events are x and $\bar{r}$, respectively, is

$$P(x)=\sum_{n=0}^{N} p(n,x)p(N-n,\bar{r}).$$

$p(k,a)$ is the Poisson probability distribution for k events when the expected number is a. The confidence limit for the expected number x_0 of true events is obtained by integrating $P(x)$ over x from $x=0$ to x_0. The results quoted are for $N=6$ and $\bar{r}=5$.

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[12]The results are quoted as upper limits for obvious reasons. However, the value obtained from Fig. 1 is 0.85 μb/sr^2, using the parabolic correction for noncoplanarity.

Acceleration of Cosmic Rays in Supernova Remnants

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If the electromagnetic wave theory of pulsars is correct, then particle acceleration occurs within the nebulous debris of the supernova explosion. Model calculations show that each event produces about 10^{49} erg in ionic cosmic rays. Particle energies range from 10^9 to 10^{16} eV (with spectral index ~ 2 before corrections for losses), and the chemical composition is mostly that of matter processed through several stages of nuclear burning. Both the electronic and the ionic components of the galactic cosmic rays can be maintained by particles accelerated in supernova remnants.

We know cosmic rays are produced in supernova remnants. The relativistic electrons are seen directly, emitting their characteristic synchrotron radiation. The better observed nebulae typically contain about 10^{48} erg in electrons having energies of 10^7 to 10^{14} eV. The pool of galactic cosmic-ray electrons can be maintained by the electronic content of dissolving supernova remnants (see below). Furthermore, one can show[1] that both the very high- and the relatively low-energy electrons must have been created somewhere within the nebulae; they cannot be survivors of the original explosion.

There is no direct evidence for relativistic ions in supernova remnants, but the indirect arguments are strong. First, whatever process accelerates electrons is likely to accelerate the associated ions. Second, supernova explosions can provide the requisite energy (see below). Finally, and most significantly, both the chemical abundances found in galactic cosmic rays, and the abundances inferred at the source, are considerably different from that seen elsewhere. Relative to the solar system, cosmic rays are progressively richer with larger atomic number; the C-N-O group of primaries at the source is overabundant by 10^1 and the iron group by 10^2 relative to hydrogen and helium.[2] It is not easy to see how material originally on ordinary stars or in interstellar space could be simultaneously accelerated and sorted into the observed cosmic-ray abundance distribution. Supernova remnants, however, are thought to contain much gas which has been processed in the stellar interior to higher A, Z; the filaments in the Crab nebula are known, for example, to have a ratio of helium to hydrogen higher than that found in galactic nebulae.[3]

The idea that supernovea contribute significantly to the cosmic-ray backgound is, of course, not new.[4,5] However, a new element was added with the discovery of pulsars, and the initial steps taken towards understanding them. It now appears that many supernovae leave, as stellar remnants, rotating magnetic neutron stars capable of emitting electromagnetic energy[6,7] and very high-energy particles.[7-10] Two of the outstanding problems in this subject—the origin of the electromagnetic field and that of the very high-energy short-lived particles in the Crab nebula—have been plausibly solved by electromagnetic theories.[6-10] According to these theories, between 10^{50} and $10^{52.5}$ erg is available per supernova event in the form of rotational energy. Much of this energy is channeled via strong electromagnetic fields into relativistic particle motion, the maximum energies attainable being $10^{18}-10^{23}$ eV, depending on details of the adopted theory.

These mechanisms cannot produce the moderately relativistic particles which comprise the bulk of the cosmic rays. In our earlier work[8,9] we considered single particles injected by an unspecified process from the neutron star into the very strong electromagnetic-wave field thought to surround the pulsar and found that, if the pulsar effective surface field is about 10^{12} G, particle energies less than 10 GeV can only be reached when the period has increased to about $10^{1.5}$ sec and little rotational energy remains to be disposed of. The other suggested mechanisms are similarly limited. Furthermore, the composition of material removed from the surface of neutron stars, while entirely conjectural, is most unlikely to have the observed cosmic-ray abundances.

However, there is another region where acceleration can occur. The wave energy is probably not exhaused in accelerating particles from the vicinity of the neutron star.[11] When these strong waves reach matter at the nebular filaments, they can still accelerate particles to relativistic energies. We have described elsewhere[12] the motion of charged particles injected into wave fields starting from various initial conditions; here we apply that work to the problem of cosmic-ray origin in supernova remnants.

Consider a pulsar with effective surface field B_s rotating with angular velocity Ω. In the surrounding magnetic dipole wave we characterize the rms electric field amplitude $E=B$ by the parameter $\nu\equiv eB/mc\Omega$. Then, at a distance r from the pulsar,

$$\nu=\nu_0(\lambda/r)(1-\sin^2\theta\,\sin^2\chi)^{1/2}, \tag{1}$$

$$\nu_0\equiv\frac{e}{mc}\langle B_s\rangle r_c^{\,3}c^{-3}\Omega^2=\begin{pmatrix}4.15\times10^{-9}\\ 2.26\times10^{-12}\end{pmatrix}\sqrt{L}\quad\begin{matrix}\text{(electrons)}\\ \text{(protons)}\end{matrix}. \tag{2}$$

Here $\langle B_s\rangle$ is defined by $m_\perp\equiv\langle B_s\rangle r_s^{\,3}$, where $m_\perp$ is the component of the magnetic dipole perpendicular to $\vec{\Omega}$; r_s is the stellar radius; e and m, are the charge and mass of the particle, respectively; θ, is the polar angle at the point of particle injection, $\chi\equiv\Omega t-r/\lambda$, is the phase of the wave at injection; and L is the magnetic dipole luminosity of the pulsar in ergs per second.

Let a particle injected at rest at r reach the relativistic energy γ after accelerating to a radius $r+\Delta r$. There are two limiting cases: (1) If $r/\lambda\lesssim\nu_0^{\,2/3}$, then the particle is "phase locked" and the energy at $r+\Delta r$ is

$$\gamma=\gamma_1\equiv(\tfrac{3}{2}\nu_0)^{2/3}(1-\sin^2\theta\,\sin^2\chi)^{1/3}[\Delta r/(r+\Delta r)]^{2/3},\quad r/\lambda<\nu_0^{\,2/3}. \tag{3}$$

For particles starting near the neutron star,[8] Eq. (3) applies. For case (2), $r/\lambda>\nu_0^{\,2/3}$, $\Delta\chi$ is large even for small $\Delta r/r$, and γ is[13]

$$\gamma=\gamma_2\equiv\tfrac{3}{4}\nu_0^{\,2}(\lambda^2/r^2)1+\tfrac{1}{2}\cos^2\theta-\sin^2\theta\,\cos^2\chi),\quad \Delta r/\lambda>\nu_0^{\,2}\lambda^2/r^2,\quad r/\lambda>\nu_0^{\,2/3}. \tag{4}$$

If $\Delta r/\lambda<\nu_0^{\,2}\lambda^2/r^2$, case (1), γ_1, applies. The expressions for γ_1 and γ_2 overlap at $r/\lambda=6^{1/6}\nu_0^{\,2/3}/2$ so, with neglect of the weak phase dependence, for large Δr,

$$\gamma=\min[(\tfrac{3}{2}\nu_0)^{2/3},\,3\nu_0^{\,2}\lambda^2/4r^2]\ \text{if}\ \Delta r/r>\min[\nu_0^{\,2}\lambda^3/r^3,\,1]. \tag{5}$$

Equation (5) involves two approximations: (a) Radiative reactions have been neglected, which is valid for particles starting from rest since $E=B$, and the radiative reaction is reduced to a negligible value. (b) If $r/\lambda<\nu_0^{\,1/3}$, the near field should be included and $E\neq B$; if one neglects radiative reactions, one finds that γ lies in the range $(r/\lambda)^2<\gamma<\nu\lambda/r$, depending on the injection phase. For $r/\lambda>\nu_0^{\,1/3}$, the case considered in this paper, Eq. (5) is valid.

We now calculate the spectra from a single event. Previous treatments have assumed injection to be from the pulsar magnetosphere, and the energy of the accelerated particles was always found to be very large. Now consider particles injected at $r\gg\lambda$, the source of injection being the expanding cloud of debris from the supernova comprising the observed nebular filaments and interfilamentary gas. The motion of this remnant is initially given by the supernova explosion itself and subsequently controlled by the pressure of the pulsar radiation.[1] It sweeps interstellar matter and magnetic field away, forming a cavity filled with radiation, accelerated particles, and perhaps filamentary debris; the radius R of the nebular remnant as a function of time is determined by direct integration.[1] This function $R(t)$ then represents the injection position r of Eqs. (3)–(5). This fixes $\gamma(t)$ as a function of injection time. Note that acceleration of the two types of particles is different since the mass enters in the expression for ν. At any time t, an equal number of protons and electrons $\mathfrak{N}(t)dt$ are injected and undergo acceleration [cf. Eq. (5)] by the strong waves that fill the cavity. Propagation is assured according to the criterion derived by Max and Perkins[14] if $n_e(r) < 10^{10} R^{-1/2}$, a limit easily satisfied when the material is predominantly in filaments. We further assume that a large fraction of the pulsar luminosity is converted into relativistic particle motions. Thus

$$\mathfrak{N}(t)(\gamma_e m_e c^2 + \gamma_i m_i c^2) = L, \tag{6}$$

where e and i refer to electrons and protons, respectively. From $\mathfrak{N}(t)$ and $\gamma(t)$ we derive the creation spectrum $N(\gamma)$,

$$N_{i,e}(\gamma_{i,e})\, d\gamma_{i,e} = \gamma(t)\, dt. \tag{7}$$

We have evaluated the cosmic-ray spectra in this approximation for the case of pulsars with initial rotations $\Omega_i = 10^{3.3}$, $10^{3.7}$, and 10^5 sec^{-1}; $\langle B_s \rangle = 2.6 \times 10^{12}$ G; and initial energy in the cavity of 10^{50} erg. Figure 1 displays $\epsilon \equiv \gamma(t) m c^2$, $\Omega(t)$, and $R(t)$ and the derived spectra for $\Omega_i = 10^{3.7}$. The initial portion corresponds to the phase-locked case $\gamma = \gamma_1$. Since ν_0 is greater for electrons, the phase-locked condition lasts longer for them than for protons. The proton spectrum is determined as follows. For a very short interval ($t \lesssim 10^{5.5}$, t in seconds always), protons injected in the nebula are phase locked ($\gamma = \gamma_1$), and receive about the same energy, $10^{14.5}$–$10^{16.0}$ eV in these integrations, as they acquired when injected from the pulsar magnetosphere and described earlier.[8] Only during this period do protons receive more energy than electrons. Then, for the interval $10^{5.5} < t < 10^{7.5}$, the electrons remain phase locked and so are produced at a roughly constant rate. Now $R \propto t^{-1}$ to $t^{-3/2}$, so γ goes as t^{-2} to t^{-3} and the spectral index is $-\frac{3}{2}$ to $-\frac{4}{3}$. Accelerated protons receive energies in the range $10^{11.5}$ to $10^{15.0}$ eV during this phase. For the interval $10^{7.5} \lesssim t \lesssim 10^{8.5}$, electrons receive energy $\gamma = \gamma_2$. Since the pulsar luminosity remains high, $R \propto t^{3/2}$ and, for both species, $\gamma \propto R^{-2} \propto t^{-3}$. The spectral index approaches $-\frac{7}{3}$ during this interval which covers the time when most of the neutron star's rotational kinetic energy is drained; protons are accelerated to energies of approximately $10^{9.0}$ to $10^{11.5}$ eV. After $t \gtrsim 10^{8.5}$, the pulsar kinetic energy has been reduced to about $10^{51.5}$ erg, and cosmic-ray production in the nebula is becoming less important.

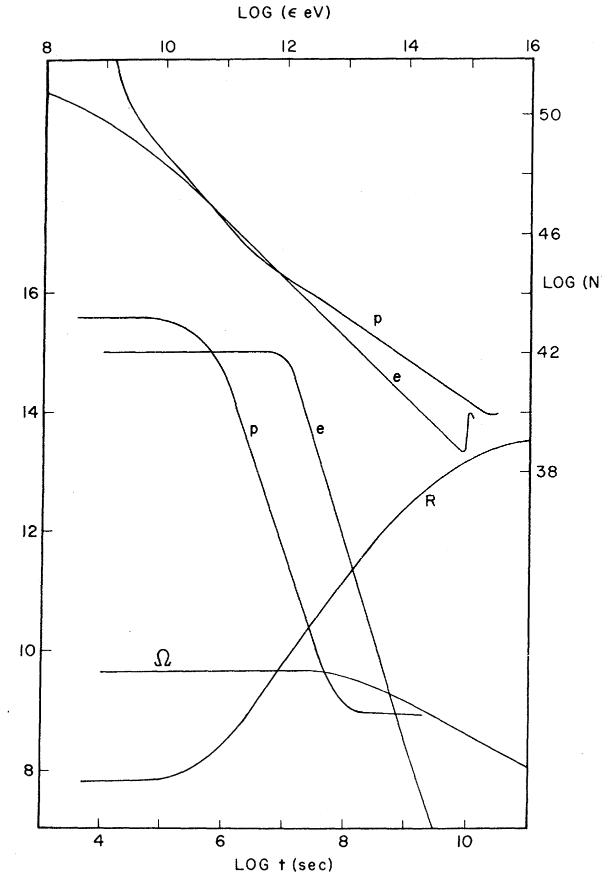

FIG. 1. Evolution of supernova remnants, lower four curves. Left-hand and lower scales give $\log\epsilon_e(t)$, $\log\epsilon_i(t)$ (ϵ in eV), $\log R(t) - 6$, and $\log\Omega(t) - 6$ as functions of $\log t$; cgs units. Resultant spectra, upper two curves using right-hand and lower scales; $N d\gamma$ is the number of particles created in interval $d\gamma$.

The total number of relativistic ions emitted during the event is of order 10^{51-52} with a total energy of $10^{48-49.5}$ erg, depending on initial conditions.

We now sum over supernova events. The calculations of injection spectra given above are irrelevant for the electrons which suffer severe synchroton losses. Instead we note that the observed energy in electrons ($\epsilon > 10^9$ eV) is about 10^{48} erg for nebulae of age about 10^{10} sec.[5] The spectral index of the observed radio synchrotron emission is typically about 0.7, corresponding to a particle spectral index of 2.4. The rate of disruption of such nebulae is the same as the birth rate of pulsars,[15] $Q = 5\times10^{11}$ parsec^{-2} yr^{-1}, giving an energy release rate of $10^{-27.5}$ erg cm^{-3}, where the effective thickness of the layer containing supernova remnants has been taken to be $W = 200$ parsec. We can compare this with the input needed to maintain the electronic component of the cosmic rays. The average galactic density in relativistic electrons determined from the background synchrotron emission (field $B_{is} = 10^{-5.5}$ G assumed) has a spectral index of 2.6 and an energy density[4] ($\epsilon > 10^{9}$ eV) of $10^{-13.5}$ erg cm^{-3}. Estimating the average electron lifetime at 10^{14} sec, we see that an injection rate of $10^{-27.5}$ erg cm^{-3} sec^{-1} is required—in excellent agreement with the amount of energy released by supernova remnants.

We now calculate the energy density of ionic cosmic rays. Let the cosmic-ray energy released into the interstellar medium be $E_{50}\times10^{50}$ erg/event. The energy input rate is then $10^{-13.0}QE_{50}/W$ or $10^{-25.6}E_{50}$ erg cm^{-3} sec^{-1}. But the observed local density of $10^{-12.0}$ erg cm^{-3} in relativistic particles having an estimated lifetime of about $10^{14.0}$ sec requires an input of $10^{-26.0}$ erg cm^{-3} sec^{-1}. Thus the pool of galactic cosmic rays can be maintained if $E_{50} = 0.5$, which is within the range of values calculated in this paper.

Under the assumption that any galaxy is a typical producer of cosmic rays, one can perform calculations giving the total production in the metagalaxy. Production was doubtless higher in the past; we assume an evolution proportional to $(1+z)^6$ to an initial epoch of $z \sim 2.3$. These numbers are indicated if quasars are indeed connected with "supernova-rich" phases in galaxies, but probably represent a crude approach to the truth in any case. The result is that the total average density at the present epoch is $u = 10^{-14.7}\times E_{50}$ erg cm^{-3} so that only if $E_{50} \sim 10^{2.7}$ will the universe be filled with cosmic rays at the density observed in the galaxy. This seems unlikely, but it is probably that some small fraction of the observed cosmic rays are metagalactic in origin and thus very old.

Several corrections to the zeroth-order solution to the problem given above must be mentioned: plasma effects, adiabatic decompression, synchrotron or nonlinear inverse Compton radiation,[12] and spallation losses. Except for synchrotron losses, which do significantly degrade the electrons, none of these effects (to be discussed in detail in a subsequent publication) alter our general conclusions.

In summary, we find that rotating neutron stars can produce the bulk of the galactic cosmic rays if we accept the electromagnetic theory of pulsars. Specifically, the simple integrations presented here show that particles accelerated within the supernova remnants have approximately the required total energy, energy distribution, and chemical composition.

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