those of thermal equilibrium in a field of 20.6 kOe at a temperature of 1.15°K, and had the following values:

$$P(H) = 1.83\%, P(^{6}Li) = 0.36\%,$$

 $P(^{7}Li) = 1.18\%.$ (10)

Using the reflexion (111), most favorable in that case, it was found that

$$\frac{g_{+}}{g_{-}} = 1 + 2 \frac{\beta PIw(^{6}\text{Li}) + \beta PIw(^{7}\text{Li}) - \beta PI(\text{H})}{\overline{a}_{\text{Li}} - \overline{a}_{\text{H}}} \cdot (11)$$

In (11), w stands for the relative isotopic abundance of ⁶Li and ⁷Li (0.074 and 0.926, respectively), and the deuterium contribution has been omitted; also,

$$\bar{a}_{\rm H} = -3.723 \pm 0.003 \, {\rm F},$$

 $\bar{a}_{\rm Li} = -1.94 \, {\rm F} \, ({\rm natural \ Li}),$
 $\beta_{\rm H} = 58.16 \, {\rm F},$
 $\beta PI \, ({\rm H}) = 5.32 \times 10^{-2} \, {\rm F},$
(12)

by far the largest contribution to the second term in (11). $\beta({}^{6}\text{Li})$ is unknown, but in view of the smallness of $Pw({}^{6}\text{Li})$ its contribution to (11) can be safely neglected. The magnitude of $\beta I({}^{7}\text{Li})$ can be deduced from its total (1.4 b) and coherent (0.6 b) scattering cross sections to be of the order of

$$\beta I (^{7} \text{Li}) \simeq \pm 3.9 \text{ F},$$
 (13)
 $\beta I P w (^{7} \text{Li}) \simeq 0.42 \ 10^{-2} \text{ F}.$

Using (12) and (13) we predict for (11)

$$\mathfrak{G}^+/\mathfrak{G}^- \simeq 1 - (5.9 \pm 0.46) \times 10^{-2},$$
 (14)

the last term in the parenthesis being the estimated contribution of ⁷Li. The experiment was done on a LiH single crystal irradiated in order to reduce the relaxation time down to 10 min. The experimental result is

$$g^+/g^- = 1 - (5.7 \pm 0.8) \times 10^{-2}$$
 (15)

The agreement between (14) and (15) is gratifying as an overall test of the experimental procedure.

It is worth pointing out that β ⁽¹⁹F) as given by (9) is sufficiently small to warrant a correction for the magnetic scattering by the nuclear magnetic moment of ¹⁹F! The amplitude for the latter is easily computed to be

$$\beta(^{19}\mathrm{F})_{\mathrm{magn}} \cong +0.015 \mathrm{F},$$

which is about 10% of the measured value (9) but smaller than the statistical uncertainty. The corrected value for β ⁽¹⁹F) should thus be

 β (¹⁹F) = -0.135 ± 0.02 F.

We gratefully acknowledge the assistance of all the members of the magnetic resonance group, of the reactor staff, and of Mr. Bédère who kindly supplied the LiH crystal.

*Present address: Department of Physics, Florida State University, Tallahassee, Fla. 32306.

¹M. Koester, N. Nucker, W. Nistler, and D. Trustedt, European-American Nuclear Data Committee Report No. 127, 1970 (unpublished), p. 20.

²V. Gillet and J. M. Normand, to be published.

³Y. Ito and C. G. Shull, Phys. Rev. <u>185</u>, 961 (1969).

⁴C. G. Shull and R. P. Ferrier, Phys. Rev. Lett. <u>10</u>, 295 (1963).

⁵M. Chapellier, V. H. Chau, and M. Goldman, Phys. Lett. <u>25A</u>, 6, 262 (1968).

⁶G. E. Bacon, *Neutron Diffraction* (Clarendon Press, Oxford, England, 1962), 2nd ed.

⁷R. M. Moon, T. Riste, and W. C. Koehler, Phys. Rev. <u>181</u>, 920 (1969).