[12]R. M. Sternheimer, Phys. Rev. 84, 244 (1951), and 86, 316 (1952).
[13]W. J. O'Sullivan, W. A. Robinson, and W. W. Simmons, Bull. Amer. Phys. Soc. 5, 413 (1960); see also W. W. Simmons and C. P. Slichter, Phys. Rev. 121, 1580 (1961).
[14]P. Raghavan and R. S. Raghavan, Phys. Rev. Lett. 27, 724 (1971).

Generalized Scaling Laws for the Electroproduction of Hadrons*

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The light-cone structure suggested by the quark model is used to propose a generalized scaling law in electroproduction with detection of one final hadron. At fixed momentum transfer between initial and final hadrons, the cross section scales in two variables: the usual Bjorken variable and the fractional longitudinal momentum of the final hadron.

The scaling laws proposed by Bjorken[1] for the inelastic form factors W_1 and νW_2 are consistent with all the experimental information available at present.[2] If the scaling laws are in fact correct in the limit $q^2 \to -\infty$, the electromagnetic current commutator has a simple structure near the light cone. Namely, the leading light-cone singularity of the commutator has the same form as it does in models where the current is constructed from products of free charged fields. This has been pointed out recently by many authors.[3] In the present Letter, we show that the light-cone structure suggested by the quark model[4] leads to interesting generalized scaling laws in electroproduction when a final hadron is detected. We first present the results, then outline the derivation. A more extensive discussion will be presented elsewhere.

Consider the process $e + X \to e' + X' + \mathrm{MM}$, where X and X' stand for hadrons, with a fixed momentum transfer t between the initial and final hadrons (see Fig. 1). We define the cross sections for the initial hadron to absorb a transverse or longitudinal virtual photon and produce the detected hadron plus missing mass by

$$\frac{1}{4\pi\alpha}\frac{2p\cdot q}{M}\left[E'\frac{d^3\sigma_{T,L}}{d^3p'}\right] = F_{T,L}(q^2, \varphi, x_{\mathrm{B}}, x_{\mathrm{F}}, t), \qquad (1)$$

where M is the target mass, φ is the azimuthal angle between the virtual photon's polarization vector and the scattering plane, and p, q, and p' are the four-momenta of the initial hadron, virtual photon, and detected final hadron, respectively. The variables are given by $x_{\mathrm{B}} = -q^2/2p\cdot q$, $x_{\mathrm{F}} = 2p'\cdot q/s$, $t = (p'-p)^2$, and $s = (p+q)^2$. An average over initial and a sum over final hadron spins is understood. Then we have for the generalized scaling law

$$\lim_{\substack{-q^2,\, s\to\infty \\ x_{\mathrm{B}},\, x_{\mathrm{F}},\, t \text{ fixed}}} F_T(q^2, \varphi, x_{\mathrm{B}}, x_{\mathrm{F}}, t) = \tilde{F}_T(x_{\mathrm{B}}, x_{\mathrm{F}}, t). \qquad (2)$$

The corresponding limit for F_L vanishes to $O(q^{-2})$. Note that the limiting function no longer depends on φ.

The variable x_{B} is the usual Bjorken scaling variable. The variable x_{F}, first introduced by Feynman,[5] is the fractional longitudinal momentum of the detected final hadron in the virtual photon–initial hadron c.m. frame, with the direction of the initial hadron taken as positive. Holding t fixed implies that the perpendicular momentum of the produced hadron as well as its energy in the rest frame of the initial hadron are finite, while x_{F} must be positive. This region is referred to in the case of purely hadronic collisions as the target framentation region. Equation (2) then represents a generalization of Feynman scaling[5] or the hypothesis of limiting fragmentation of Benecke *et al.*[6] to deep inelastic scattering. It is

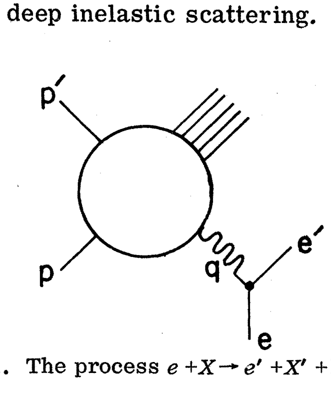

FIG. 1. The process $e + X \to e' + X' + \mathrm{MM}$.

equivalent to the statement that $(1/\sigma_T)(E'd^3\sigma_T/d^3p')$ approaches a limit in the target fragmentation region of deep inelastic scattering, where σ_T is the total cross section for absorption of transverse virtual photons.

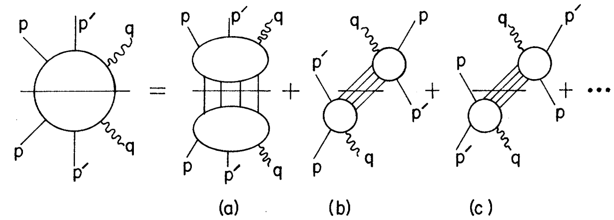

FIG. 2. The three-body absorptive part. (a) Connected part, (b) semiconnected part giving the desired cross section, (c) remaining semiconnected parts.

Previous work on deep inelastic scattering with detection of a final hadron has been done in the field-theory parton model.[7] There a much different kinematic region is considered in which s, q^2, $p'\cdot q$, and t all become infinite with x_B, $p'\cdot q/p\cdot q$, and $t/p\cdot q$ fixed. A scaling law is found to hold in this region which differs from Eq. (2) by having an additional power of $p\cdot q$, i.e., it is $(p\cdot q)F_T$ which approaches a limit. To arrive at Eq. (2) we construct the current from quark fields, $J_\mu{}^a(x) = :\bar\psi(x)\gamma_\mu \times\frac{1}{2}\lambda a\psi(x):$, and extract the leading singular behavior in the commutator near the light cone:

$$[J_\mu{}^a(x), J_\nu{}^b(0)] = d_{abc}s_{\mu\nu\alpha\beta}[:\bar\psi(x)\gamma^\alpha\tfrac{1}{2}\lambda c\psi(0): - :\bar\psi(0)\gamma^\alpha\tfrac{1}{2}\lambda c\psi(x):]\partial^\beta\delta(x^2)\epsilon(x_0)/4\pi + \cdots, \tag{3}$$

where $s_{\mu\nu\alpha\beta} = g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha} - g_{\mu\nu}g_{\alpha\beta}$. The omitted terms are either less singular as $x^2\to 0$ or cannot contribute to the process in question because they either involve axial currents or are antisymmetric in the SU(3) labels a and b. The bilocal operators in Eq. (3) are to be interpreted as formal sums of their Taylor expansions, e.g.,

$$:\bar\psi(x)\gamma_\alpha\tfrac{1}{2}\lambda c\psi(0): = \sum_{n=0}^{\infty}\frac{1}{n!}x^{\mu_1}\cdots x^{\mu_n}O_{\mu_1\cdots\mu_n,\alpha}{}^c(0). \tag{4}$$

The $O_{\mu_1\cdots\mu_n,\alpha}{}^c$ are a set of local field operators which include the hadronic currents as well as an infinite chain of operators of increasing spin. The matrix elements of the $O_{\mu_1\cdots\mu_n,\alpha}$ depend on the details of strong interaction dynamics and are therefore difficult or impossible to calculate. However, the scaling laws depend only on the form of the light-cone expansion and not on the numerical values of the various matrix elements. It should be noted that the right-hand side of Eq. (3) is not automatically conserved in general. The slight modifications necessary to insure conservation have been discussed by Gross and Treiman.[8]

To relate the commutator to the cross section for the process $q+p\to p'+\mathrm{MM}$, we follow Mueller[9] and first consider the total absorptive part of the forward process $q+p+p'\to q+p+p'$. We introduce quantities $\mathfrak{F}_{T,L}$ defined by

$$\mathfrak{F}_{T,L}(q^2,\varphi,x_B,x_F,t) = (2\pi)^3EE'\epsilon_{T,L}{}^{*\mu}\int d^4x\, e^{iq\cdot x}\langle \mathrm{in}\, pp'|[J_\mu{}^Q(x), J_\nu{}^Q(0)]|pp'\,\mathrm{in}\rangle\epsilon_{T,L}{}^\nu, \tag{5}$$

where Q denotes the usual combination of SU(3) indices appropriate for the electromagnetic current and we sum and average over hadron spins as usual. Completely disconnected parts are subtracted off. The cross sections of interest are contained in the semiconnected parts of $\mathfrak{F}_{T,L}$ [see Fig. 2(b)],

$$\mathfrak{F}_{T,L}(q^2,\varphi,x_B,x_F,t)_{\mathrm{sc}} = F_{T,L}(q^2,\varphi,x_B,x_F,t). \tag{6}$$

We also define the matrix elements of the bilocal operators in Eq. (3):

$$EE'\langle \mathrm{in}\, pp'|[:\bar\psi(x)\gamma_\mu\tfrac{1}{2}\lambda c\psi(0): - :\bar\psi(0)\gamma_\mu\tfrac{1}{2}\lambda c\psi(x):]|pp'\,\mathrm{in}\rangle = A_1{}^c(p\cdot x, p'\cdot x, t, x^2)p_\mu + A_2{}^c(p\cdot x, p'\cdot x, t, x^2)p_\mu'. \tag{7}$$

Here we again subtract off completely disconnected parts. It is implicit in the light-cone expansion of Eq. (3) that the bilocal operators contain no singular behvior as $x^2\to 0$. Therefore to leading order as $q^2\to-\infty$, we may set $x^2=0$ in $A_{1,2}{}^c$. The procedure of Gross and Treiman to insure current conservation only adds terms proportional to q_μ or q_ν in the matrix element of Eq. (5), and so if we work in the usual gauge satisfying $\epsilon\cdot q=0$, these extra terms may be omitted. Substituting the light-cone expansion of Eq. (3) into Eq. (5) we then easily derive the scaling law

$$\lim_{\substack{-q^2,s\to\infty\\ x_B,x_F,t\ \mathrm{fixed}}}[\mathfrak{F}_T(q^2,\varphi,x_B,x_F,t)] = d_{QQc}\pi^2\int_{-\infty}^{+\infty}d\xi\, d\xi'[\tilde A_1{}^c(\xi,\xi',t) + x_F(1-x_B)\tilde A_2{}^c(\xi,\xi',t)]\delta(-x_B+\xi+x_F(1-x_B)\xi'), \tag{8}$$

where we have introduced the Fourier transforms[10] of $\tilde{A}_{1,2}{}^c$,

$$A_{1,2}{}^c(p\cdot x, p'\cdot x, t, 0) = (2\pi)^{-2}\int_{-\infty}^{+\infty} d\xi\, d\xi'\, e^{i\xi p\cdot x} e^{i\xi' p'\cdot x} A_{1,2}(\xi, \xi', t). \tag{9}$$

The longitudinal quantity $\mathfrak{F}_L$ vanishes to $O(q^{-2})$ in this limit.

The generalized scaling law Eq. (2) follows from Eq. (8) if the leading light-cone singularity of the full matrix element is also present in the relevant semiconnected part. While a rigorous proof of this statement has not yet been constructed, the following arguments suggest that Eq. (2) is correct and therefore that the leading light-cone singularity is present in semiconnected parts.

(a) In perturbation theory with nucleons coupled to neutral vector gluons, Eq. (2) holds for each particle in the theory up to factors of $\log|q^2|$. For W_1 and νW_2, ignoring the logarithmic factors of perturbation theory leads to the Bjorken scaling laws. Applying the same procedure in the present context then leads to Eq. (2) as the generalized scaling law.

(b) From conservation of energy and logitudinal momentum, we have the following exact sum rule:

$$\lim_{-q^2\to\infty} \int_0^1 \int \sum F_T(q^2, \varphi, x_{\rm B}, x_{\rm F}, t)\, d^2p_\perp dx_{\rm F} = 2\pi W_1(x_{\rm B}), \tag{10}$$

where the sum is carried out over the distinct stable hadrons which can be produced. For $x_{\rm F} > 0$,

$$t = M'^2 + M^2(1 - x_{\rm F} + x_{\rm F}x_{\rm B}) - (M'^2 + p_\perp^2)/x_{\rm F}(1 - x_{\rm B}) + O(1/s).$$

If perpendicular momenta are limited in electroproduction as they are in hadronic collisions, the dominant contribution to the sum rule will come from finite $p_\perp$ and therefore finite t. Equation (2) is quite consistent with the sum rule and allows it to be satisfied in the simplest possible way, namely, the limit can be taken inside the integral and the limit of the integrand exists for each distinct stable hadron.

(c) Near the lower end of the missing-mass spectrum, $M_x^2/s = 1 - x_{\rm F} \to 0$. In this kinematic region, since t is finite while $s\to\infty$, exchange mechanisms are expected to control the cross section. In Fig. 3 we consider the case where the initial and final hadrons are a proton and a neutron, respectively, so that an important contribution to the cross section will come from π exchange. The presence of the leading light-cone singularity of Eq. (3) in the single π matrix element will guarantee its presence in the semiconnected part and hence lead to Eq. (2) for the single π exchange contribution to the cross section. The generalization to the case where the exchange is Reggeized can be made by realizing that this argument applies to every particle along the Regge trajectory. This type of argument lends support to Eq. (2) in the kinematic region near $x_{\rm F} = 1$.

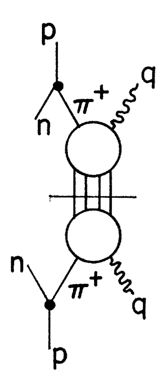

FIG. 3. π-exchange contribution for the case where initial and final hadrons are a proton and neutron, respectively.

To conclude, we propose Eq. (2) as a generalized scaling law in electroproduction with detection of a final hadron. Confirmation of Eq. (2) for the case of nucleons, π's, K's, etc., would show that the leading light-cone singularity is present in semiconnected parts. If, in addition, the transverse cross section dominates in each case and the SU(3) structure of Eq. (8) is confirmed, the quark-model light-cone algebra would be strongly supported.

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[1]J. D. Bjorken, Phys. Rev. 179, 1547 (1969).

[2]The latest results are contained in E. D. Bloom *et al.*, SLAC Report No. SLAC-PUB-796, 1970 (to be published). For earlier work see E. D. Bloom *et al.*, Phys. Rev. Lett. 23, 930 (1969); M. Breidenbach *et al.*, Phys. Rev. Lett. 23, 935 (1969); W. Albrecht *et al.*, Nucl. Phys. B13, 1 (1969).

[3]The importance of the light cone in electroproduction at large q^2 is stressed from various points of view by

B. L. Ioffe, Phys. Lett. 30B, 123 (1969); R. A. Brandt, Phys. Rev. Lett. 22, 1149 (1969), and 23, 1260 (1969), and Phys. Rev. D 1, 2808 (1970); R. Jackiw, R. Van Royen, and G. West, Phys. Rev. D 2, 77 (1970). The method of light-cone expansions used here originates with Y. Frishman, Phys. Rev. Lett. 25, 966 (1970); R. A. Brandt and G. Preparata, Nucl. Phys. B27, 541 (1971). See also H. Fritzsch and M. Gell-Mann, California Institute of Technology Report No. CALT-68-297, 1971 (unpublished).

[4]Fritzsch and Gell-Mann, Ref. 3; J. Cornwall and R. Jackiw, Phys. Rev. D 4, 367 (1971).

[5]R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969).

[6]J. Benecke, T. T. Chou, C. N. Yang, and E. Yen, Phys. Rev. 188, 2159 (1969).

[7]S. Drell and T. M. Yan, Phys. Rev. Lett. 24, 855 (1970); P. V. Landshoff and J. C. Polkinghorne, Nucl. Phys. B33, 221 (1971).

[8]D. Gross and S. Treiman, Phys. Rev. D 4, 1059 (1971).

[9]A. H. Mueller, Phys. Rev. D 2, 2963 (1970).

[10]The limits on the ξ, ξ' integrations in Eqs. (8) and (9) are formal. The actual supports of $\tilde{A}_{1,2}{}^c$ are determined by the mass spectrum.

Multiplicity Growth and Leading Particle Energy Loss*

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The relation between the multiplicity growth and the energy-loss spectrum of an incident particle, as implied by Poisson emission with a classical spectrum, appears to be obeyed experimentally in pp collisions.

The Brookhaven National Laboratory experiments[1] on the behavior of the final proton in highly inelastic pp collisions at a few tens of GeV revealed a spectrum of puzzling simplicity: To a first approximation the emerging proton can have any value of longitudinal momentum with equal probability, as long as that momentum is large. That is, the cross section $d\sigma/dP_L$ in $p+p \to p+$anything for fast final protons is, aside from the narrow elastic and quasielastic structures at the maximum momentum, approximately independent of P_L. (It is worth noting that this statement, or that of any power-law behavior $d\sigma/dP_L \sim P^a$, holds in all frames reached by Lorentz transformations leaving the proton moving relativistically in the same direction.)

Subsequent experiments[2] have confirmed these results. In Fig. 1 we show the CERN results at 19 GeV/c. Since that time, some preliminary understanding of the situation at very high energy, beyond the region where resonances and quasi two-body reactions dominate the scattering channels, has been achieved.

The average multiplicity[3] appears to increase logarithmically at very large energy so that, ignoring a constant term,

$$\bar{N} = C \ln S. \tag{1}$$

The constant C is approximately 1 if we multiply the experimental charged multiplicity of 0.7 ± 0.1 by $\frac{3}{2}$ to attempt to account for undetected π^0's. The multiparticle production spectra may be dominated by "soft pions,"[4] whose probability distribution peaks for very low energy. These features are reminiscent of a simple "bremsstrahlung picture," an idea entertained, with just these points in mind, at least as far back as 1942,[5] and elaborated recently in the context of "scaling" by Feynman.[6] In "bremsstrahlung" the abrupt acceleration of the charge[7] leads to a spectrum of radiated photons containing on the average equal amounts of energy per unit frequency interval,

$$d\bar{E}(\omega)/d\omega = C, \tag{2}$$

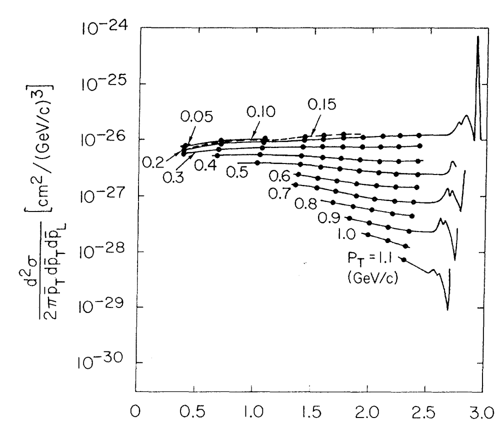

FIG. 1. Proton energy-loss spectrum observed for various small transverse momenta at 19 GeV.