for collisionless absorption and the mean energy of the heated electrons can be estimated to be

$$\langle \mathcal{E} \rangle = (m_e \omega_0 P_A / n_c')^{1/2},$$

where P_A is the absorbed power per unit surface area and n_c' is the electron density gradient at the critical surface.

Recent one- and two-dimensional simulations including relativistic particle dynamics have also shown that when the laser wave field is sufficiently intense that the cyclotron frequency $\omega_{ce} > \omega_{pe}$ for a significant distance on the overdense side of the critical surface, a significant fraction of the incident energy is imparted to the electrons in a manner similar to that seen above by the oscillating radiation pressure, i.e., the $\vec{V} \times \vec{B}$ force. Here, however, polarization and angle of incidence are not so important, and the resulting electron energy distribution shows less of the two-temperature structure seen above.

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Sound Velocity in a Nematic Liquid Crystal*

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We have made sound-velocity measurements in the liquid crystal p-methoxy-benzylidine, p-n-butyl-aniline. Near the nematic-isotropic phase transition we observe a dispersion in the sound velocity. Quantitative measurements are presented on the sound-velocity anisotropy in an oriented liquid crystal.

It was found recently that the attenuation of ultrasonic waves shows an angular dependence in nematic liquid crystals,[1-3] the attenuation being higher for the propagation direction along an applied magnetic field and smaller for propagation perpendicular to the field. In all these papers it was noted that the corresponding velocity anisotropy, if any, was very small, typically 0.1% and smaller. Therefore, phase-sensitive detection is necessary in order to make a quantitative study of this effect.

We would like to report here on the first such velocity measurements using the liquid crystal p-methoxy-benzylidine, p-n-butyl-aniline (MBBA).[4] Our apparatus for the measurement of sound velocity has been described elsewhere,[5] and is very well suited for conditions where the attenuation is high. The measurement consists of a phase comparison, where the phase change of a pulse-modulated echo is compared with the stable cw source. For our measurements the resolution was typically 3 parts in 10^6 for relative changes in velocity. The sample was placed in brass cylinders using several different lengths (2.5, 4.8, and 7.3 mm), and the temperature was varied and stabilized by a heater with feedback control. The temperature stability was better than 0.1°C.

We first show our experimental results and then attempt to interpret them. In Fig. 1 we

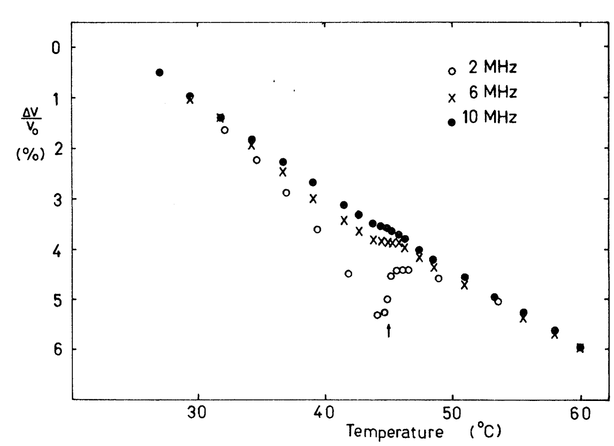

FIG. 1. Relative velocity changes for different frequencies as a function of temperature. Arrow indicates attenuation maxima at 45°C.

show measurements of the relative velocity for three different frequencies (2, 6, and 10 MHz) as a function of temperature without magnetic field. The absolute sound velocity at 22°C for 2 MHz is 1.54×10^5 cm/sec. One sees clearly a strong effect of dispersion near the nematic-isotropic phase transition T_n. Whereas the 2-MHz curve still shows a pronounced dip at the transition, it is much less pronounced at 6 MHz and almost completely eliminated at 10 MHz. Such dispersive effects have been observed before at gas-liquid transitions[6] and at magnetic transitions.[7] If there is such dispersion present, one expects a frequency dependence of the attenuation which is different from ω^2. While this has not yet been observed for MBBA, it has been noted in para-azoxyanisole (PAA).[8] Although the coupling of the sound wave to the order parameter (or a related quantity) is not known, one can estimate the relaxation time for this quantity near T_n. From Fig. 1 we get ($\omega\tau_0\sim1$ for $\nu\sim6$ MHz) $\tau_0\sim3\times10^{-8}$ sec. Because of the first-order nature of the transition and because we do not know whether a monodispersive relaxation model describes the effect, we do not attempt to determine the temperature dependence of τ_0.

In Fig. 2 we show the velocity anisotropy at $T=21$°C, $\nu=10$ MHz and with an applied field of 5 kOe. This anisotropy is established within seconds at fields higher than 1 kOe. No field dependence was observed between 0.5 and 5 kOe; the field was only used for aligning the molecules. In the case of Fig. 2 we observe a total anisotropy $\Delta=[v(\theta=0)-v(\theta=90°)]/v$ of 12×10^{-4}. The solid line is the function $\cos^2\theta$ which gives a good fit to the experiment. It is seen that possible $\cos^2 2\theta$ terms would be at least an order of magnitude smaller. The same features seem to apply to the attenuation in PAA and para-azoxyphenetole (PAP).[3] Our experimental results in Fig. 2 suggest that the elastic energy contains a term $Ck^2+C'(\hat{n}\cdot\vec{k})^2$, where $\hat{n}$ is the director and $\vec{k}$ the wave vector. C' is of the order of $25\times10^{-4}C$ for 10 MHz.

The temperature and frequency dependences of the velocity anisotropy Δ are shown in Fig. 3. One notices for $T\lesssim35$°C roughly a linear dependence of Δ on frequency in our frequency range: $\Delta=(1.25\pm0.05)\times10^{-10}\nu$. The temperature dependence of Δ is rather weak in this temperature region. For our frequency range and for temperatures such that $35°\text{C}<T<T_n$, Δ increases and converges to an approximately frequency-independent value of $\sim16\times10^{-4}$ just below T_n. It then falls to 0 within ~0.2°C. The high attenuation around T_n makes it very difficult to measure Δ with sufficient accuracy to study the exact behavior near T_n. We also found a strong dependence of Δ and T_n on sample purity. Δ changes by as much as 50% with less pure samples from different manufacturers.

The experimental anisotropy in the sound velocity indicates that at finite frequencies a liquid crystal has an anisotropic compressibility. This anisotropy can be explained if at these frequencies a liquid crystal in some respects behaves like a solid and the free energy contains terms like[9]

$$F=\tfrac{1}{2}k_1(u_{xx}+u_{yy})^2+k_2(u_{xx}+u_{yy})u_{zz}+\tfrac{1}{2}k_3u_{zz}^2, \tag{1}$$

where the k's are elastic constants, and the u_{ij} are the elastic strains. We have chosen the z

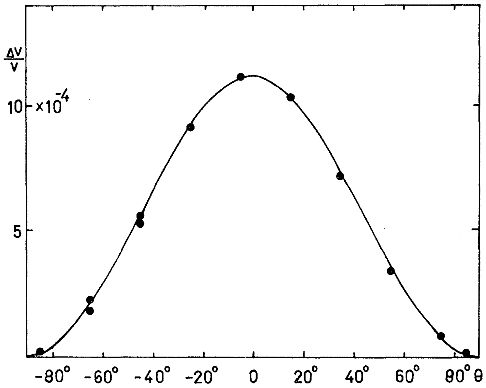

FIG. 2. Angular dependence of sound velocity. $T=21$°C, $\nu=10$ MHz, and $H=5$ kOe. θ is the angle between the field direction and propagation direction. Solid line is $12.5\times10^{-4}\cos^2\theta$.

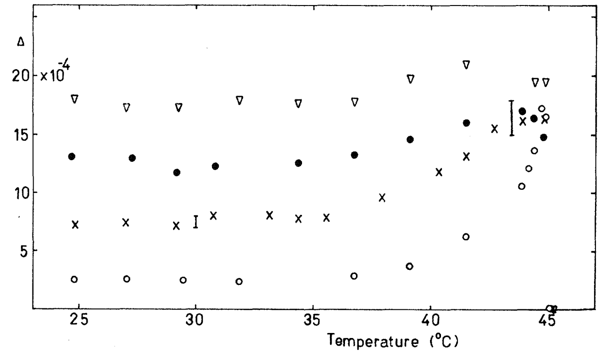

FIG. 3. Temperature dependence of the velocity anisotropy Δ for various frequencies: open circles, 2 MHz; crosses, 6 MHz; closed circles, 10 MHz; and triangles, 14 MHz. The bars indicate typical errors for different temperatures.

axis to be along the director $\hat{n}$, and Eq. (1) is consistent with the uniaxial symmetry of the liquid crystal. The condition $k_2^2 = k_1 k_3$ eliminates propagating shear modes, and the sound velocity is given by

$$c^2 = (1/\rho)[k_1 + (k_1 + k_3)\cos^2\theta], \tag{2}$$

which is of the form found experimentally in Fig. 2. However, the elastic constants must have an important frequency dependence, and as $\omega \to 0$ we would expect that $k_1 - k_3 \to 0$. If this were not the case, it would cost a finite energy to change the shape of a liquid crystal, the volume being kept constant. This is not consistent with our present ideas of the structure of a liquid crystal. The frequency dependence of the elastic constants could arise out of some structural relaxation process in the liquid crystal.[10] If we assume a single relaxation time τ_s for this process, then

$$k_3(\omega) - k_1(\omega) = k\omega^2\tau_s^2/(1+\omega^2\tau_s^2), \tag{3}$$

where k is a constant. It would be expected that $k\tau_s$ is of the order of a viscosity coefficient, i.e., 0.1 P.[10] The relative anisotropy in the velocity of sound is thus

$$\Delta = (k/2\rho c^2)\omega^2\tau_s^2/(1+\omega^2\tau_s^2). \tag{4}$$

Taking $k\tau_s \sim 0.1$ P, $\rho \sim 1$ g/cm^3, $c = 1.5\times10^5$ cm/sec, and $\omega/2\pi = 9$ MHz, we find that $\Delta \sim 10^{-4}\, \omega\tau_s/[1+(\omega\tau_s)^2]$. This is in rough agreement with experiment for $\omega\tau_s \sim 1$. This leads to a relaxation time $\tau_s \sim 2\times10^{-8}$ sec which is very much longer than that found in ordinary liquids. The experimental frequency dependence of Δ is only in rough agreement with (4) which might indicate a spectrum of relaxation times. Figure 3 indicates that τ_s increases with temperature by a factor of ~3 from $T = 30$ to 44.4°C.

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Electron-Phonon Interactions, *d* Resonances, and Superconductivity in Transition Metals

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A simple theory is developed for calculating the electron-phonon coupling constant λ.

Both for weak- and for strong-coupling superconductors, the important parameter which determines the transition temperature T_c is the electron-phonon coupling constant[1] λ. For simple metals one can reliably estimate λ by using pseudopotentials.[2,3] However, no such first-principle understanding of λ exists in the case of transition metals. As is well known, a transition-metal ion, in a solid, must be represented as a resonant scatterer of electrons and therefore cannot be described by a weak pseudopotential. Consequently, to evaluate λ one must perform a full band-structure calculation for the Fermi surface and the wave functions. As such calculations are fraught with extreme difficulties none as yet has been attempted. Our main purpose