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## Quadrupole Moment of the 660-keV "Rotational" Level of <sup>117</sup>In

R. S. Raghavan and P. Raghavan

*Physik-Department der Technischen Universität München, 8046 Garching bei München, Germany*

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The quadrupole interaction frequency  $\nu_Q = e^2qQ/h$  of the 660-keV ( $I = \frac{3}{2}^+$ ) level of <sup>117</sup>In has been measured by the method of time-differential perturbed angular correlations in an alloy of In<sub>0.99</sub>Cd<sub>0.01</sub> at 4.2°K. From the known quadrupole interaction of <sup>115</sup>In in the same alloy at 4.2°K and the atomic beam value of  $Q(^{115}\text{In})$ , the quadrupole moment of this level is deduced to be  $|Q| = 0.64(4)$  b, in remarkable agreement with the theoretical value of 0.636 b derived from a description of this level as a rotational state of a  $K = \frac{1}{2}^+$  band.

The nuclear properties of odd indium isotopes have received considerable attention in recent years both experimentally and theoretically.<sup>1</sup> In particular, the excited states of <sup>117</sup>In have revealed very interesting features. The appearance of positive-parity levels as low as 660 keV ( $\frac{3}{2}^+$ ) and 749 keV ( $\frac{1}{2}^+$  or  $\frac{3}{2}^+$ ) connected by an  $E2$  transition which is enhanced to 100 times the single-particle strength prompted Bäcklin, Fogelberg, and Malmskog<sup>2</sup> to suggest that these levels form the beginning of a  $K = \frac{1}{2}^+$  rotational band coexisting with the single-proton-hole states below and levels arising from the coupling of a single proton hole with phonons, above. All available data on these levels<sup>3</sup> support this interpretation. In this framework a definite quadrupole moment (QM) is predicted for the 660-keV level. In this Letter we report the experimental determination of this quantity; the result is in excellent agreement with the predicted value, providing strong support for the rotational nature of this level.

The usual problem in direct measurements of the spectroscopic QM is the lack of reliable estimates of the electric field gradient (EFG) although the quadrupole interaction (QI) frequency itself could be measured accurately. In the present work we have eliminated the need to know the EFG by taking advantage of the following facts: (a) Extensive nuclear quadrupole resonance (NQR) data on <sup>115</sup>In in In(Cd) alloys at 4.2°K exist. NQR data for the alloy rather than for pure In are crucially necessary since it is very difficult in practice to make a carrier-free activity of the <sup>117</sup>Cd parent and introduce it into In metal. On the

other hand, the alloy sources are simple and easily prepared. (b) The QM of <sup>115</sup>In is known from atomic-beam hfs measurements. If the QI frequency of the 660-keV level of <sup>117</sup>In could be measured at 4.2°K in a source of In(Cd) alloy of an identical composition as in the NQR work, the ratio of the QI frequencies yields the ratio of QM's,  $Q(^{117}\text{In})/Q(^{115}\text{In})$ , from which  $Q(^{117}\text{In})$  could be deduced.

We have measured the QI frequency of the level in question by the method of time-differential perturbed angular correlations (TDPAC). The TDPAC function for the present case can be written as<sup>4</sup>

$$W(\theta, t) = 1 + A_2 G_2(t) P_2(\cos\theta), \quad A_4 = 0,$$

where  $\theta$  is the angle between the two  $\gamma$  rays in coincidence,  $A_2$  is the angular correlation coefficient, and  $G_2(t)$  describes the perturbation. For the case of a pure, static, axially symmetric, randomly oriented EFG and for a nuclear spin  $I = \frac{3}{2}$ , the perturbation factor  $G_2(t)$  is given by

$$G_2(t) = \frac{1}{3}(1 + 4 \cos\omega_0 t),$$

where  $\omega_0 = (3e^2qQ)/2I(2I-1)\hbar$  and  $q$  is the EFG at the nuclear site. The time spectrum of the  $\gamma$ - $\gamma$  coincidences connecting this level then shows the exponential decay modulated by  $G_2(t)$ , the amplitude of the modulation being determined by  $A_2$ , the experimental factors which reduce the ideal  $A_2$ , and the angle  $\theta$ . The experiment consists in observing these modulations and extracting  $\omega_0$  therefrom. The QI frequency  $\nu_Q = e^2qQ/h$  is then simply given by the relation  $\nu_Q = \omega_0/\pi$ .

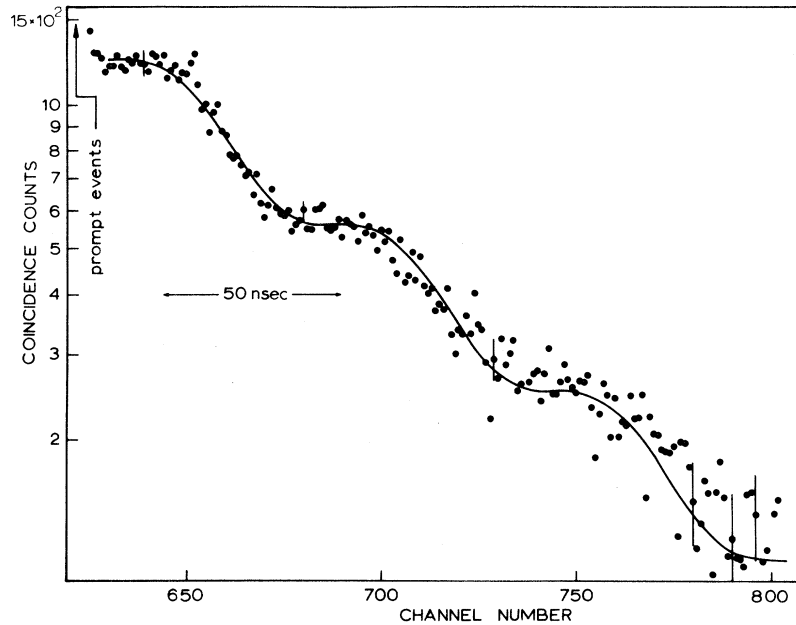


FIG. 1. Decay curve of the 660-keV level in  $^{117}\text{In}$  modulated by the QI in  $\text{In}_{0.99}\text{Cd}_{0.01}$  at  $4.2^\circ\text{K}$ . The solid line is the least-squares fit by the function  $N = N_0 \exp(-\lambda t) [1 + A_2 G_2(t) P_2(\cos \theta)]$ , where  $\theta = 180^\circ$  and  $G_2(t)$  is the theoretical function for an axially symmetric, static, randomly oriented EFG for spin  $I = \frac{3}{2}$ .

The 660-keV level of  $^{117}\text{In}$  is readily amenable to TDPAC work; the  $A_2$  of the 89–345-keV  $\gamma$ - $\gamma$  correlation is large ( $A_2 = -0.36$ ) and the lifetime of the level<sup>5</sup> ( $T_{1/2} = 53.5$  nsec) is easily in the range of practical time spectrometry. The sources of  $^{117}\text{Cd}$  (2.7 h) and  $^{117\text{m}}\text{Cd}$  (3.5 h) were produced by neutron irradiation of 87.2% enriched  $^{116}\text{Cd}$  metal for a few minutes and melted with pure In in vacuum so as to make alloys of 1 at.% Cd in In. Experiments were also carried out with sources in pure Cd metal directly after the irradiation. The 89–345-keV  $\gamma$ - $\gamma$  coincidence time spectra at  $\theta = 180^\circ$  were recorded using two NaI(Tl) detectors with standard electronics and a multichannel analyzer. The system time resolution under experimental conditions was 3.2 nsec. Experiments were done at  $4.2^\circ\text{K}$  as well as at  $295^\circ\text{K}$  for the alloy and at  $295^\circ\text{K}$  for the Cd metal source.

Figure 1 shows the results for the alloy source at  $4.2^\circ\text{K}$ . The solid line is the least-squares fit by the function  $N = N_0 \exp(-\lambda t) W(\theta, t)$  after subtraction of the random coincidences. The excellent fits (see Figs. 1 and 2) obtained with a single frequency characteristic of the QI of an  $I = \frac{3}{2}$  level resolve any doubt regarding the spin of this level. The modulation frequency  $\omega_0$  and the QI frequency  $\nu_Q$  for all the measurements are summarized in Table I.

Thatcher and Hewitt<sup>6</sup> have made NQR measure-

ments of the QI frequencies  $\nu_Q(115)$  of  $^{115}\text{In}$  in In metal containing up to 4.3 at.% concentration of Cd at  $4.2^\circ\text{K}$ . For 1 at.% concentration, a composition used in the present work, they report  $\nu_Q(115) = 43.2(1)$  MHz. For the 660-keV excited state of  $^{117}\text{In}$  in the alloy source at  $4.2^\circ\text{K}$ , we measure (see Table I)  $\nu_Q(117^*) = 32.1(5)$  MHz. Thus the ratio  $Q(117^*)/Q(115)$  is derived to be 0.743(15). The value of  $Q(115)$  is quoted to be 1.16 b in the *Table of Isotopes*.<sup>7</sup> However this is subject to corrections (28%) due to configuration mixing of the electronic states.<sup>8,9</sup> The corrected value  $Q(115) = 0.834$  b,<sup>10</sup> rather than the value 1.16 b, is experimentally supported by recent muonic hfs work on  $^{115}\text{In}$ .<sup>11</sup> After applying a Sternheimer correction<sup>12</sup> (3.2%) one computes  $Q(115) = 0.861(45)$  b, where the error is derived by assuming  $\approx 10\%$  accuracy for the corrections applied. Combining this value with the experimental ratio  $Q(117^*)/Q(115) = 0.743(15)$ , we obtain  $|Q(117^*)| = 0.64(4)$  b. The major part of the error here arises from the error assigned to  $Q(115)$ .

Bäcklin, Fogelberg, and Malmkog<sup>2</sup> propose that the 660-keV level ( $\frac{3}{2}^+$ ) is the first rotational level of a  $K = \frac{1}{2}^+$  band based on the 749-keV level ( $\frac{1}{2}^+$  or  $\frac{3}{2}^+$ ) with a decoupling parameter of  $-2.2$ . The  $B(E2)$  of the 89-keV transition connecting these two levels can be used to calculate the intrinsic QM of this band and they obtain  $Q_0 = 3.18$  b. From the projection formula  $Q = Q_0 [3K^2 - I(I+1)] /$

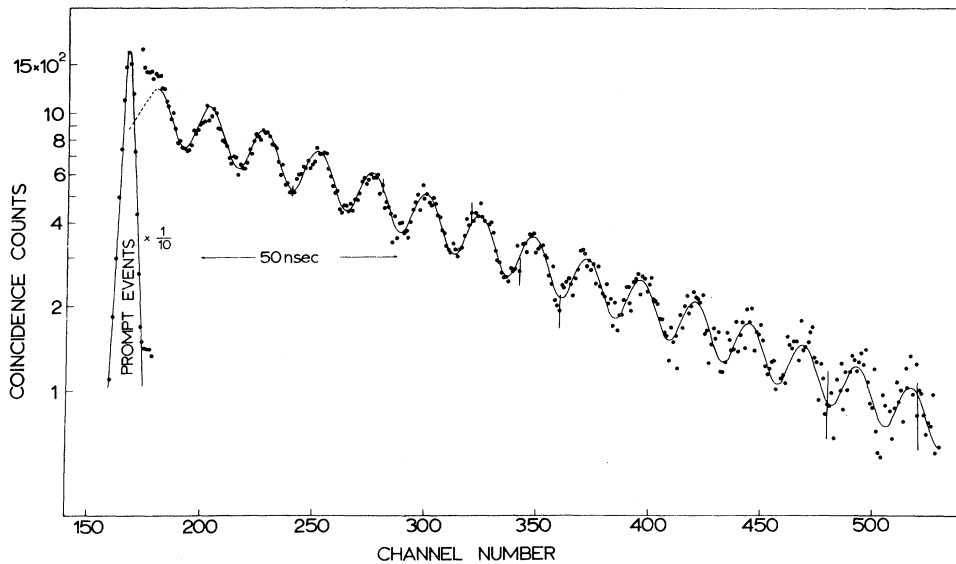


FIG. 2. Same as in Fig. 1 for  $^{117}\text{In}$  in Cd metal at 295°K.

$(I+1)(2I+3)$ , the spectroscopic QM of the  $\frac{3}{2}^+$  level can be computed to be  $|Q_{th}(117^*)| = 0.636$  b. The agreement of this number with the experimental value above is striking and lends considerable credibility to the description of these levels as rotational. Since the  $E1$  transition from the "rotational" 660-keV level to the "spherical"  $p_{1/2}$  state is hindered by a factor of  $5 \times 10^6$ , it becomes interesting to ask to what extent the isomerism of the 660-keV level arises from its deformed shape.

Finally we remark that the QI in the In(Cd) alloy source at 295°K (see Table I) is reduced by 30% as compared to the value at 4.2°K, a behavior similar to that found previously for In metal.<sup>13</sup> The QI of  $^{117}\text{In}$  in Cd metal at 295°K, determined here for the first time (see Fig. 2), is stronger than in the In(Cd) alloy by more than a factor of 6 and may be useful for the measurement of the QM's of other In levels. From the solid-state point of view it is interesting that the EFG in In in Cd is about 50% stronger than that at Cd in Cd.<sup>14</sup> These aspects will be dealt with in more

TABLE I. Summary of results of the QI of the 660-keV level of  $^{117}\text{In}$ .

Host	Temperature (°K)	$\omega_0$ ( $10^6$ rad/sec)	$\nu_Q$ (MHz)
$\text{In}_{0.99}\text{Cd}_{0.01}$	4.2	100.8(15)	32.08(48)
$\text{In}_{0.99}\text{Cd}_{0.01}$	295	68.3(7)	21.74(22)
Cd	295	455.8(14)	145.08(45)

detail in a later paper.

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*Note added in proof.*—As this paper was under publication we were informed of a similar measurement by H. Haas and D. A. Shirley [University of California Radiation Laboratory Report No. UCRL-20426 (unpublished)] who obtain  $Q(117^*) = 0.58(6)$ , in good agreement with our result.

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## Generalized Scaling Laws for the Electroproduction of Hadrons\*

John D. Stack

Physics Department, University of Illinois, Urbana, Illinois 61801

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The light-cone structure suggested by the quark model is used to propose a generalized scaling law in electroproduction with detection of one final hadron. At fixed momentum transfer between initial and final hadrons, the cross section scales in two variables: the usual Bjorken variable and the fractional longitudinal momentum of the final hadron.

The scaling laws proposed by Bjorken<sup>1</sup> for the inelastic form factors  $W_1$  and  $\nu W_2$  are consistent with all the experimental information available at present.<sup>2</sup> If the scaling laws are in fact correct in the limit  $q^2 \rightarrow -\infty$ , the electromagnetic current commutator has a simple structure near the light cone. Namely, the leading light-cone singularity of the commutator has the same form as it does in models where the current is constructed from products of free charged fields. This has been pointed out recently by many authors.<sup>3</sup> In the present Letter, we show that the light-cone structure suggested by the quark model<sup>4</sup> leads to interesting generalized scaling laws in electroproduction when a final hadron is detected. We first present the results, then outline the derivation. A more extensive discussion will be presented elsewhere.

Consider the process  $e + X \rightarrow e' + X' + MM$ , where  $X$  and  $X'$  stand for hadrons, with a fixed momentum transfer  $t$  between the initial and final hadrons (see Fig. 1). We define the cross sections for the initial hadron to absorb a transverse or longitudinal virtual photon and produce the detected hadron plus missing mass by

$$\frac{1}{4\pi\alpha} \frac{2p \cdot q}{M} \left[ E' \frac{d^3\sigma_{T,L}}{d^3p'} \right] = F_{T,L}(q^2, \varphi, x_B, x_F, t), \quad (1)$$

where  $M$  is the target mass,  $\varphi$  is the azimuthal angle between the virtual photon's polarization vector and the scattering plane, and  $p$ ,  $q$ , and  $p'$  are the four-momenta of the initial hadron, virtual photon, and detected final hadron, respectively. The variables are given by  $x_B = -q^2/2p \cdot q$ ,  $x_F = 2p' \cdot q/s$ ,  $t = (p' - p)^2$ , and  $s = (p + q)^2$ . An aver-

age over initial and a sum over final hadron spins is understood. Then we have for the generalized scaling law

$$\lim_{\substack{-q^2, s \rightarrow \infty \\ x_B, x_F, t \text{ fixed}}} F_T(q^2, \varphi, x_B, x_F, t) = \tilde{F}_T(x_B, x_F, t). \quad (2)$$

The corresponding limit for  $F_L$  vanishes to  $O(q^{-2})$ . Note that the limiting function no longer depends on  $\varphi$ .

The variable  $x_B$  is the usual Bjorken scaling variable. The variable  $x_F$ , first introduced by Feynman,<sup>5</sup> is the fractional longitudinal momentum of the detected final hadron in the virtual photon-initial hadron c.m. frame, with the direction of the initial hadron taken as positive. Holding  $t$  fixed implies that the perpendicular momentum of the produced hadron as well as its energy in the rest frame of the initial hadron are finite, while  $x_F$  must be positive. This region is referred to in the case of purely hadronic collisions as the target fragmentation region. Equation (2) then represents a generalization of Feynman scaling<sup>5</sup> or the hypothesis of limiting fragmentation of Benecke *et al.*<sup>6</sup> to deep inelastic scattering. It is

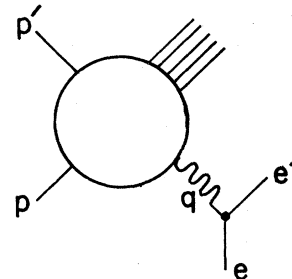


FIG. 1. The process  $e + X \rightarrow e' + X' + MM$ .