Some Implications of a Recent Test of Muonic-Atom Theory*

George A. Rinker, Jr., and Marvin Rich
Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico 87544
(Received 29 November 1971)

We find that the recent discrepancies observed by Dixit *et al.* in muonic-atom transition energies can be reproduced by various simple potentials.

Dixit, Kessler, and co-workers[1-3] have recently completed a series of very high precision measurements of muonic-atom transition energies in a variety of elements. Among these transitions are a number that should be largely unaffected by nuclear and electronic effects, which are generally believed to be the largest sources of uncertainty in the theoretical treatment of the problem. For this reason, these transitions provide a test of what is presumed known about muonic atoms. If the reported experimental numbers are taken at face value, the theory appears to be inadequate, as significant and systematic discrepancies were found.

Since these discrepancies were not seen in a similar experiment conducted by Backenstoss *et al.*,[4] the experimental situation is not absolutely clear at the present time. However, it is interesting to note that an earlier experiment by Anderson *et al.*[5] detected a similar discrepancy in the $4f_{7/2} \rightarrow 3d_{5/2}$ transition energy in ^{206}Pb and ^{208}Pb. This discrepancy was studied by Ford and Wills,[6] who were not able to find a prescription which would remove it. The more recent experiment by Kessler and co-workers[2] has verified their earlier measurement. In addition, a recent experiment by Martin *et al.*[7] had a similar difficulty with the $3p_{3/2}$-$3d_{5/2}$ splitting in ^{208}Pb. These results lend credibility to the idea that the present discrepancies are real.

The purpose of this Letter is to investigate the behavior of the present discrepancies in terms of the relevant physical parameters. Before we do so, however, it is worthwhile to consider for a moment the theoretical contributions to the muon energy levels which have already been included.

The finite-nuclear-size correction is calculated to be smallest for those elements in which the discrepancies are largest (Ba and Pb). This effectively rules it out as a possible source of the disagreement for any but highly exotic assumptions about the nuclear charge distributions.

We investigated the "exotic assumption" suggested in Ref. 3 (the nuclear halo originally postulated by Barrett *et al.*[8]) by attaching various tails to nuclear charge distributions appropriate for ^{208}Pb. Diffuse, constant tails which extended to radii of from 10 to 40 fm were tried, as well as exponential tails with relatively long falloff distances. We invariably found that a tail which removed the $5g \rightarrow 4f$ discrepancies prevented a reasonable fit to the data for the lower-lying transitions, within the limits of the charge distributions which were used for the bulk of the nucleus. Although we cannot rule out the possibility that a tail might provide the answer, it does not seem likely. Even more exotic suggestions, like a shell of charge of 50 fm radius, might do the trick but seem even less physical.

Effects of electron screening are evidently ruled out by the two sets of transitions observed in Ba ($5g \rightarrow 4f$ and $4f \rightarrow 3d$). The screening was calculated[1] to be larger for the outer ($5f \rightarrow 4f$) set, as one would expect. However, the inner transitions exhibit a much larger discrepancy.[9] This indicates that whatever the difficulty is, its source is probably the nucleus and not the outer atom.

The other theoretical corrections which have been included have been studied extensively and with good success[10] both in electronic and muonic atoms. These are vacuum polarization,[11-15] Lamb shift,[8,16-18] nuclear polarization,[19-21] and nuclear recoil.[22,23] There appears to be no good reason to doubt that they have been accounted for correctly here.

In an effort to find any trends in the data, we have attempted to fit the observed discrepancies by means of perturbing potentials of the form

$$V(r) = C_{mn} Z^m |V_C(r)/Ze^2|^n$$

over the volume occupied by the muon, where m and n may be 1, 2, or 3, C_{mn} is an adjustable parameter, and $V_C(r)$ is the electrostatic (Coulomb) potential generated by the average nuclear charge distribution. Since $V_C(r)$ varies like r^2 for small r and like r^{-1} for large r, the above potential approximates $C_{mn}Z^m/r^n$ over the region of primary interest, while avoiding an unphysical singularity at the origin. This choice does not exhaust all possible potentials, of course, but it is simple, convenient, and representative of many electromagnetic effects.

We did not consider any explicitly spin-dependent interactions, since the data do not indicate any inconsistencies in the fine-structure splittings. This may be seen by observing that the difference in energies of a pair of transitions in a given element is equal to the difference in the splittings of the upper and lower levels. The experimental and theoretical values for these numbers are nearly the same.

The energy shifts due to these potentials were calculated in first-order perturbation theory with numerically derived muon wave functions and appropriate nuclear charge distributions. After C_{mn} was adjusted to give a least-squares fit to the discrepancies for each choice of m and n, we calculated the shifts which would be produced in the $3d_{3/2} \rightarrow 2p_{1/2}$ and $2p_{3/2} \rightarrow 1s_{1/2}$ transitions in Pb.

These energies are listed in columns 5–13 of Table I, along with the values of C_{mn} and χ^2 obtained. Column 14 contains the best fit for a perturbing potential proportional to the lowest-order vacuum polarization potential, as was suggested in Ref. 3.

The most striking aspect of Table I is that the potentials linear in Z (which includes the lowest-order vacuum polarization) do not provide a credible fit to the data, while the potentials nonlinear in Z do. This might conceivably be explained in terms of a nonlinear electrodynamics,[24] or more conventionally, in terms of quantum-electrodynamic (QED) graphs with more than one photon-nucleus vertex. The potentials which vary as r^{-3} evidently do not have a long enough range, as they cause large perturbations in the lower-lying energy levels which probably cannot be absorbed by the nuclear distribution of nuclear polarization uncertainties. [We note that those effects which can be approximately formulated in terms of a δ-function potential (Lamb shift, heavy particle exchange,[25] muon finite size[26]) are even worse in this respect.] The energy shift due to such effects is essentially proportional to the overlap between the muon and the nucleus. Since this is larger by a factor of about 5×10^4 for the $1s_{1/2}$ state in Pb than for the $4f_{5/2}$ state, a 100-eV shift of the latter would lead to a 5-MeV change in the former.

Of the potentials in Table I, we are thus left with four possibilities which fit the Anderson data about equally well. In addition, all of these approximately remove the earlier discrepancies.[5,7]

If we are to believe that similar potentials may

TABLE I. Comparison of the discrepancy ΔE between theoretical and experimental muonic-atom transition energies with best fits for several simple potentials with asymptotic forms $C_{mn}Z^m/r^n$. All energies are in electron volts. The χ^2 per degree of freedom is given in the first line. The final two transitions for lead were not included in the fits.

Z	Transition	E_{exp}	ΔE $(E_{th}-E_{exp})$	$C_{11}\frac{Z}{r}$	$C_{21}\frac{Z^2}{r}$	$C_{31}\frac{Z^3}{r}$	$C_{12}\frac{Z}{r^2}$	$C_{22}\frac{Z^2}{r^2}$	$C_{32}\frac{Z^3}{r^2}$	$C_{13}\frac{Z}{r^3}$	$C_{23}\frac{Z^2}{r^3}$	$C_{33}\frac{Z^3}{r^3}$	Vacuum Polarization
$\chi^2/19$ ⟶				1.88	0.54	0.70	1.88	0.37	0.44	2.72	0.62	0.26	1.75
C_{mn} (eV-fm^n) ⟶				121	2.38	0.0355	2890	58.8	0.910	63300	1480	2.42	0.0339
20 Ca	$3d_{3/2}\rightarrow2p_{1/2}$	158 173	8±18	27	10	3	25	10	3	25	12	4	25
	$3d_{5/2}\rightarrow2p_{3/2}$	156 830	15±16	26	10	3	24	10	3	23	11	4	24
22 Ti	"	191 921	0±19	32	14	5	33	15	5	37	19	7	32
		189 967	10±18	32	14	4	32	14	5	34	17	6	31
26 Fe	"	269 427	35±20	46	23	9	56	30	12	73	44	19	50
		265 705	22±17	44	23	9	53	28	11	65	40	17	48
38 Sr	$4f_{5/2}\rightarrow3d_{3/2}$	200 254	21±20	34	25	14	26	20	12	18	16	10	29
	$4f_{7/2}\rightarrow3d_{5/2}$	198 708	4±18	33	25	14	25	20	11	17	15	9	28
47 Ag	"	308 428	44±20	52	48	34	50	48	35	42	46	36	52
		304 759	35±18	51	47	33	48	46	35	39	43	33	50
48 Cd	"	321 973	39±19	55	52	37	53	52	39	46	52	41	54
		317 977	29±18	54	50	36	51	50	37	43	48	38	53
50 Sn	"	349 953	47±21	60	59	44	61	62	48	55	64	52	61
		345 226	50±19	58	57	43	58	59	46	51	59	48	59
56 Ba	"	441 299	99±22	76	83	69	87	99	86	89	116	106	82
		433 829	114±20	73	80	67	82	93	81	80	105	96	79
	$5g_{7/2}\rightarrow4f_{5/2}$	201 260	31±17	34	37	31	22	25	21	11	14	13	26
	$5g_{9/2}\rightarrow4f_{7/2}$	199 902	22±16	34	37	31	21	24	21	10	14	13	25
82 Pb	"	437 687	137±22	75	121	148	71	118	149	54	104	139	122
		431 285	122±19	73	117	144	67	112	142	50	96	129	117
	$3d_{3/2}\rightarrow2p_{1/2}$			408	658	805	1390	2320	2940	4030	7730	10400	740
	$2p_{3/2}\rightarrow1s_{1/2}$			585	944	1160	3120	5200	6610	12900	24800	33200	1270

arise from electromagnetic effects, we must be able to approximately construct the constants C_{mn} for them from the parameters available. Limiting ourselves to e^2, α, and $\lambda_\mu/2\pi$, we find that $C_{21} \approx 0.03e^2\alpha^2$, $C_{31} \approx 0.06e^2\alpha^3$, $C_{22} \approx 0.4e^2\alpha^2\lambda_\mu/2\pi$, and $C_{32} \approx 0.9e^2\alpha^3\lambda_\mu/2\pi$. This indicates that higher-order QED effects cannot be ruled out *a priori* as sources of the present discrepancies. It is the range of the effect more than its order in α that is important.

The last two lines of Table I give only a rough idea of the trouble which might arise in other transitions due to the potentials considered. A more thorough investigation requires a consistent simultaneous fit of all the known levels. We have attempted such a fit using the Z^2/r^2 potential in Table I, but were unable to obtain a good representation (with χ^2 per degree of freedom $\lesssim 1$) of all the data by merely adjusting the parameters of the charge distribution. However, this does not rule out the possibility that some similar effect might provide most of the answer. The transitions which were fit determine, at best, only the long-range part of the effect (on a muonic-atom scale), and the short-range behavior may be more important for many of the states.

It is also of some interest to see if the above potentials are compatible with other precision tests of QED, such as the Lamb shift and $2p$ fine-structure splitting in hydrogenlike electronic atoms, since any serious conflict might lead one to suspect that the present effect is unique to muonic atoms or is due to experimental error. In fact, we have found that no such conflict exists, as reasonable extrapolations of the potentials to electronic atoms do not violently disturb the Lamb shift or $2p$ splitting.

One of us (G.R.) would like to thank Madhu Dixit and Herbert Anderson for making their data available prior to publication, and for useful discussions.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]M. S. Dixit, Ph.D. thesis, University of Chicago, 1971 (unpublished).

[2]D. Kessler, in Proceedings of the Muon Physics Conference, Fort Collins, Colorado, 6–10 September 1971 (unpublished).

[3]M. S. Dixit, H. L. Anderson, C. K. Hargrove, R. J. McKee, D. Kessler, H. Mes, and A. C. Thompson, Phys. Rev. Lett. 27, 878 (1971).

[4]G. Backenstoss, S. Charalambus, H. Daniel, Ch. von der Malsburg, G. Poelz, H. P. Povel, H. Schmitt, and L. Tauscher, Phys. Lett. 31B, 233 (1970).

[5]H. L. Anderson, C. K. Hargrove, E. P. Hincks, J. D. McAndrew, R. J. McKee, R. D. Barton, and D. Kessler, Phys. Rev. 187, 1565 (1969).

[6]K. W. Ford and J. G. Wills, Phys. Rev. 185, 1429 (1969).

[7]P. Martin, G. H. Miller, R. E. Welsh, D. A. Jenkins, and R. J. Powers, Phys. Rev. Lett. 25, 1406 (1970).

[8]R. C. Barrett, S. J. Brodsky, G. W. Erickson, and M. H. Goldhaber, Phys. Rev. 166, 1589 (1968).

[9]It has been pointed out to one of us (G.R.) by W. Greiner (private communication) that significant changes in the populations of the electron states while the muon is in a $4f$ level could negate this argument. However, the relatively short lifetimes of the muon levels makes this seem improbable.

[10]S. J. Brodsky and S. D. Drell, Annu. Rev. Nucl. Sci. 20, 147 (1970).

[11]J. Schwinger, Phys. Rev. 75, 651 (1949).

[12]E. A. Uehling, Phys. Rev. 48, 55 (1935).

[13]R. Glauber, W. Rarita, and P. Schwed, Phys. Rev. 120, 609 (1960).

[14]R. J. McKee, Phys. Rev. 180, 1139 (1969).

[15]B. Fricke, Z. Phys. 218, 495 (1969).

[16]T. Appelquist and S. J. Brodsky, Phys. Rev. A 2, 2293 (1970).

[17]G. W. Erickson and D. R. Yennie, Ann. Phys. 35, 271, 447 (1965).

[18]R. C. Barrett, Phys. Lett. 28B, 93 (1968).

[19]M. Y. Chen, Phys. Rev. C 1, 1167, 1176 (1970).

[20]H. F. Skardhamer, Nucl. Phys. A151, 154 (1970).

[21]R. K. Cole, Jr., Phys. Rev. 177, 164 (1969).

[22]E. E. Salpeter, Phys. Rev. 87, 328 (1952).

[23]H. Grotch and D. R. Yennie, Rev. Mod. Phys. 41, 350 (1969).

[24]J. Rafelski, L. P. Fulcher, and W. Greiner, Phys. Rev. Lett. 27, 958 (1971).

[25]J. L. Gammel, LASL Report No. LA-DC-12224 (unpublished); H. Fearing, to be published.

[26]F. Iachello and A. Lande, Phys. Lett. 35B, 205 (1971).