both $Cl^-$ and $O^{2-}$. Expressed in the *common* coordinate system ($xyz$) we find, via (9), a spin-pair exchange

$$\mathcal{H}_{ab} \sim -80(s_{ax}s_{bx}+s_{az}s_{bz})+40s_{ay}s_{by}+40(s_{ax}s_{bz}-s_{az}s_{bx}), \tag{10}$$

with coefficients in degrees Kelvin.

Further measurements have been made on closely related complexes $Cu_4OBr_6[OP(C_6H_5)_3]_4$ and $Cu_4OCl_6(C_5H_5N)_4$ with qualitatively similar findings.[9] Agreement with orbitally degenerate theory can again be obtained with exchange parameters conforming qualitatively with physical expectations.

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[1]I. Dzialoshinski, J. Phys. Chem. Solids 4, 241 (1958).
[2]T. Moriya, Phys. Rev. 120, 91 (1960).
[3]K. Takeda, S. Matsukawa, and T. Haseda, J. Phys. Soc. Jap. 30, 1330 (1971).
[4]P. Bloembergen, K. G. Tan, F. H. J. Lefevre, and A. H. M. Bleyendaal, J. Phys. (Paris), Colloq. 32, 878 (1971).
[5]W. P. Wolf, J. Phys. (Paris), Colloq. 32, 26 (1971).
[6]F. Hartmann-Boutron, J. Phys. (Paris) 29, 212 (1968).
[7]A. P. Ginsberg, R. C. Sherwood, R. W. Brooks, and R. L. Martin, J. Amer. Chem. Soc. 93, 5927 (1971).
[8]J. Bertrand, Inorg. Chem. 6, 495 (1967).
[9]M. E. Lines, A. P. Ginsberg, R. L. Martin, and R. C. Sherwood, to be published.
[10]P. W. Anderson, Phys. Rev. 115, 2 (1959).

# Delayed Neutron Emission from $^{137}I$

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The $\beta$-delayed neutron energy spectrum from $^{137}I$ has been studied with a high-resolution neutron spectrometer. The neutron energy distribution is found to exhibit prominent line structure. The overall shape of the distribution indicates that $\gamma$-ray emission strongly competes with $d$-wave neutron emission over a large excitation energy interval. The line structure is shown to be in general agreement with spin-dependent level-density calculations.

Delayed neutron spectrometry is a potentially fruitful technique for precise level-density measurements above the neutron binding energy. No work has been reported previously on this subject because of the difficulty of producing delayed neutron precursors in sufficient strength, and the limited energy resolution of most fast neutron spectrometers. The recent development of a $^3He$ ionization chamber[1] with high detection efficiency and superior energy resolution for fast neutrons, in combination with an on-line mass spectrometer, has permitted the investigation of neutron-emitting energy levels in $^{137}Xe$ following $\beta$ decay from $^{137}I$.

The OSIRIS on-line isotope separator[2] was used to separate by mass the fission products from a $^{235}U$ sample located near the core of a 1-MW reactor. Ions with mass 137 were collected on magnetic tape and transferred to the remote neutron spectrometer, giving an initial source strength of about 3 $\mu$Ci and a fast neutron counting rate of more than 1 count/sec. Fresh samples were collected simultaneously with the measurement and transferred to the counting station at 60-sec intervals.

The experimental pulse-height distribution is shown in Fig. 1 for delayed neutrons emitted from excited levels in $^{137}Xe$. The prominent line structure seems to suggest neutron emission from widely spaced levels, particularly towards the low-energy end of the spectrum. An interpretation of the structure as arising from statistical fluctuations[3] would appear to be less probable. Analysis of the pulse-height distribution in terms of the energy-dependent spectrometer response function gives the individual energy components, shown in Fig. 1 by vertical lines with appropriate relative amplitudes. Two series of peaks are

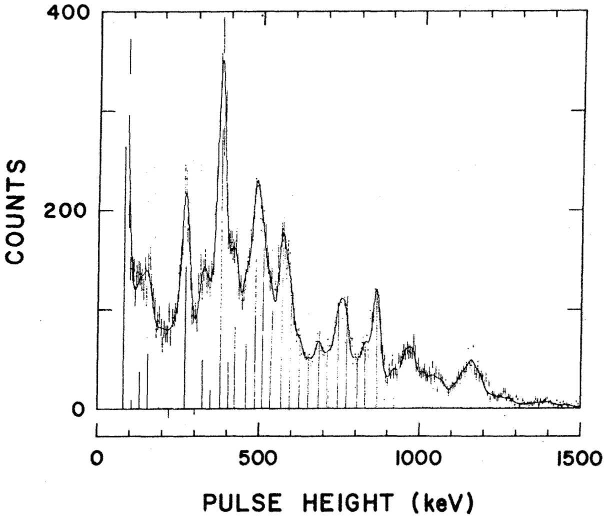


FIG. 1. Experimental pulse-height distribution of delayed neutrons from $^{137}$Xe. Error bars on data indicate statistical uncertainty. Solid curve is sum of energy-dependent response functions, whose energy and relative amplitudes are shown as vertical lines.

evident in the experimental spectrum. The most prominent series occurs at neutron energies 270, 380, 488, 570, 685, 756, 863, 965, and 1140 keV, and many of the lower-energy peaks are widely spread and remarkably well defined. A second series of peaks with much lower intensity can be seen at energies 155, 325, 425, and 460 keV and a large number of higher values. The overall average intensity distribution can be described as rising rapidly to a maximum at 350–400 keV, followed by a gradual reduction in intensity with increasing energy up to about 1300 keV.

The energy resolution of the spectrometer was 30 keV full width at half-maximum for neutrons up to 1500 keV, and the experimental widths of the peaks at 270, 380, and 863 keV are very close to this value. The other prominent peaks are broadened or asymmetrical, suggesting the presence of two or more closely spaced neutron-emitting levels.

In this Letter we shall discuss both the general shape of the neutron energy distribution and its fine structure. The diagrammatic energy-level scheme shown in Fig. 2 indicates that the shape of the spectrum is determined by the probability of populating levels in a given energy range in $^{137}$Xe coupled to the probability that these levels will decay by neutron emission to the ground state of $^{136}$Xe. The first excited state of $^{136}$Xe is at 1.32 MeV and will hardly be attainable judging from the estimated $Q_\beta$ value of $^{137}$I and neutron binding energy of $^{137}$Xe.

The $^{137}$Xe level population density can be written

$$\omega(E)\,dE = \text{const}\sum_{J,\pi}|M_{J\pi}|^2 \times f(Z+1,Q_\beta - E)\rho_{J\pi}(E)\,dE, \qquad (1)$$

where $M_{J\pi}$ is the matrix element for the transition to a level of spin $J$ and parity $\pi$, and $f$ is the Fermi function. $\rho_{J\pi}(E)$ is the density of levels of spin $J$ and parity $\pi$.

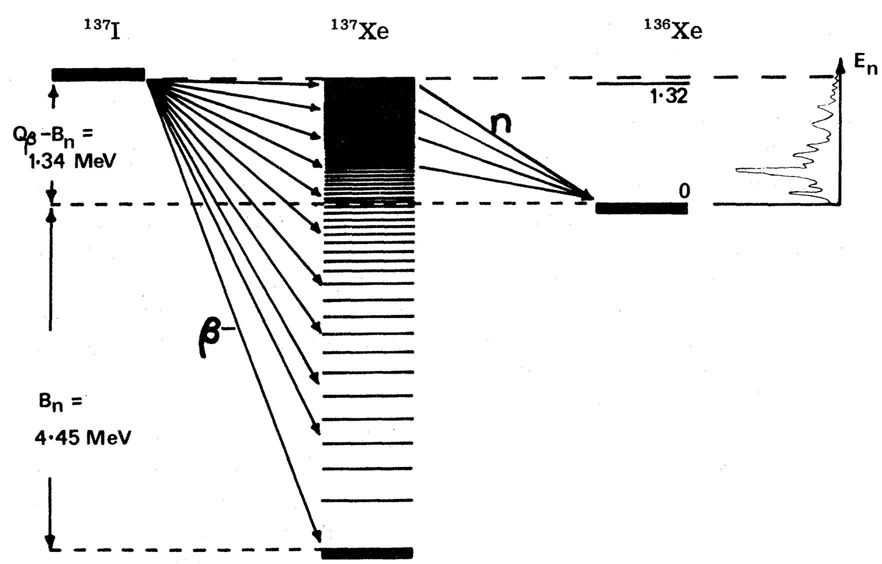


FIG. 2. Energy-level diagram for $\beta$-delayed neutron emission from $^{137}$Xe.

Following Jahnsen, Pappas, and Tunaal[4] we assume that $|M|^2$ is independent of energy, but changes by about 2 orders of magnitude from allowed to first-forbidden $\beta$ transitions. An alternative approach, in which $|M|^2\rho$ is assumed to be independent of energy, has been suggested by the decay of heavy neutron-deficient nuclei to highly excited daughter states[5] and in delayed proton emission.[6] However, the applicability of these results to the case of heavy neutron-rich nuclei is far from obvious, in view of the very different energy relationship between the isobaric analog state and the neutron-emitting levels. Indeed, recent measurements[7] indicate that the product $|M|^2\rho$ increases with energy for neutron-rich nuclei, although usually not as rapidly as the level density.

We replaced the Fermi function by the approximation $(Q_\beta - E)^5$ and employed the Fermi gas model[8] to obtain a level-density expression of the form

$$\rho(U) = \tfrac{1}{12}\pi^{1/2}a^{-1/4}U^{-5/4}\exp(2\sqrt{aU}), \tag{2}$$

where $\rho(U)$ is the total level density at an excitation $U$ above the degenerate level.

With the assumption of an energy-independent $|M|^2$, the level population density $\omega(E)$ is a smooth function, rising from zero at the paring energy $P(Z)+P(N)$ to a maximum value, and decreasing to zero at the energy $Q_\beta$. Blatt and Weisskopf[9] suggested a value of 9 $\mathrm{MeV}^{-1}$ for the level-density parameter $a$, although more recently[10] a value closer to 15.5 $\mathrm{MeV}^{-1}$ has been proposed. A prediction[11] by semiempirical shell-correction systematics gives an intermediate value of 13 $\mathrm{MeV}^{-1}$ for this parameter. However, in no case can the observed maximum neutron intensity at energy 4.85 MeV ($B_n$ +0.38 MeV) be attributed to the maximum in the level population density, which occurs between 2.8 and 3.4 MeV for values of $a$ in the range 9 to 15.5 $\mathrm{MeV}^{-1}$.

Allowed $\beta$ transitions from the $\frac{7}{2}^+$ ground state of $^{137}$I can lead to $d$-wave neutron emission from $\frac{5}{2}^+$ levels and $g$-wave neutron emission from $\frac{7}{2}^+$, $\frac{9}{2}^+$ levels in $^{137}$Xe. Competition by $\gamma$ emission is expected[4] to be severe for neutrons up to 300–400 keV in the former case, and up to much higher energies in the latter case. Consequently we conclude that the sharp drop in intensity of neutrons below 250 keV is due to $\gamma$ competition, rather than to the energy dependence of the level population density.

Several spin-dependent level-density expressions have been developed,[8,10,12] giving the fraction of the states with spin $J$. We have adopted Ericson's form,

$$\rho(E,J) = \frac{2J+1}{2(2\pi)^{1/2}\sigma^3}\rho(E)\exp\left[-\frac{J(J+1)}{2\sigma^2}\right] \tag{3}$$

together with a value[12] of 4 for the spin cutoff parameter $\sigma$. Combining Eqs. (2) and (3), assuming equal probabilities for states with even and odd parity,[8] and employing the value of 9 $\mathrm{MeV}^{-1}$ for the parameter $a$, we obtain an average separation for $\frac{5}{2}^+$ levels in $^{137}$Xe ranging from 133 to 27 keV, over the neutron energy range from 0 to 1.34 MeV. These figures agree well with the observed neutron energy distribution, and provide a measure of support for the initial interpretation of neutron emission from individual, widely spaced levels.

In summary, the $\beta$-delayed neutron energy spectrum from $^{137}$I has been determined, and the maximum observed energy of about 1300 keV is in agreement with the $Q_\beta$ and $B_n$ values. The sharp falloff in neutron intensity below 300 keV is shown to be due to $\gamma$ competition and not to the shape of the $\beta$ strength function. A series of prominent peaks in the spectrum is interpreted as allowed $\beta$ decay to $\frac{5}{2}^+$ levels in $^{137}$Xe, followed by $d$-wave neutron emission to the ground state of $^{136}$Xe. A second series of peaks with lower intensity is interpreted as first-forbidden $\beta$ decay to $\frac{3}{2}^-$ levels in $^{137}$Xe, followed by $p$-wave neutron emission. Level spacings are compatible with simplified calculations using the Fermi gas model of the nucleus.

It should be pointed out that the remarkably low density of neutron-emitting levels in $^{137}$Xe is probably due to a combination of factors. With only one neutron outside a closed shell, both the level density parameter and the neutron binding energy have low values. Furthermore, because of the high spin value of $^{137}$I, the transition rules for allowed $\beta$ decay and the dependence of $\gamma$ competition on neutron wave number, only $\frac{5}{2}^+$ levels contribute effectively to neutron emission. It is hardly to be expected that other delayed neutron emitters will show this combination of factors, and consequently they will tend to exhibit more complex or even continuous neutron energy spectra.

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[1]S. Shalev, Technion Report No. TNSD-R/412, 1970 (unpublished); J. M. Cuttler, S. Shalev, and Y. Dagan, Trans. Amer. Nucl. Soc. 12, 63 (1969).

[2]S. Borg, I. Bergstrom, G. B. Holm, B. Rydberg, L. de Geer, B. Grapengiesser, E. Lund, and L. Westgaard, Nucl. Instrum. Methods 91, 109 (1971).

[3]G. Hansen, private communication (1971).

[4]T. Jahnsen, A. C. Pappas, and T. Tunaal, *Delayed Fission Neutrons* (International Atomic Energy Agency, Vienna, 1968), p. 35.

[5]C. L. Duke, P. G. Hansen, O. B. Bielsen, and G. Rudstam, Nucl. Phys. A151, 609 (1970).

[6]I. Bacso, D. D. Bogdanov, S. Darocsy, V. A. Karnaukhov, and L. A. Petrov, Yad. Fiz. 7, 1153 (1968) [Sov. J. Nucl. Phys. 7, 689 (1968)].

[7]K. B. Nielsen, H. Johansen, and G. Rudstam, unpublished.

[8]T. Ericson, Advan. Phys. 9, 425 (1960).

[9]J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (Wiley, New York, 1952).

[10]N. Cindro, Rev. Mod. Phys. 38, 391 (1966).

[11]J. W. Truran, A. G. W. Cameron, and E. Hilf, CERN Report No. CERN-70-30, 1970 (unpublished).

[12]J. R. Huizenga and R. Vandenbosch, Phys. Rev. 120, 1305 (1960).

# Validity of Strutinsky's Theory of Renormalization

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The validity of Strutinsky's theory of renormalization is examined by comparing its predictions with those of a Hartree-Fock (HF) calculation for the nucleus $^{32}S$. Very good agreement is obtained if the single-particle states from the HF calculation are used in the Strutinsky calculation. For a conventional Strutinsky calculation using a phenomenological Nilsson potential, agreement with HF results is not as good. The latter agreement is improved if one treats the quadrupole moment of the density rather than that of the average potential as the independent variable.

Recently, a theory of renormalization to take into account the effects of shells on the deformation energy has been proposed by Strutinsky.[1,2] The theory has since been adopted in a number of calculations of nuclear structure throughout the periodic table.[2-7]

In essence the theory of renormalization postulates that the total energy of a system is given as in the liquid-drop model but with a correction term which can be derived in a prescribed way from the single-particle spectrum of the average field. As a result, relative equilibrium minima are correlated with the presence of generalized "shells." The validity of such an approach has been discussed in great detail in terms of fluctuations in density, in quadrupole moments, and in the forces acting at the equilibrium deformation.[3] Here we wish to examine the validity of the theory by comparing its predictions with those of a Hartree-Fock (HF) calculation.

Previous work along similar lines has been undertaken by Bassichis *et al.*[8] who did a HF calculation for $^{108}Ru$ and $^{108}Xe$. They obtain a qualitative correlation between the HF results and the results from a calculation with renormalization (henceforth called a Strutinsky calculation). However, in their work the comparison of the HF and Strutinsky calculations is made difficult by the complexity of the single-particle spectrum. Further, in interpreting their results, an ambiguity arises in extracting the smooth part of the liquid-drop deformation energy $E_{LD}$ from the HF results. The latter is accomplished with a free-hand drawing through and around the calculated points.

It is simpler and more constructive to look at