final states. From these considerations, the correlations in $C(y_c, y_d)$ are expected to increase with increasing energy.[11] It is also tempting to identify the observed maxima in $C(y_c, y_d)$ with "pionization"—perhaps they indicate correlated production of particles with small c.m. momenta. Differences in $C(y_c, y_d)$ for $\pi^- p$ interactions at 8 and 18.5 GeV/c might thus indicate increasing "pionization" at higher energies. Observations on $\pi^+\pi^-$ and $\pi^+\pi^+$ correlations in two-particle inclusive final states would be useful in testing these hypotheses. The present results emphasize the importance of further study of two-particle and multiparticle inclusive reactions.

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[1]K. G. Wilson, Cornell University Report No. CLNS-131, 1970 (unpublished).

[2]E. L. Berger, in Proceedings of the International Colloquium on Multiparticle Dynamics, Helsinki, May 1971 (to be published).

[3]L. Van Hove, Phys. Rep. 1C, 347 (1971).

[4]N. N. Biswas, N. M. Cason, V. P. Kenney, J. T. Powers, W. D. Shephard, and D. W. Thomas, Phys. Rev. Lett. 26, 1589 (1971).

[5]W. D. Shephard, J. T. Powers, N. N. Biswas, N. M. Cason, V. P. Kenney, R. R. Riley, and D. W. Thomas, Phys. Rev. Lett. 27, 1164 (1971).

[6]We note that $\pi^- p$ events with ≤ 2 charged secondaries and $\pi^+ p$ events with ≤ 4 charged secondaries cannot contribute to Reactions (1) and (2). Studies of rapidity distributions as a function of final-state multiplicity show that these events are primarily responsible for the humps at positive y in Fig. 1.

[7]E. L. Berger and A. Krzywicki, Phys. Lett. 36B, 380 (1971).

[8]As the multiplicity of the final state increases, the reduced average kinetic energy per particle leads to a reduced probability for production of particles with large $|y|$. Thus the $f(s, \vec{P}_c)$, which include all multiplicities, fall less rapidly with increasing $|y|$ than the $G(s, \vec{P}_c, \vec{P}_d)$, which cannot include events of the lowest multiplicities (see Ref. 6). This leads to negative values for C.

[9]Other definitions of a correlation function are possible. R. H. Arnold, ANL Report No. ANL/HEP 7114, 1971 (to be published), suggests a correlation function $C' = C(s, \vec{P}_c, \vec{P}_d)/[\sigma_\infty^{-2} f(s, \vec{P}_c) f(s, \vec{P}_d)]$. We have examined the behavior of this function. The negative values for $|y_d| \geq 1$–2 are emphasized in C', since the rapid decrease in the single-particle distribution, which makes C small at large $|y|$, is divided out. The positive peaks in Fig. 3 are still obvious in plots of C'.

[10]G. Goldhaber, S. Goldhaber, W. Lee, and A. Pais, Phys. Rev. 120, 300 (1960).

[11]E. L. Berger, private communication.

Particle Creation in Isotropic Cosmologies*

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The simplest covariant generalization of the scalar wave equation leads to significant pion creation and annihilation processes near an isotropic Friedmann-type singularity (such processes are negligible for particles of nonzero spin). Estimates for a plausible initial state yield pion creation of the same order of magnitude as obtained by Zeldovich near an anisotropic Kasner-type singularity.

Zeldovich has considered a cosmological mechanism for the production of particles near a Kasner singularity.[1] Arguments based on a Newtonian model and analogy with electromagnetic pair production indicated that anisotropic expansion is necessary for significant particle production to occur. From another point of view, the above conclusion is a consequence of the conformal invariance[2] of the equations governing the quantized particle fields in the limit when the (rest) mass can be neglected. Such a limit holds near a cosmological singularity because the particles created are predominantly relativistic. The conformal invariance of the equations then results in a suppression of particle creation in the isotropic case.[3-5] (That is why a finite result was obtained in Ref. 5 when the fermion creation was extrapolated back to the singularity.) For nonzero spins, the simplest fully covariant generalizations of the equations of motion are indeed conformally invariant when the mass can be neglected. Therefore, significant creation of neutrinos, electrons, photons, and other particles of nonzero spin by the space-time metric will not occur in an isotropic expansion. For spin 0, the fully covariant generalization of the Klein-Gordon equation which becomes conformally invariant when the mass can be neglected is[6]

$$(g^{jk}\nabla_j\nabla_k - m^2 + \tfrac{1}{6} g^{jk} R_{jk})\varphi = 0. \qquad (1)$$

However, the simplest fully covariant generalization of the scalar wave equation, namely

$$(g^{jk}\nabla_j\nabla_k - m^2)\varphi = 0, \qquad (2)$$

is not conformally invariant when m can be neglected. In Ref. 4, Sec. F, a relation was found between the Einstein equations in a Friedmann universe and the creation of spin-0 particles,[7] which depends critically on the use of Eq. (2). That relation, and the simplicity of Eq. (2), might tend to favor it, in our opinion, as the equation governing the physical pion field.[8] Of course, only consistency with observation can in the long run decide between the two equations. We will show that the particle creation resulting from (2) near an isotropic Friedmann-type singularity is of the same order of magnitude as that resulting from (1) or (2) near an anisotropic Kasner singularity.[9]

In a space-time which is a vacuum solution of Einstein's equations, the scalar curvature vanishes, and (1) and (2) become identical. Therefore, anisotropy is indeed necessary to produce significant particle creation in a vacuum solution of the unmodified Einstein field equations. However, if the presence of matter or processes such as quantum fluctuations of the metric[10] give rise to scalar curvature, then significant production of pions obeying Eq. (2) will occur near isotropic cosmological singularities, without the need for anisotropy. It is clear that these considerations may also be of significance for isotropic gravitational collapse. In general, whether a net creation or annihilation of pairs[11] will occur depends on the initial state, since the formalism is time-reversal invariant. For an incoherent mixture of states initially containing definite numbers of particle-antiparticle pairs, there will be a net creation of pairs [Ref. 4, Eq. (53)], whereas for a class of coherent superpositions of such states there will be a net annihilation of pairs. For the present estimates, we suppose that the initial state is an incoherent mixture of the kind mentioned above, or a state in which matter, but no antimatter, is initially present, so that a net creation of pions will occur.

A rough estimate of the particle creation resulting from Eq. (2) near an isotropic Friedman-type singularity can be made by a method used in Ref. 1. For simplicity, we consider the class of line elements of the form

$$ds^2 = dt^2 + R(t)^2(dx^2 + dy^2 + dz^2). \qquad (3)$$

This line element is sufficient to yield estimates of the quantities involved. A more thorough investigation, with the general Robertson-Walker line element and an explicit analysis of the reaction back of the created matter on the expansion via the Einstein equations, is under way. It was shown in Ref. 4 (Sec. D) that the gravitationally induced pair creation is analogous to excitation of an oscillator or pendulum when the length l of its string is changed.[12] For the pendulum, the probability of excitation is significant when $|\dot{l}/l|$ is larger than the characteristic frequency ω. Similarly, one expects roughly that the probability of production of a pair of particles of enerby ω will be significant when $|\dot{R}/R|$ is larger than ω. Following Zeldovich (Ref. 1), we assume that the singularity is not quite reached, but rather that for a time of the order of the Planck time $t_P \equiv \sqrt{G}$, the quantity $|\dot{R}/R|$ is roughly of the order of t_P^{-1}, and that the particle production after t_P is relatively negligible. For typical particles, such as pions, m is much smaller than t_P^{-1}, so that the created particles are predominantly relativistic, and the momentum is essentially equal in magnitude to the energy ω. Then, since the production is isotropic, the density of states per unit energy and per unit volume is of the order $\omega^2 d\omega$. Assuming that on the average roughly one pair is created in each available mode, the number n per unit volume present at time t_P, just after the period of significant production, is

$$n \sim \int_0^{t_P^{-1}} \omega^2 d\omega \sim t_P^{-3}. \qquad (4)$$

Since the average energy of the created particles is of the order t_P^{-1}, the energy density ρ at that time is

$$\rho \sim t_P^{-4}. \qquad (5)$$

These magnitudes are the same as those obtained in Ref. 1 for the Kasner model. The energy density present at time t_P is consistent with a Friedmann expansion after that time (when the particle creation is relatively negligible), since the Einstein equation

$$8\pi G\rho = 3(\dot{R}/R)^2 \tag{6}$$

becomes

$$G t_P^{-4} \sim t_P^{-2}, \tag{7}$$

which is consistent. Thus, the energy density at the present epoch could conceivably be explained on the basis of such an early production process, as noted by Zeldovich in the anisotropic case.

An alternate estimate can be based on the mathematical equivalence of the present problem with a quantum-mechanical scattering problem. When the field satisfying Eq. (2), with line element (3) and periodic boundary conditions imposed in a cube of coordinate length L, is written as[13]

$$\varphi(\vec{x},t) = [LR(t)]^{-3/2}\sum_{\vec{k}} 2^{-1/2} \times [A_{\vec{k}} e^{i\vec{k}\cdot\vec{x}} h(\vec{k},t)^* + \mathrm{H.c.}], \tag{8}$$

where the $A_{\vec{k}}$ are constant annihilation operators, then h satisfies the equation

$$\ddot{h} + \left[\frac{k^2}{R^2} + m^2 - \frac{3}{4}\left(\frac{\dot{R}}{R}\right)^2 - \frac{3}{2}\frac{\ddot{R}}{R}\right] h = 0. \tag{9}$$

To apply scattering theory, note the similarity to the Schrödinger equation

$$\psi'' + (\epsilon - U)\psi = 0, \tag{10}$$

with $\psi \to h$, $x \to t$, $\epsilon = m^2$, and the potential

$$U = \frac{3}{4}\left(\frac{\dot{R}}{R}\right)^2 + \frac{3}{2}\frac{\ddot{R}}{R} - \frac{k^2}{R^2}. \tag{11}$$

We suppose that initially $R(t)$ corresponds to a contracting Friedmann universe. During the contraction phase (with $t \ll 0$) the potential U is zero or negative for all k, and approaches zero as $t \to -\infty$ [examples of typical $R(t)$ are $(-t)^{2/3}$ and $(-t)^{-1/2}$]. In the region $|t| < t_P$, we assume that the reaction back of the induced particle creation on the expansion, or some other nonlinear effect,[14] causes $R(t)$ to smoothly change over from contraction to expansion. During that process, clearly, $\ddot{R}$ must become positive, so that for a range of values of k the potential U will be positive. We assume that when $|t| < t_P$, the quantity $\frac{3}{4}(\dot{R}/R)^2 + \frac{3}{2}\ddot{R}/R$ is roughly of order t_P^{-2}. As t increases, U again becomes zero or negative and approaches zero as $t \to \infty$. For all k such that $k^2/R(t)^2$ is small with respect to t_P^{-2}, the potential U will have a positive barrier or bump of approximate height t_P^{-2} and width t_P centered at $t = 0$. The major part of the particle production corresponds to the quantum-mechanical scattering from this potential barrier.

With m of the order of the pion mass, the solution of Eq. (9) for $|t| \gg m^{-1}$ is given to excellent approximation by the WKB result, so that for $t \ll -m^{-1}$ we let

$$h = \omega^{-1/2} \exp\left(i\int^t \omega\, dt'\right), \tag{12}$$

with

$$\omega = [k^2/R(t)^2 + m^2]^{1/2}. \tag{13}$$

Then $A_{\vec{k}}$ is the annihilation operator for the particles initially present. For $t \gg m^{-1}$, h will have the form

$$h = \alpha\omega^{-1/2} \exp\left(i\int^t \omega\, dt'\right) + \beta\omega^{-1/2} \exp\left(-i\int^t \omega\, dt'\right), \tag{14}$$

with α and β independent of t. Current conservation in the scattering problem implies that

$$|\alpha|^2 - |\beta|^2 = 1. \tag{15}$$

Then, as shown in Ref. 4, the annihilation operator for particle present when $t \gg m^{-1}$ is

$$B_{\vec{k}} = \alpha^* A_{\vec{k}} + \beta A_{-\vec{k}}^{\dagger}. \tag{16}$$

Neglecting the effect of the initial density of matter,[15] we describe the system by the state vector with no particles initially present. The particle number density for $t \gg m^{-1}$ is then given by[4]

$$n(t) = [2\pi^2 R(t)^3]^{-1}\int_0^\infty dk\, k^2 |\beta(k)|^2. \tag{17}$$

A straightforward calculation using, for example, the WKB connection formulas for a potential of the type described, then yields results in agreement with a number density at time t_P (just after the period of significant production or scattering) given roughly by Eq. (4). The above calculation can be viewed as an estimate of either the particle creation occurring in an actual contraction followed by an expansion, or the particle creation which might be expected in a self-consistent model with a nearly singular initial state.

An exact analysis of the same scattering problem for the time-separated part of Eq. (1) with m set equal to zero yields precisely zero particle production, since the *exact* solutions for all t are (Ref. 4, Sec. H)

$$h \propto R(t)^{+1/2} \exp\left(\pm i\int^t kR^{-1}\, dt'\right). \tag{18}$$

Thus, there is no mixing of the positive- and negative-frequency parts of the field, and no particle production (or scattering). With m in Eq. (1) equal to the pion mass, one would expect very roughly that only a number density of order m^{-3} and energy density of order m^{-4} would be produced in a time interval of order m^{-1} near the singularity. Therefore, Eq. (1) cannot lead to an energy density of the observed order of magnitude in an isotropic model.

For an entirely consistent treatment of the particle creation resulting from Eq. (2) near an isotropic cosmological singularity, the general Robertson-Walker line element must be used, since the global spatial curvature is affected by the created matter. However, the above estimates would nevertheless be expected to be valid in order of magnitude. They indicate that the particle creation resulting from Eq. (2) near an isotropic cosmological singularity is worth investigating further as a simple model bearing on the problem of explaining the origin of matter in terms of the underlying space time. Investigations of the gravitationally induced particle creation may lead to fundamental new insights in cosmology.

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[1]Ya. B. Zeldovich, Pis'ma Zh. Eksp. Teor. Fiz. 12, 443 (1970) [JETP Lett. 12, 307 (1970)].

[2]Conformal invariance here refers to form invariance under a coordinate-dependent change of the metric, $g_{jk} \rightarrow \Omega^{-2} g_{jk}$, and an appropriate renormalization of the field, as discussed by R. Penrose, in *Relatively, Groups and Topology,* edited by C. DeWitt and B. DeWitt (Gordon and Breach, New York, 1964), p. 565.

[3]L. Parker, Phys. Rev. Lett. 21, 562 (1968).

[4]L. Parker, Phys. Rev. 183, 1057 (1969).

[5]L. Parker, Phys. Rev. D 3, 346 (1971).

[6]We use units with $\hbar = c = 1$. The metric signature is +2, and ∇_j is a covariant derivative. Additional interactions are neglected, since gravitational effects would be expected to be predominant near the singularity. Those additional interactions should be considered in the next stage.

[7]This relation is that the particle creation rate for $m=0$ vanishes in a radiation-filled universe, and that it vanishes most rapidly as a function of increasing m in a dust-filled universe. Our statement of the latter result in Ref. 4 was somewhat ambiguous, since particle production naturally vanishes in general as m approaches infinity.

[8]Equation (1) and the corresponding alteration in the gravitational theory were discusses by C. G. Callan, Jr., S. Coleman, and R. Jackiw, Ann. Phys. (New York) 59, 42 (1970). It would seem that the suppression of infinites in the matrix elements of their modified energy-momentum tensor is related to the suppression of particle creation by Eq. (1) in conformally flat space-times.

[9]Consistency of the particle production with conservation laws is extensively discussed by Ya. B. Zeldovich, in *Magic without Magic: John Archibald Wheeler,* edited by John R. Klauder (Freeman, San Francisco, 1972).

[10]J. A. Wheeler, in *Battelle Recontres,* edited by C. DeWitt and J. A. Wheeler (Benjamin, New York, 1968).

[11]For example, if the final no-particle state is taken as the initial state, then complete annihilation results.

[12]We ignore questions as to whether the effective oscillator frequency becomes imaginary near the singularity because, as our second argument below confirms, the estimated order of magnitude is not affected.

[13]Reference 4, Sec. A. The charged pion field is treated analogously, as in L. Parker, Ph. D. thesis, Harvard University, 1966 (unpublished).

[14]Such effects are discussed in B. S. DeWitt, Phys. Rev. 162, 1254 (1967); T. V. Ruzmaikina and A. A. Ruzmaikin, Zh. Eksp. Teor. Fiz. 57, 680 (1970) [Sov. Phys. JETP 30, 372 (1970)]; V. Ts. Gurovich, Dokl. Akad. Nauk SSSR 6, 1300 (1971) [Sov. Phys. Dokl. 15, 1105 (1971)].

[15]For the kinds of initial states under present consideration, it is clear from Ref. 4, Eq. (53), that the initial presence of matter will tend to enhance the pion creation.

[16]C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1972), Chap. 44.

[17]Since completion of this paper, the following reference, which summarizes and refers to the very important recent work of Ya. B. Zeldovich and collaborators has come to my attention: Ya. B. Zeldovich, Comments Astrophys. Space Phys. 3, 179 (1971). Their consideration of Eq. (1), whether or not it applied to actual pions, should in any event yield the main effects of the equations of nonzero spin. The program sketched in the present paper is complementary to theirs, in the sense that it should yield the cosmological effects of the possible breaking of the conformal invariance by the pion field [i.e., if it satisfies Eq. (2)].