TABLE I. Quantities obtained from experiments: $^{28}$Si$(p,p')^{28}$Si, Ref. 1; $^{28}$Si$(d,p)^{29}$Si, Ref. 2; and $^{24}$Mg$(d,p)^{25}$Mg, Ref. 3. The theoretical values of the ratio $n_d/n$ and the width $\langle\Gamma_d\uparrow\rangle$ are calculated by using Eqs. (16) and (14), respectively.

| | $\frac{\langle\Gamma_\mu\rangle}{D}$ | $\langle\Gamma_\mu\rangle$ (keV) | $(n)_{\text{expt}}$ | $\left(\frac{n_d(n_d-1)}{2}\right)_{\text{expt}}$ | $(n_d)_{\text{expt}}$ | $\left(\frac{2D}{\pi\langle\Gamma_\mu\rangle}\right)^{\frac{1}{2}} \leq \left(\frac{n_d}{n}\right)_{\text{theor}} \leq 1$ | $\left(\frac{n_d}{n}\right)_{\text{expt}}$ | $\langle\Gamma_d\uparrow\rangle$ (keV) |
|---|---|---|---|---|---|---|---|---|
| $^{28}$Si$(p,p')^{28}$Si | 2 | 110 | 9 | 22 | 7 | $0.58 \leq \left(\frac{n_d}{n}\right)_{\text{theor}} \leq 1$ | 0.8 | 200 |
| $^{28}$Si$(d,p)^{28}$Si | 15 | 30 | 5 | 3 | 3 | $0.21 \leq \left(\frac{n_d}{n}\right)_{\text{theor}} \leq 1$ | 0.6 | 243 |
| $^{24}$Mg$(d,p)^{25}$Mg | 20 | 40 | 5 | 3 | 3 | $0.18 \leq \left(\frac{n_d}{n}\right)_{\text{theor}} \leq 1$ | 0.6 | 432 |

(a) The width $\langle\Gamma_d\uparrow\rangle$ depends so strongly on the ratio $n_d/n$ that the exact ratio $n_d/n$ is needed to determine $\Gamma_d\uparrow$. In order to get $(n_d/n)_{\text{expt}}$ as close to $n_d/n$ as possible, an experiment with a large value of $(n)_{\text{expt}}$ should be performed. (b) By counting the number $(n_d)_{\text{expt}}$ only the large values of $C_{cc'}(0)$ can be included. Therefore, the width $\langle\Gamma_d\uparrow\rangle$ obtained in this way is the width for decay out of a *single* doorway state. (c) The number $(n_d)_{\text{expt}}$ depends upon the calculation of $C_{cc'}(0)$, so that the calculations have to eliminate the influence from some gross structure resonance or modulation effect.[9]

I would like to express my appreciation to Professor W. F. Hornyak and Professor J. B. Marion for the opportunity to work in this laboratory and also to Professor M. L. Roush and Dr. W. Reichart for their interesting and valuable suggestions. Thanks are also due to M. Daston and L. Lee for their hospitality.

*National Science Council Fellow of Republic of China. On leave from National Tsing Hua University, Taiwan, China. Present address: Department of Physics, National Tsing Hua University, Hsinchu, Taiwan, China.

[1]A. C. Shotter, P. S. Fisher, and D. K. Scott, Nucl. Phys. 159, 577 (1970).

[2]C. C. Hsu, S. M. Lee, T. Tohei, and S. Morita, to be published.

[3]S. M. Lee, Y. Hiratate, K. Miura, S. Kato, and S. Morita, Nucl. Phys. 122, 97 (1968).

[4]A. M. Lane, Phys. Lett. 31B, 344 (1971), and Ann. Phys. (New York) 63, 171 (1971).

[5]*Intermediate Structure in Nuclear Reactions*, edited by H. P. Kennedy and R. Schrils (Univ. of Kentucky Press, Lexington, Ky., 1968).

[6]P. A. Moldauer, Phys. Rev. 135, 642 (1964).

[7]C. C. Hsu, Phys. Rev. C 2, 767 (1970).

[8]H. Feshbach, A. K. Kerman, and R. H. Lemmer, Ann. Phys. (New York) 41, 230 (1967).

[9]B. W. Allardyce, P. J. Dallimore, I. Hall, N. W. Tanner, A. Richter, P. von Brentano, and T. Mayer-Kuckuk, Nucl. Phys. 85, 193 (1965).

## Reactions to Unbound Final States*

U. Strohbusch,† H. J. Körner,‡ and G. C. Morrison
*Argonne National Laboratory, Argonne, Illinois 60439*

and

J. P. Schiffer
*Argonne National Laboratory, Argonne, Illinois 60439, and University of Chicago, Chicago, Illinois 60637*

(Received 26 October 1971)

It is pointed out that the strength of ($^3$He,$d$) reactions to unbound states is correlated with the penetrability for proton decay of these states. The anomalously weak transitions to $s$ states fit into this pattern. Additional data from the reaction $^{116}$Sn($^3$He,$d$)$^{117}$Sb* are presented. Some questions regarding extrapolations to other reations are raised.

Considerable recent interest in stripping reactions to unbound analog states was initiated by a Letter reporting the ($^3$He,$d$) reaction on Zr and Mo isotopes, in which McGrath *et al.*[1] found anomalies in such transitions. In particular, $l=2$ transitions were seen strongly whereas $l=0$ transitions, expected in the same nuclei, were absent or very weak. The effect was contrary to the re-

sults of distorted-wave calculations in which the form factor is taken as proportional to that of the bound parent state. Similar results[2] have been obtained for the $(d,n)$ reaction. In Ref. 2 it was pointed out that part of this anomaly consists in a strong enhancement of the $l=2$ cross sections with respect to the distorted-wave calculations.

Subsequent papers[3-8] presented calculations using a more realistic proton form factor and showed that they reproduced the experimental trends quite well. It became apparent that the reason for the enhancement is that in the surface region where the transfer is expected to take place, the tail of the analog wave function is larger than that of the parent state.

The question then arises why the $l=0$ transitions are so weak. Although distorted-wave calculations with form factors matched into the continuum reproduce these weak $l=0$ transitions, one still has no "understanding" of why this should be so.

The connection of a quasibound state with the continuum is expressed by the penetrability factor $P_l$ in $R$-matrix theory. To study the influence of $P_l$ on the cross section we have calculated the proton penetrabilities for the unbound analog states observed[1] in the $(^3\mathrm{He},d)$ reactions on Zr and Mo. For $l=0$ protons they are 3–5 times those for $l=2$. In Fig. 1 the ratio $\sigma_{\mathrm{exp}}/\sigma_{\mathrm{calc}}$ is

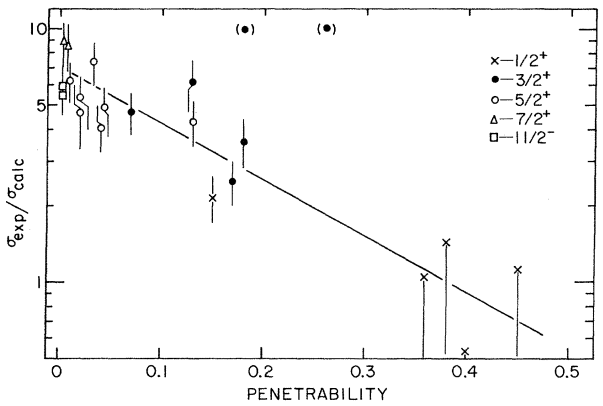


FIG. 1. Enhancement $\sigma_{\mathrm{exp}}/\sigma_{\mathrm{calc}}$ versus the proton penetrability for $(^3\mathrm{He},d)$ transitions to unbound analog states ($\sigma_{\mathrm{exp}}$ taken from Ref. 1). The error bars indicate only errors of the experimental cross sections and do not include the errors in the spectroscopic factors. The solid line merely indicates an average trend. The points in parentheses correspond to transitions to $2d_{3/2}$ states in $^{93}$Nb and $^{95}$Nb. Their departure from the average trend probably is due to the contribution of an unresolved $g_{7/2}$ state, as has been checked for the corresponding parent state in $^{93}$Zr by a high-resolution $(d,p)$ measurement.

plotted as a function of $P_l$ with

$$\sigma_{\mathrm{calc}} = 4.42 \frac{S}{2T_0+1} \frac{2I_f+1}{2I_i+1} \frac{\sigma^{\mathrm{DW}}}{2j+1}$$

and $P_l = kR(F_l^2+G_l^2)^{-1}$ with $R=1.4A^{1/3}$. Here $S$ is the spectroscopic factor of the parent state and $\sigma^{\mathrm{DW}}$ the distorted-wave Born approximation (DWBA) cross section[9] calculated assuming the form factor of the transferred proton to be the same as that of the neutron in the corresponding parent state. The experimental cross sections are taken from Ref. 1.

The main feature of this figure is a clear trend of decreasing enhancement of the experimental cross section relative to the calculation from ~8 to <1 with increasing penetrability. The $s$ states are at the high-penetrability side, and it is characteristic that the only $s$-state transition seen in the $(^3\mathrm{He},d)$ experiments (the one to $^{93}\mathrm{Tc}^*s_{1/2}$) is the one associated with the smallest $l=0$ proton penetrability; for all others only a limit could be set.[1] The trend of the curve is already established by the $l\geq 2$ states and is continued by the points corresponding to $l=0$.

This behavior is confirmed by the results obtained at this laboratory for $(^3\mathrm{He},d)$ reactions on $^{116}$Sn and $^{117}$Sn isotopes. Some spectra obtained with the Enge split-pole spectrograph are shown in Fig. 2. The only transition clearly observed is that to the $\frac{11}{2}^-$ analog state in $^{117}$Sb. An angular distribution for this is also shown in Fig. 2. The solid curve represents a standard DWBA calculation multiplied by 8. This enhancement factor with the corresponding penetrability ($2\times10^{-3}$) fits in well with the curve in Fig. 1, even though, strictly speaking, such a curve should be $Z$ dependent. The $l=0$ and even the $l=2$ transitions are not seen or are so weak that only an upper limit (20 $\mu$b/sr) for the cross section could be estimated. If Fig. 1 is taken as a guide, no larger cross section could be expected for these transitions. Of course, this situation is the reverse of that in elastic proton scattering in which only the $\frac{11}{2}^-$ state is unobserved.[10] Thus stripping reactions seem to be useful in locating high-spin analog resonances.

In Fig. 3 again $(^3\mathrm{He},d)$ cross sections divided by DW calculations with the parent form factor are shown, but only for the $l=2$ transitions in the Mo isotopes. They are now plotted as a function of the binding energy of the transferred proton. Since $l$ and $Z$ are the same for all the data, this is similar to the plot in Fig. 1 with proton energy replacing the proton penetrability. This form of

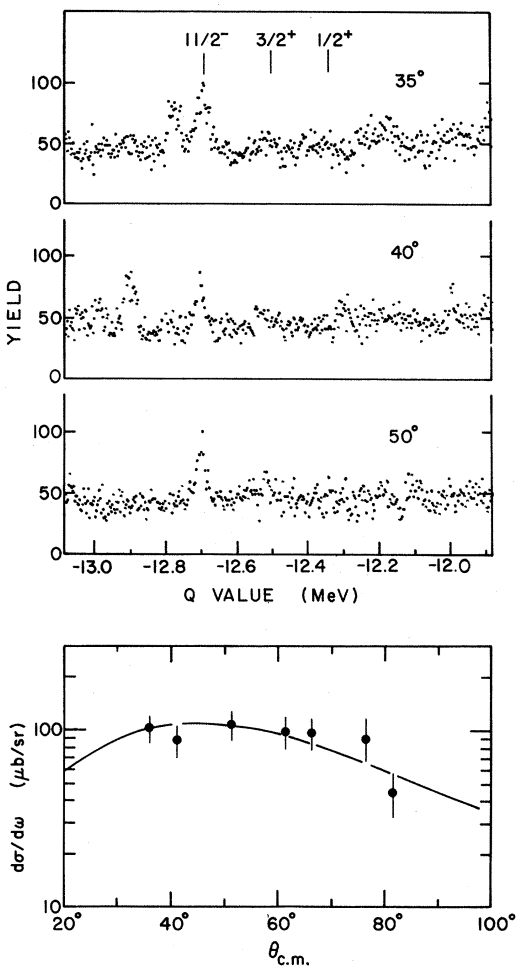


FIG. 2. Spectra for $^{116}$Sn($^3$He,$d$) and angular distribution for the transition to the $\frac{11}{2}^-$ analog in $^{117}$Sb obtained at $E_{^3\mathrm{He}} = 24$ MeV.

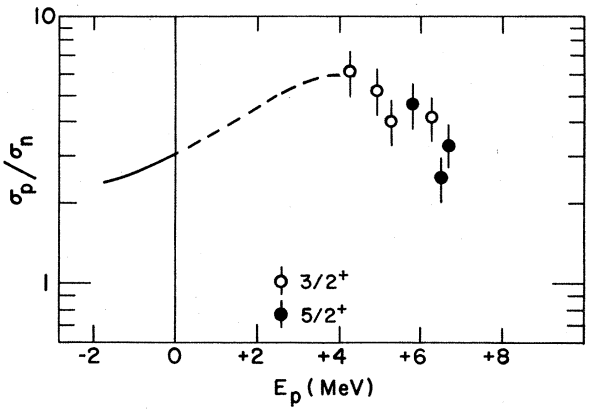


FIG. 3. Mo($^3$He,$d$) cross section $\sigma_p$ compared to DWBA cross section $\sigma_n$ as a function of the proton energy $E_p$. The cross section $\sigma_n$ was obtained from a DWBA calculation with the parent-neutron form factor. For the solid curve at negative $E_p$, the values of $\sigma_p$ are DWBA calculations with a bound-proton form factor —the binding energy of the proton differing from that of the neutron by the Coulomb energy difference typical of Mo. For the points, the values of $\sigma_p$ are experimental (Ref. 1).

plot allows the display to be extended to bound protons. The solid curve at negative values of $E_p$, calculated for the hypothetical case where the proton state would be bound, already shows an enhancement increasing with increasing proton energy; the dashed curve is strictly an interpolation. The enhancement becomes larger for unbound states (as indicated by the experimental points on the right-hand side of this figure) but then begins to decrease with increasing proton energy.

The initial effect of less binding is to increase the form factor in the surface region where most of the reaction cross section originates, and this gives rise to the observed enhancement. This increase seems to continue into a limited region of positive proton energy. At higher positive proton energies the observed cross section starts to fall off, suggesting that the relevant (surface) part of the form factor is decreasing. The plot of the data in Fig. 1 suggests a close connection between this decrease and the penetrability.

One may ask why such a qualitative connection between penetrability and cross section is useful when "correct" calculations can reproduce the experimental results. The reason is not only that the calculations are laborious, but that they give one little feeling for what trends to expect in going to different states or nuclei. For lighter nuclei (i.e., for smaller Coulomb energies) the difference in the surface amplitudes of the parent and the analog wave functions and hence the enhancement are expected to be smaller. This is consistent with the trend we observe in our preliminary results obtained for ($^3$He,$d$) on Ni and Zn isotopes. Enhancements of ~4 were observed for $l = 1$, 3, and 4 transitions to unbound states with the penetrability always less than $2 \times 10^{-2}$.

The charge-exchange ($p$,$n$) reaction also may proceed to unbound analog states. Effects similar to the ones in ($^3$He,$d$) would occur and modify the relevant form factor. All calculations fitting such quasielastic ($p$,$n$) data have been in terms of a parent (bound) form factor.[11,12] In a heavy nucleus there may be a factor-of-2 effects which would then drastically alter the parameters of symmetry potentials deduced from these data. One should also consider the role of binding in pickup reactions in which the final state becomes

unbound. The form factor for the picked-up particle could be regarded as a bound-state wave function, but the potential binding it is itself decaying—as is the potential causing the distortion in the outgoing channel.

One may further wonder what corresponding effects may occur in the realm of particle physics, where the high-energy reactions leave decaying resonances as products.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

†On leave from Physikalisches Institut der Universität Freiburg, Freiburg, Germany.

‡On leave from the Technische Universität München, München, Germany.

[1]R. L. McGrath, N. Cue, W. R. Hering, L. L. Lee, Jr., B. L. Liebler, and Z. Vager, Phys. Rev. Lett. 25, 682 (1970).

[2]S. A. A. Zaidi, C. L. Hollas, J. L. Horton, P. J. Riley, J. L. C. Ford, Jr., and C. M. Jones, Phys. Rev. Lett. 25, 1503 (1970).

[3]B. J. Cole, R. Huby, and J. R. Mines, Phys. Rev. Lett. 26, 204 (1971).

[4]D. H. Youngblood and R. L. Kozub, Phys. Rev. Lett. 26, 572 (1970).

[5]C. M. Vincent and H. T. Fortune, Phys. Rev. C 2, 782 (1970).

[6]S. A. A. Zaidi and W. R. Coker, Phys. Rev. C 4, 236 (1971).

[7]R. L. Kozub and D. H. Youngblood, Phys. Rev. C 4, 535 (1971).

[8]W. R. Coker and S. A. A. Zaidi, Center for Nuclear Studies, University of Texas, Austin, report, 1971 (unpublished).

[9]The code DWUCK has been used. We are grateful to Dr. P. D. Kunz for making this code available to us.

[10]P. Richard, C. F. Moore, J. A. Becker, and J. D. Fox, Phys. Rev. 145, 971 (1966).

[11]G. R. Satchler, R. M. Drisko, and R. H. Bassel, Phys. Rev. 136, B637 (1964).

[12]C. J. Batty, B. E. Bonner, E. Friedman, C. Tschälar, L. E. Williams, A. S. Clough, and J. B. Hunt, Nucl. Phys. A116, 643 (1968).

# Low-Lying $0^+$ States and $(p,t)$ Strengths in the Actinides*

W. I. van Rij and S. H. Kahana
*Brookhaven National Laboratory, Upton, New York 11973*
(Received 4 November 1971)

We present a noncollective description of the low-lying $0^+$ states strongly excited in $(p,t)$ reactions in the actinides. The description is based on the possible weakness of the oblate-prolate pairing force, suggested by Griffin, Jackson, and Volkov, and the low density of oblate states near the Fermi surface in these nuclei.

Maher *et al.*[1] have found that a low-lying $0^+$ state observed in the actinides at about 900 keV is excited strongly in $(p,t)$ reactions. These authors have further indicated that a description of this state as a pairing or $\beta$ vibration is likely to fail. The uniform appearance of this $(p,t)$ excited state ($0_1^+$), together with its appreciable electromagnetic decays to the ground-state band, led Maher *et al.* to postulate the existence of a new and stable type of collectivity. More recently Griffin, Jackson, and Volkov[2] made the interesting suggestion that such a new collectivity may arise if the nuclear pairing between particles in prolate levels and those in oblate levels is weak. These authors carried out a model calculation for two neutrons, which perhaps demonstrates the required collectivity. However, a few-particle calculation is unlikely to be appropriate for nuclei possessing a high single-particle level density. Further, the existence of a collective state to explain the observed phenomena is perhaps unnecessary. The $(p,t)$ strength to the $0_1^+$ state, though an appreciable fraction of the ground-state strength, may result from the population of just one or perhaps a few of the Nilsson levels. The observed $E0$ and $E2$ decays which are of single-particle magnitude seem to indicate such a possibility.

In this Letter a model is presented describing the $0_1^+$ state to first approximation as a pair of holes in the oblate $\frac{1}{2}[501]$ Nilsson level. These holes are built into a pairing ground state. The single-particle nature of the oblate level thus constructed guarantees a large $(p,t)$ excitation, an excitation enhanced because the spherical $3p_{1/2}$ level giving rise to the $\frac{1}{2}[501]$ level possesses several nodes and contributes strongly to two-neutron transfer. To account for the wide dissemination of the $0_1^+$ state in the actinides and for its general absence in the rare earths, it is necessary to perform a many-body calculation, paying attention to the underlying single-particle