

Positive Linear Dispersion in the Velocity of Sound in He II

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(Received 24 November 1971)

The dispersion in the phase velocity of first sound in He II at 1.38°K was measured between 20 and 60 GHz (1–3°K). The results show direct experimental verification of positive dispersion in the phonon branch. The dispersion in the sound velocity was measured to be $C(\nu) = C_0[1 + (6.3 \pm 0.7) \times 10^{-13}\nu + (0 \pm 8) \times 10^{-25}\nu^2]$.

Evidence¹⁻⁵ has recently been accumulating to suggest that the dispersion of the sound velocity in He II is positive in the low-frequency region. In addition, the recent analysis by Molinari and Regge⁵ of the neutron scattering data has shown that odd-power as well as even-power terms most probably must be included in the local expansion for the energy of an elementary excitation of order higher than the first order in the momentum. We present here direct experimental evidence that confirms both of these points. We find that the data on the dispersion in the sound velocity of He II can best be fitted by using a positive linear term in the expansion. Also, we give data on the dispersion of the phase shift which occurs at a liquid-helium-solid interface.

Acoustic interferometry is the classical way to study dispersion and phase shifts with high precision, and we have shown how this can be done for sound waves in He II in the frequency range of 20 to 60 GHz (1 to 3°K). This is the most interesting energy range for studying the dispersion; however, it is too low in energy for precise neutron measurements and too high for more standard low-frequency acoustic techniques. The interferometer⁶ in these experiments is formed by adsorbed films of liquid helium on freshly cleaved surfaces of SrF₂ crystals. The film thickness is determined by the chemical potential of the gas in the vicinity of the crystal. Monochromatic acoustic waves are generated and detected through the paramagnetic impurity ion, divalent thulium, which is present in trace amounts in these crystals.

Richmond and Ninham⁷ have shown that our original data on helium film thickness are in good agreement with the theory of Lifshitz and collaborators^{8,9} on the van der Waals force. Using a computer program supplied by Parsegian and Ninham¹⁰ for calculating this potential, we have been making more detailed comparisons with more precise data. This analysis of the helium-film data will soon be published. One interesting aspect of the more detailed analysis is the

residual differences between the calculated thickness d_V , as obtained from the measured potential of the helium gas, and the simple acoustic thickness d_A , as derived from the relationship

$$d_A = NC_0/\nu, \quad (1)$$

where N is the order of the resonant mode in the film ($\frac{1}{4}, \frac{3}{4}, \dots$), ν the frequency, and C_0 the sound velocity of the bulk liquid which is taken to be 236.6 m/sec. The conclusions of this paper are not dependent on small changes in the value of C_0 . Some of the residuals, $d_V - d_A$, are shown plotted in Fig. 1. The important thing to be derived from Fig. 1 is that the residuals are frequency dependent. For the same film thickness d_A the van der Waals potential increases or the true film thickness decreases as the frequency increases. This phenomenon can be explained by a frequency-dependent phase shift $d_0(\nu)$. The residuals converge at thicknesses of about 100 Å and then steadily increase, which is explained by a positive correction to the velocity of sound.

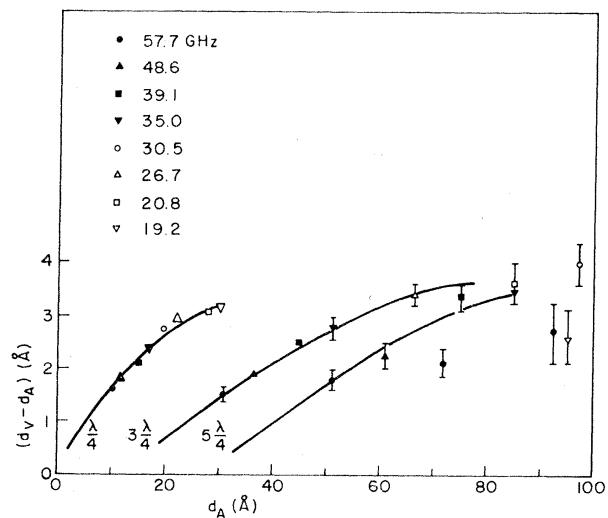


FIG. 1. The difference between the calculated thickness d_V and the acoustic thickness d_A as a function of d_A for different frequencies for unsaturated helium films at 1.38°K.

Thus, the true film thickness in terms of the acoustic measurements is given by

$$d = NC_0[1 + \delta(\nu)]/\nu + d_0(\nu), \quad (2)$$

where $\delta(\nu)$ is the dispersion correction to the phase velocity in the bulk liquid and $d_0(\nu)$ is the frequency-dependent phase shift which occurs near the liquid-solid interface.

For the same film thickness d , the difference in the acoustic thickness calculated for two different modes corresponding to two different frequencies ν_1 and ν_2 , $\nu_1 < \nu_2$, is given to a good approximation by

$$\begin{aligned} \Delta d &= d_A(\nu_2)[1 - N_1\nu_2/N_2\nu_1] \\ &= d_A(\nu_2)[\delta(\nu_1) - \delta(\nu_2)] + d_0(\nu_1) - d_0(\nu_2). \end{aligned} \quad (3)$$

By modulating the film thickness using standard lock-in techniques, the center frequencies of the two modes can be measured with a precision of better than 1 part in 10^3 . This provided a precise method whereby the dispersion in thick saturated films could be measured. The modulation was achieved by noting that the film thickness on the crystal is determined by the height of the crystal above the free surface of the bulk liquid. In our apparatus, the bulk liquid was at the bottom of a 1-mm-i.d. tube which had a small heater at its bottom. It was observed that by modulating the heater with a square wave at frequencies around a few hertz, the film thickness could be modulat-

ed up to amplitudes of a few angstroms. The modulation of the film thickness would only occur at certain resonant frequencies of an unidentified nature. The results of measurements using the technique are given in the first half of Table I.

It is not possible to make similar measurements with very thin films since the number of modes is restricted. However, an equivalent measure of Δd can be obtained using data similar to that given in Fig. 1 by taking the difference between each measured point and a linear interpolation between the values obtained at the highest frequency used in the experiment (57.74 GHz). Errors in calculating d_V are eliminated by this procedure. An error may arise from the possibility that the simple technique implicitly used in this work is not suitable for the quarter-wave resonance at 57.74 GHz. However, the changes introduced if this point is used are not sufficient to change the conclusions of this paper.

The quantity Δd is plotted in Fig. 2 as a function of the film thickness for data obtained near two frequencies, $\nu_1 \sim 35$ GHz and $\nu_2 \sim 55$ GHz. This plot confirms the simple model and demonstrates the overall consistency of the data. If no phase shift existed, Δd would be 0 for $d_A = 0$. The slope of the curve shows that the coefficient of the dispersion must be positive since $\nu_1 - \nu_2$ or $\nu_1^2 - \nu_2^2$ would be negative. It is also seen that it takes a film thickness of about 100 Å for the positive dispersion in the bulk velocity to just cancel the negative dispersion in the phase shift.

The data given in Table I have been analyzed using Eq. (1) and the expansion $\delta(\nu) = A\nu + B\nu^2$ for the dispersion and the expansion $d_0(\nu) = d_0 + C_1\nu + C_2\nu^2$ for the phase shift. A , B , C_1 , and C_2

TABLE I. Experimental data used to obtain the dispersion in the sound velocity of liquid He II at 1.38°K in the energy range of 1 to 3°K.

WAVE NO. N_1	FREQ. (GHz) ν_1	WAVE NO. N_2	FREQ. (GHz) ν_2	d_A (Å)	Δd (Å)
7/4	33.47	11/4	52.66	123.6	-0.16 ± 0.1
5/4	19.47	9/4	36.66	145.0	-0.11 ± 0.1
7/4	35.04	11/4	55.09	118.2	-0.06 ± 0.1
7/4	33.39	11/4	52.56	123.8	-0.19 ± 0.1
9/4	32.31	15/4	54.16	163.9	-0.93 ± 0.15
9/4	33.50	15/4	56.13	158.1	-0.82 ± 0.15
9/4	37.75	13/4	54.55	140.5	-0.57 ± 0.15
5/4	20.95	9/4	37.77	140.9	-0.17 ± 0.15
5/4	20.87	9/4	37.58	141.7	-0.07 ± 0.15
9/4	38.15	13/4	55.31	139.0	-0.51 ± 0.1
1/4	19.16	----	57.74	30.9	2.1 ± 0.2
1/4	20.8	----	57.74	28.5	2.0 ± 0.2
1/4	26.7	----	57.74	22.2	1.7 ± 0.2
1/4	30.5	----	57.74	19.4	1.5 ± 0.2
1/4	35.1	----	57.74	16.8	1.3 ± 0.2
1/4	39.13	----	57.74	15.1	0.8 ± 0.2

SATURATED FILMS

UNSATURATED FILMS

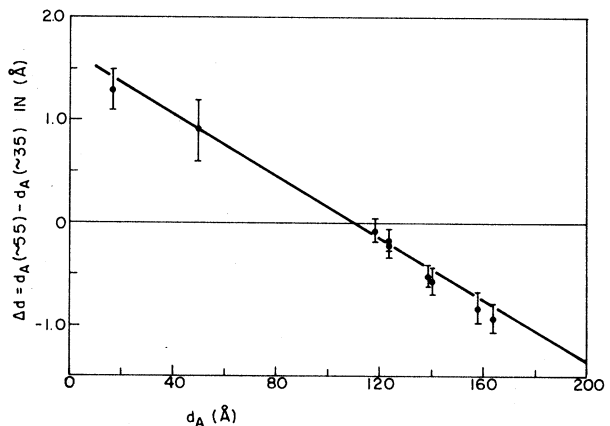


FIG. 2. The difference between d_A at about 55 GHz and d_A around 35 GHz as a function of the acoustic thickness d_A for a constant thickness d .

are the coefficients to be determined using Eq. (3) and the data. Using the method of least squares, we find that the dispersion in the sound velocity is given by $\delta(\nu) = (6.3 \pm 0.7) \times 10^{-13}\nu + (0 \pm 8) \times 10^{-25}\nu^2$, with ν given in units of hertz. The values from Molinari and Regge⁵ in these units are $\delta(\nu) = 7.3 \times 10^{-13}\nu - 4.7 \times 10^{-24}\nu^2$. The agreement in the linear term is rather remarkable, while not much can be said about the small quadratic term. The data are not consistent with an expansion for the dispersion of $\delta(\nu) = A\nu^2 + B\nu^4$. Thus, the experimental results give direct evidence that the dispersion is positive and that the linear term must be included in the expansion.

The values for the coefficients C_1 and C_2 for the phase shift are $C_1 = (-0.12 \pm 0.01) \times 10^{-9}$ and $C_2 = (6 \pm 1) \times 10^{-22}$. Using these coefficients the relative values of the phase shift can be obtained, while the absolute scale is obtained by a simple linear extrapolation to zero thickness d of data similar to that given in Fig. 1. A phase shift of 3.6 Å is obtained at 20 GHz, and a value of 0.8 Å at 60 GHz. The very rapid decrease in the phase shift suggests that the energy of the elementary excitations in helium is suppressed in the vicinity of the liquid-solid interface where the helium is under large pressures. More evidence for the deformation of the phonon spectrum has recently been given by Jäckle and Kehr.¹¹ If this is the case, this may be a natural explanation for the Kapitza-resistance dilemma.

We have been informed by Regge¹² that the following information was inadvertently dropped

from the paper of Molinari and Regge.⁵ The coefficients given in Eq. (3) of their paper were incorrectly rounded off. The correct fit is

$$\epsilon(p) = C\hbar p(1 + 0.5465p - 1.3529p^2 + 0.2595p^3 + 0.1860p^4 - 0.0522p^5)^{1/2}.$$

These coefficients may change by as much as 30% within a standard deviation. However, the corresponding errors are very strongly correlated, and independent rounding off is not allowed.

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Stationary States of Two-Dimensional Turbulence

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(Received 22 November 1971)

A class of exact statistical equilibria for the Navier-Stokes equation not including viscosity in two dimensions is noted and the corresponding fluctuation spectrum calculated. These solutions may account for the phenomena recently observed in numerical simulations of two-dimensional turbulence by Deem and Zabusky, including the existence of two distinct regimes of turbulence and the relaxation of one of these towards equipartition of vorticity.

There is considerable interest in two-dimensional hydrodynamic turbulence, partly because it may be relevant to meteorology and oceanography, but mainly because it is amenable to investigation by high-resolution numerical methods.¹⁻³

Deem and Zabusky¹ have recently reported such

numerical calculations and have interpreted their results in terms of the existence of two regimes. In the first regime (case 1) the equilibrium scalar-model energy spectrum takes the form

$$E(k) \propto k^{-\mu} \quad (1)$$