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[10]An exact formulation of the renormalization group is mentioned in Ref. 1.

# Acoustoelectric Periodic Structure

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The interaction of a conducting pseudoperiodic grating with volume acoustic modes of a piezoelectric crystal has been tentatively explained in terms of an acoustoelectric periodic structure exhibiting exponential and periodic couplings.

The physical appearance of the interaction between a pseudoperiodic metallic grating, deposited along a pure-mode axis of a piezoelectric crystal, and volume acoustic modes was documented in a recent experimental investigation.[1] The present note suggests that this kind of interaction can be regarded as an acoustoelectric periodic structure[2] exhibiting exponential and periodic couplings.

The sample used in the present study consisted of two 25-pair pseudoperiodic gratings with a spatial period of $2\pi/k_T = 32\times10^{-6}$ m (where $k_T$ is the grating wave number) deposited $1.5\times10^{-3}$ m apart along the $Z$ axis of a $Y$-cut oriented $LiNbO_3$ crystal. For further details the reader is referred to Ref. 1. The transmission spectrum of the sample is shown in Fig. 1(a). The spectrum can be related to the quasistatic dispersion relation for the acoustic wave propagation along the $Z$ axis of a semi-infinite crystal, which can be written as

$$D(\omega,k_z)\equiv D_{\mathrm{R}}(\omega,k_z)D_s(\omega,k_z)D_l(\omega,k_z)\equiv(\omega^2-v_{\mathrm{R}}^2k_z^2)(\omega^2-v_s^2k_z^2)^2(\omega^2-v_l^2k_z^2)=0, \tag{1}$$

where $\omega$ is the angular frequency, $k_z$ is the longitudinal wave number, $v_{\mathrm{R}}$ is the velocity of the Rayleigh surface wave, $v_s$ is the velocity of a degenerate set of pure-shear volume modes, and $v_l$ is the velocity of the volume longitudinal piezoelectric mode. The periodic perturbation introduced in the crystal by conducting gratings results in the periodicity of the dispersion equation which can be written in a coupled form as

$$D^c(\omega,k_z)\equiv D_{\mathrm{R}}^c(\omega,k_z+Lk_T)D_s^c(\omega,k_z+Mk_T)D_l^c(\omega,k_z+Nk_T)=0, \tag{2}$$

where $L$, $M$, and $N$ are integers. This periodicity accounts for couplings between each of the acoustic modes as shown in Fig. 1(b), which was obtained by a graphical procedure applicable to periodic structures.[2] In Fig. 1(b) the Brillouin zone along the $Z$ axis is shown for the three acoustic branches of which one is degenerate. The remarkable feature of the coupled dispersion equation is that forbidden bands give rise to enhanced couplings. Each splitting is due to an interaction of two branches of a specific acoustic mode yielding for real $\omega$ two complex wave numbers, one of which is responsible for a growing mode. Gaps containing two splittings at the zone edge will generate two growing modes propagating in the opposite directions. The modes will grow exponentially in space along the grating structure ($Z$ direction) on account of the energy supplied to the grating, which is termed exponential coupling. Two more points are of interest here. First, the coupling due to splittings located at the center of the zone, $|k|\approx 0$, are rather small, since the growth parameter $|\mathrm{Im}(k)|d \ll 1$, where $d$ is the length of the grating, while at the edge of the zone, $|k|\simeq k_T$, the couplings are strong

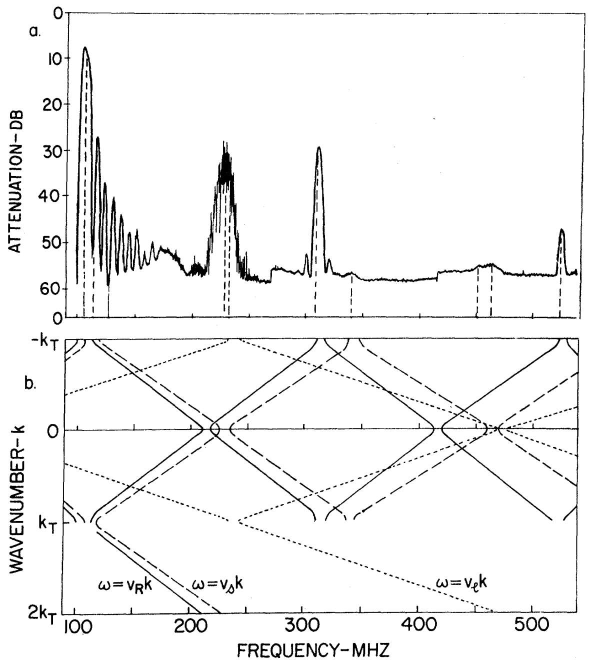


FIG. 1. (a) Transmission spectrum of the piezoelectric sample. The rf power of 20 mW corresponds to the 0-dB level. (b) The coupled dispersion equation of the longitudinal volume mode, nonpiezoelectric shear modes, and Rayleigh surface wave corresponding to velocities $v_l$, $v_s$, and $v_R$, respectively. Only the first Brillouin zone is shown. The grating wave number is $k_T = 1.97 \times 10^5$ m$^{-1}$.

because of a larger value of the growth parameter. For large values of $\omega$ and wave numbers near the zone edge, the couplings are weak because of imperfections in the periodicity of the structure, intrinsic losses, and the finite length of the grating. The second point of interest is the enhanced coupling at $\omega \simeq 2\pi \times 230$ MHz. First, we note that $k_z^s - k_z^l \simeq k_T$, where $k_z^s$ and $k_z^l$ are the wave numbers of the degenerate set of shear modes and the longitudinal piezoelectric mode, respectively. The effect of this relation on Eq. (2) is to translate the branches of $D_s^c$ and $D_l^c$ in respect to one another for $k_T$. In this process some of the splittings of $D_s^c$ and $D_l^c$ will overlap at the zone edge and the four complex solutions will interact, which is termed periodic coupling (with periodicity parameter $k_T$). The complex acoustic wave at $\omega \simeq 2\pi \times 230$ MHz will have all three components of polarization which are due to two shear degenerate modes and a longitudinal mode. The energy of the wave grows exponentially along the length of the grating, as described before, but in an oscillatory manner with a continuous interchange of energy between the longitudinal mode and shear modes. At $\omega \simeq 2\pi \times 460$ MHz the periodic coupling does not take place since the splittings of $D_s^c$ and $D_l^c$ do not overlap, and since one of the modes has always weak exponential coupling by virtue of being located at the center of the zone, $|k| \approx 0$.

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