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[10] The substitution $l = R_{\rm ion}$ is made mainly to simplify the formal result. The results are not essentially different with a more elaborate estimate for $l(\rho)$.

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Effective Viscosity of Dilute Solutions of Liquid He^3 in He^4 between 20 mK and 1 K

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We have made detailed measurements of the effective viscosity of dilute He^3-He^4 solutions between 20 mK and 1 K, by observation of the damping of torsional oscillations of a quartz crystal. The results are strikingly different, in magnitude and in temperature dependence, from theoretical predictions and from previous preliminary results.

Although the hydrodynamic properties of liquid solutions of He^3 in He^4 are of prime importance both for the theory of He^3 quasiparticle interactions and for the design of heat exchangers for He^3 dilution refrigerators, comparatively little experimental work has been done to measure their viscosity. We have now measured this quantity for solutions with nominal He^3 molar concentrations from 1% to 10% in the temperature range 20 mK to 1 K. The low-temperature results are strikingly different both in temperature dependence and in magnitude from the theoretical predictions of Baym and co-workers[1] based on the Bardeen-Baym-Pines effective interaction, being lower by a factor of about 25 at 20 mK. They are also lower than the lowest-temperature measurements of Webeler and Allen,[2] and the preliminary results of Black, Hall, and Thompson.[3]

To derive the viscosity, we measured the logarithmic decrement of torsional oscillations of a cylindrical quartz crystal immersed in the liquid, a technique used previously by Welber[4] for He^4, by Betts *et al.*[5] for He^3, and by Webeler and Allen[2] for He^3-He^4 solutions. From classical hydrodynamics,

$$\eta_e \rho_n = (M/S)^2 (f/\pi)(\Delta - \Delta_0)^2, \qquad (1)$$

where η_e is an effective viscosity; ρ_n is the normal density; M, S, and f are the mass, surface area, and resonant frequency of the crystal, respectively; Δ is the logarithmic decrement; and Δ_0 the value of Δ *in vacuo*. The crystal (0.3 cm in diameter by 5 cm long) was the same as that used by Betts *et al.* with $f \sim 40$ kHz. The decrement was determined from the decay of the crystal oscillations which were displayed on an oscilloscope and recorded photographically. Using the observed Δ and calculated values of ρ_n, η_e was calculated from Eq. (1). The equation is only valid if the mean free path of the momentum carriers is much less than the viscous penetration depth which must itself be much less than the thickness of liquid surrounding the crystal. These conditions were met for He^3 quasiparticles in our solutions. The viscometer was attached to the mixing chamber of a ^3He dilution refrigerator and its temperature measured by two cerium magnesium nitrate self-inductance thermometers.[6] The powdered cerium magnesium nitrate was in direct contact with the sample liquid, and the two thermometers agreed to within 4% at all temperatures. A 220-Ω Speer resistor was also incorporated within the sample chamber. Nominal mixture concentrations were accurate to 2%.

Figure 1 is a log-log plot of the product $\eta_e \rho_n$ against temperature for each He^3 molar concentration x. Curve A refers to the saturated He^4-rich solution, the accepted ^3He concentration of which varies from 6.4% at 20 mK to 10% at 230 mK.[7] Data for our 10% solution, for example, join this curve at about 235 mK, at which temperature it separates into two phases; the viscosity measured at lower temperatures is that of the ^4He-rich saturated solution. Data obtained for solutions with x between 6.4% and 10% join the saturation curve at lower temperatures, all of which agree closely with the established phase diagram. The effective viscosity along the phase separation line is well defined by these measurements on solutions of differing initial molar concentrations. We also show data for ^3He concentrations below the solubility limit. These do not undergo phase separation and their concentrations remain constant. Day-to-day reproducibility is well within the overall accuracy.

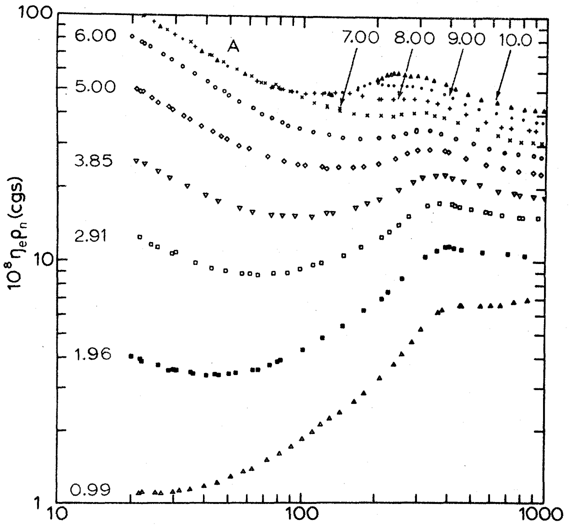

FIG. 1. A plot of the quantity $\eta_e \rho_n$ [Eq. (1)] versus temperature for the various solutions.

Many theoretical calculations have been made of ^3He quasiparticle interactions in solutions, which can be used to predict the viscosity (see Østgaard[8] for recent work and references). We compare our results here with the explicit theoretical viscosities of Baym and Saam,[1] which cover the limiting low-temperature region and the high-temperature semiclassical region around 1 K. Our data are replotted in Fig. 2 as η_e versus temperature, together with the prediction of Baym and Saam for a 5% solution. The ρ_n values used below 0.6 K were based on the inertial mass of He^3 quasiparticles as measured by Brubaker *et al.*[9]; above 0.6 K, extrapolations of these authors' results were used with corrections for the normal density of He^4. We also include in Fig. 2

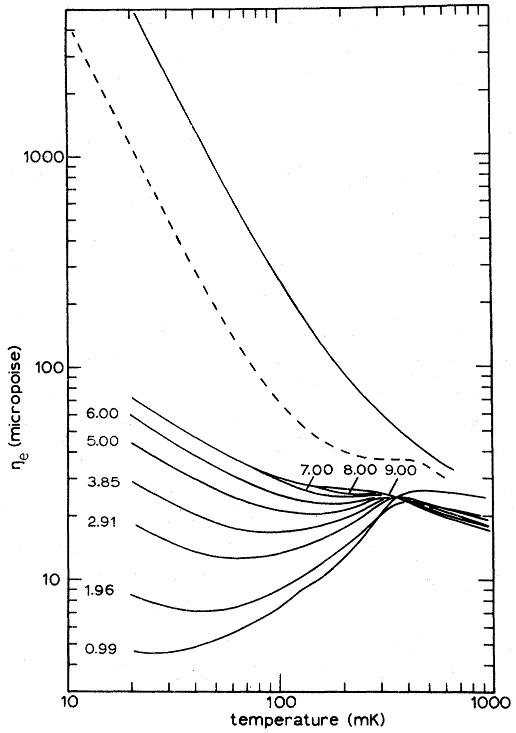

FIG. 2. The effective viscosities of He^3-He^4 solutions at various concentrations. The figures on the graph indicate the molar percentage of He^3; the 7%-, 8%-, and 9%-solution results coincide at the lower temperatures where the solution is saturated. Also shown are the experimental result for pure He^3 (solid curve) and the theoretical estimate (dashed curve) of Baym and Saam for a 5% He^3 solution.

our new results for pure ^{3}He, which are in excellent agreement with recent measurements of viscosity[3] and attenuation of first sound.[10] In the low-temperature limit, we find

$$\eta_e = 2.2/T^2, \quad \text{cgs units}. \tag{2}$$

This gives us confidence in our measurements.

In the classical region between 0.4 and 1 K, our results agree reasonably well with theory. Except for the lowest concentrations, η_e is nearly independent of concentration and the magnitude is roughly as predicted by Baym and Saam. In the intermediate region the experimental results show a much larger dip than is indicated by the theory, even allowing for the fact that at 40 kHz the effect of phonon viscosity on η_e is unclear. Comparison here is not particularly significant since this region was estimated in the theory by matching up the low-temperature and the high-temperature results. More significant is the very much smaller observed viscosity at the lowest temperatures, and the fact that its temperature dependence is not even approximately T^{-2}. The latter is particularly noticeable for the 5% mixture, in which η_e is fitted quite accurately by a power law, $\eta_e \sim T^{-0.56}$, over the temperature range 20 to 45 mK.

If these results give the true hydrodynamic viscosity, they carry implications for the design of the exit tube from the mixing chamber of a ^{3}He dilution refrigerator, one of the limitations on the performance of which is the viscous heating developed by the solution flowing in this tube. In calculations of this effect it has been assumed that the viscosity there is of order 1 mP, whereas our results would indicate values some 20 times smaller. There may, on the other hand, be a fundamental difference between the viscosity of a superfluid solution as measured by the torsional oscillation method at this frequency, and that appropriate to hydrodynamic flow, although this would have to be due to a previously unconsidered mechanism relating to the accommodation of quasiparticle momentum to the crystal surface. Recent unpublished measurements by Fisk and Hall[11] based on a vibrating-wire technique at a lower frequency do, apparently, give agreement with the Baym and Saam result for a 1.3% solution, and it is also true that calculations of convective instabilities, and direct observations of viscous heating,[12] give results which seem to be consistent with the higher viscosity and with its variation as T^{-2}. Some clarification of this situation is desirable.

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