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Experimental Inner-Electron Binding Energies in $_{100}$Fm and Limits on Electrodynamic Nonlinearities*

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Our earlier experimental value for the 1*s* binding energy in $_{100}$Fm (141.967 ± 0.013 keV) agrees with a precision relativistic Hartree-Fock calculation using linear electrodynamics to 2 ± 35 eV. This result is compared with a recent approximate calculation of the reduction (3.3 keV) in this binding energy when Born-Infeld nonlinear electrodynamics is used, and it indicates that such nonlinearities are limited to the order of 1% of Born-Infeld theory.

Atomic electron binding energies (eigenvalues) for inner shells in superheavy elements ($Z = 100$–250) were recently calculated by Rafelski, Fulcher, and Greiner[1] (RFG) on the basis of the usual linear Maxwellian (M) electrostatic interaction potential and also by using a particular nonlinear, Born-Infeld (BI) electrostatic interaction characterized by an upper limit to the electrostatic field strength. This limit E_0 ($= 1.2 \times 10^{18}$ V/cm) corresponds to the Born-Infeld requirement that the mass of the electron is entirely of electromagnetic origin. The numerical solution of the Dirac equation for each electron was obtained for a Thomas-Fermi atomic charge distribution for both interactions.

The results (of RFG) were presented only in graphical form (Fig. 2 of Ref. 1, reproduced here as Fig. 1) supplemented by a few high-Z numerical values; e.g., for $Z = 164$, $E(1s;\mathrm{M}) - E(1s;\mathrm{BI}) = \sim 770 - \sim 500$ keV $= \sim 270$ keV. From the graph it can be seen that the binding-energy differences between the linear and nonlinear theories are rapidly increasing functions of Z above $Z = 100$. As read from the graph, the Maxwellian 1*s* binding energy for $_{100}$Fm is $\sim 140 \pm 4$ keV to be compared with the recent experimental value[2] of 141.963 ± 0.013 keV, indicating reasonable graphing ac-

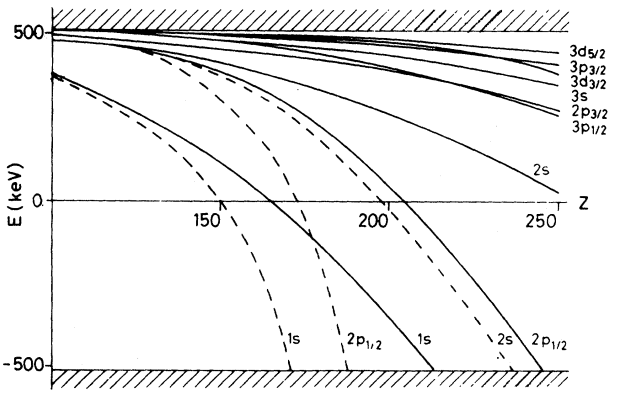

FIG. 1. (Photographic reproduction of Fig. 2 of Ref. 1.) Energy eigenvalues as a function of nuclear charge. Solid curves obtained with Born-Infeld theory; dashed curves, with Maxwell's equations. Upper shaded area is electron continuum; binding energies of orbits defined with respect to lower boundary of this (vacuum) continuum. Lower shaded area (gap $= 2mc^2$) is "positron" continuum.

TABLE I. Calculated (relativistic Hartree-Fock) $1s$ electron binding energy in $_{100}$Fm (all energies in keV).

Source	Amount
E_{1s} (neutral-atom eigenvalue, "frozen orbitals")	−143.051
Magnetic[a] (neutral-atom value, "frozen orbitals")	+0.709
Retardation[a] (neutral-atom value, "frozen orbitals")	−0.040
Rearrangement[b]	+0.088
Self-energy[c] (extrapolated from Ref. 3 values for $Z=70$–90)	+0.484
Vacuum polarization[d] (extrapolated from Ref. 3 values for $Z=70$–90)	−0.154
Electron correlation (Ref. 3)	−0.001
E_{1s} ($Z=100$)	−141.965 ± 0.025[e]
Experimental value	−141.967 ± 0.013

[a]Calculated by transverse-interaction method of Ref. 4.

[b]The rearrangement correction is the sum of three rearrangement energies: +0.111 keV, electrostatic; −0.025 keV, magnetic; and +0.003 keV, retardation. These were obtained from data for W, Hg, Pb, and Rn of Ref. 4.

[c]G. W. Erickson, Phys. Rev. Lett. 27, 780 (1971), in a calculation valid for all Z, obtained 0.462 ± 0.080 keV (read from Fig. 2 which gives self-energy term only, labeled "Lamb shift").

[d]Vacuum polarization is the only correction here obtained from a power series in Z.

[e]This error estimate is meant to include extrapolation errors in the correction terms, plus terms which are second order in α.

curacy. The small difference (M-BI) in $1s$ binding energies read from the graph is several keV at $Z=100$. This suggested to us the possibility that comparison of our recent precise binding-energy measurements[2] in $_{100}$Fm (unknown to RFG during their work) with accurate self-consistent field calculations of the binding energies could perhaps discriminate between the predictions of linear and nonlinear theories even at this "low" Z. Several recent relativistic Hartree-Fock calculations[3,4] at lower Z (70–90), including corrections for the self-energy, magnetic energy, vacuum polarization, retardation, and electron correlation energies, have shown very good agreement with experimental $1s$ binding energies to within <6 eV (0.006%) for Z values at subshell closures.

For this comparison, one of us (J.M.) calculated the $1s$ binding energy in Fm for the Maxwellian potential case. With a relativistic Hartree-Fock numerical program[4] the "frozen-orbital" eigenvalues for all shells were computed. Corrections for the Lamb shift (self-energy plus vacuum polarization) extrapolated to $Z=100$ from $Z=70$–90 values of Desiderio and Johnson,[3] for magnetic energy, for retardation and electron correlation, and for electron rearrangement converted the $1s$ eigenvalue of 143.051 keV to a binding energy of 141.965 ± 0.025 keV for the free atom (see Table I). We state the agreement with the experimental value (141.963 ± 0.013 keV for the $1s$ binding energy[5] to the Fermi surface of fermium oxide plus 0.004 keV estimated for electron work function) to be ± 0.035 keV. We note further that our experimental value fits very smoothly with a "Mosely-law" fit to $1s$ binding energies in the range $Z=90$–100.

Professor Greiner kindly supplied us the numerical values for the energy eigenvalues for orbitals of the neutral $_{100}$Fm atom, calculated using the Thomas-Fermi electron distribution and a Fermi nuclear charge distribution[6] with $C=7.7$ fm. For the $1s$ shell, the Maxwell interaction gave 142.83 keV and the Born-Infeld interaction gave 139.53 keV. Since these eigenvalues are subject to all of the corrections of the character of those listed in Table I to convert them into accurate binding energy predictions, we have, for present purposes, less concern with their absolute values than with the fate of their 3.3-keV difference during such conversion. It suffices that this difference, which is a hundredfold greater than our stated 0.035-keV agreement with a Maxwell-interaction-based correction calculation, not be radically reduced, in order that we may then infer a strong support for linear electrodynamics. We enumerate four comments on the 3.3-keV difference.

(1) In their article[1] RFG state that for somewhat lower Z, specifically $Z=82$, the M-BI $1s$ binding energy difference of ~0.25 keV is "of the order of magnitude of the vacuum-polarization correction," implying that the calculated difference is uncertain by such an amount. For $Z=100$, E_{vac}(Maxwell) = −0.154 keV (Table I). Whether or not a similar vacuum-polarization correction

is properly applicable to the nonlinear Born-Infeld case, the resulting shift of the 3.3-keV difference seems negligible.

(2) They also state, again for the case of $_{82}$Pb, for which the M-BI 1s difference is given as ~0.25 keV: "Such small changes can always be attributed to slight modifications in the nuclear charge distribution." However, the remarkable agreement with 1s binding energies achieved[3,4] for $_{74}$W, $_{80}$Hg, $_{82}$Pb, $_{86}$Rn, and here for $_{100}$Fm, indicates that the usual Fermi nuclear charge distribution used in these calculations is realistic over this large (74–100) Z range, and cannot be the source of an uncertainty of the order of a keV in the binding energy of either the Maxwell or Born-Infeld calculations, much less in their difference. Moreover, with a Dirac-Slater code, a 20% change in nuclear radius led to only a 113-eV change in 1s binding energy.[7]

(3) That the RFG calculations used the Thomas-Fermi (TF) atomic model rather than the preferred Dirac-Hartree-Fock (DHF) procedure results in a minor difference in the energy eigenvalue. For the Maxwell case the difference is only 0.22 keV (143.05 keV, cf. Table I, versus 142.83 keV, RFG). Since a similar difference will obtain in the Born-Infeld case (DHF-TF), the net effect on the M-BI difference on using DHF would seem to be a trivial change in the 3.3 keV.

(4) The largest corrections applied to the main term (cf. Table I), the magnetic and self energy, total 1.2 keV in the Maxwell case. Though substantial, the agreement with experiment to ≤ 6 eV cited for several high-Z elements strongly supports the accuracy of these terms. Again, since similar positive correction terms will be applicable to the Born-Infeld case, the 3.3-keV difference can hardly be expected to suffer a drastic reduction nearly to zero.

Thus there appears small likelihood that the best possible calculations will yield a M-BI difference for $_{100}$Fm grossly less than 3 keV. Therefore the agreement of our measured and calculated (Maxwell) 1s binding energies implies that linear electrostatic theory is an excellent approximation and that any possible nonlinearity is much smaller than that of Born-Infeld theory, i.e., $E_0 \gg 1.2\times 10^{18}$ V/cm.[8] To reveal the effect of nonlinear electrodynamics, RFG proposed experiments at Z(effective) = 164, $\cdots$, 196 (Pb-Pb and Cf-Cf scattering). From the comparisons cited here it now appears that such experiments will face a greatly reduced deviation from the predictions of linear electrodynamics, compared with the ~50% differences in the binding-energy predictions of Maxwell versus Born-Infeld at, e.g., $Z = 164$.

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[1]J. Rafelski, L. P. Fulcher, and W. Greiner, Phys. Rev. Lett. 27, 958 (1971).

[2]F. T. Porter and M. S. Freedman, Phys. Rev. Lett. 27, 293 (1971).

[3]A. M. Desiderio and W. R. Johnson, Phys. Rev. A 3, 1267 (1971).

[4]J. B. Mann and W. R. Johnson, Phys. Rev. A 4, 41 (1971).

[5]As an example of the agreement with experiment attained for other shells, the calculation for the Fm 2s binding energy gave 27.579 keV; experiment (Ref. 2) gave 27.577 ± 0.008 keV. This calculation used the same rearrangement energy correction as for 1s, and a Lamb-shift correction $\frac{1}{8}$ that of the 1s shift based on a N^{-3} dependence on principal quantum number N.

[6]In the calculations of Ref. 1 a uniformly charged sphere model was used for the nucleus.

[7]D. Liberman, private communication.

[8]W. Greiner and J. Rafelski (private communication) have calculated a 1s eigenvalue for $_{100}$Fm using a modified Born-Infeld nonlinear theory with $E_0 = 7^2 E_{0,\mathrm{BI}} = 59 \times 10^{18}$ V/cm. $E_{0,\mathrm{BI}}$ is the Born-Infeld value. The value 142.37 keV is only 0.46 below the Maxwell-case value.