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Inclusive N-Body Reaction in Nucleon-Nucleon Interactions at Ultrahigh Energies*

P. L. Jain and Z. Ahmad
High Energy Experimental Laboratory, Department of Physics, State University of New York at Buffalo, Buffalo, New York 14214
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Inclusive reactions for multiparticles produced from cosmic-ray emulsion data at $\sim 10^{12}$ eV are presented and are compared with the predictions of limiting-fragmentation distribution.

Within the last couple of years a number of papers both theoretical[1-3] and experimental[4] have appeared on the inclusive reactions of type $A+B \to C+$anything as a source of information for understanding the mechanism of multiparticle production processes. Benecke *et al.*[1] introduced the hypothesis of limiting fragmentation, and Feynman[2] predicted the behavior for the function $f(p_t, x) = (2E/\pi s^{1/2})\, d^2\sigma/dx\, dp_t^2$, where E is the energy of the observed particle in the c.m. system, the Feynman variable $x = 2p_l/s^{1/2}$, and s the square of the total energy in the c.m. system. It can be shown that these two hypotheses are equivalent at high energies.[5]

Measuring an angle and the momentum of a single particle has been rather popular, and practically all the experiments reported recently have been for a single observed particle (inclusive one-body reaction) emerging from high-energy hadron-hadron interactions. A single particle gives a rather incomplete reflection of the actual behavior of general collisions. Moreover, because only one final particle is detected in these experiments, there is no empirical way to ascertain whether the observed particle is scattered directly or is the decay product of a resonance. If it is from a resonance, a different approach is desirable.

Most of the data presented so far on single-particle production have been taken from the accelerator energy range. Recently, charged-particle multiplicities and the angular distribution of the secondary particles in pp interactions[6] for the energy range 100–800 GeV have been reported from cosmic-ray experiments. According to these authors, their angular distribution was biased against smaller angles, ≤ 2 mrad, and larger angles, $> 40°$.

We present here an analysis of 21 cosmic-ray jets[7-12] of energy $\sim 10^{12}$ eV, which were selected out of about 135 jets with ≤ 4 black prongs. The complete analysis of a jet observed in nuclear emulsion is rather tedious[7] because of the many parameters that are required for its analysis. The energy of the secondary particles were determined by multiple and relative scattering measurements. We know of no published data reporting an inclusive n-body reaction with jets studied completely in emulsion of TeV energy range. The data presented here have energy ranges between 1–100 TeV and average multiplicity ~ 17. It is known that most of the particles produced in high-energy hadron collisions are pions. Production of strange particles is less abundant, and baryon-antibaryon pair production is even more rare.

The differential cross section for a single inclusive process, i.e., $N+N \to$ (one identified particle) +anything, is written as

$$d\sigma = dp^3 f(p_t, p_l, s)/E = \pi\, d(p_t^2)\, dp_l f(p_t, p_l, s)/E \tag{1}$$

where E is the energy and p_t and p_l are the transverse and longitudinal momenta of the observed particle in the c.m. system. Feynman has suggested that the structure function $f(p_t, p_l, s)$ scales at high energy, i.e., as $s\to\infty$, it becomes only a function of p_t and the ratio $2p_l/s^{1/2}=x$ such that $\lim_{s\to\infty} f(p_t, p_l, s) \to f(p_t, x)$:

$$d^2\sigma = \pi f(x, p_t) \times [x^2 + 4(p_t^2 + m^2)/s]^{-1/2} dx\, d(p_t^2). \quad (2)$$

In the analysis of jets we measure the angular distribution of the charged secondaries, and we define

$$X = \log_{10}(\tan\theta_L),$$

$$Y = \log_{10}(\gamma_C \tan\theta_L) = \log_{10}\left(\frac{\tan\theta_L}{\tan\theta_C}\right) = \log_{10}\left(\frac{(p_t/p_l)_L}{(p_t/p_l)_C}\right),$$

where $\gamma_C = (1-\beta_C^2)^{-1/2}$ is the Lorentz factor of the c.m. system with respect to the lab frame and θ_L is the angle of a shower particle in the lab frame. At cosmic-ray energies this angle is closely related to the rapidity variable $R = \frac{1}{2}\ln[E+p_l)/(E-p_l)]$. For $p_l \gg p_t \gg m_\pi$, $R_L \to \ln(2/\tan\theta_L) \to R_C + \ln(2\gamma_C)$, where $R_C = -\ln(\gamma_C \tan\theta_L)$; here R_C and R_L are the center-of-mass and lab rapidities, respectively. The test of the limiting fragmentation of the target particle is to measure the rapidities and to see whether these distributions become independent of the primary energy. The distribution of $\log_{10}(\gamma_C\tan\theta_L)$ for 21 events[13] is shown in Fig. 1(a). The distribution is rather flat at the center $-2 \le R_C \le 2$. The width of this depends upon the dispersion of the distribution in the X coordinate, i.e., $\alpha = [\langle \sum (X-\bar{X})^2\rangle]^{1/2}$. The width α of the X distribution, i.e., $\log_{10}(\gamma_C\tan\theta_L)$, is a measure of limiting behavior, and we observe that for large primary energies the width of the distribution $R_{max} - R_{min} \approx \ln\gamma_C$. In Fig. 1(b) we have plotted α as a function of primary energy and we see that the growth of α is proportional to $\log_{10}\gamma_C$. We have also measured the inelasticity η (i.e., fractional energy loss) of the individual events, where $\eta = E_s/E_0$; E_0 is the kinetic energy of the primary particles in the lab system and E_s is the sum of the energies of all the charged particles multiplied by a correction factor of 1.5 for neutral particles.[14] We notice that the value of η decreases very slowly with E^{beam} in the TeV energy range.

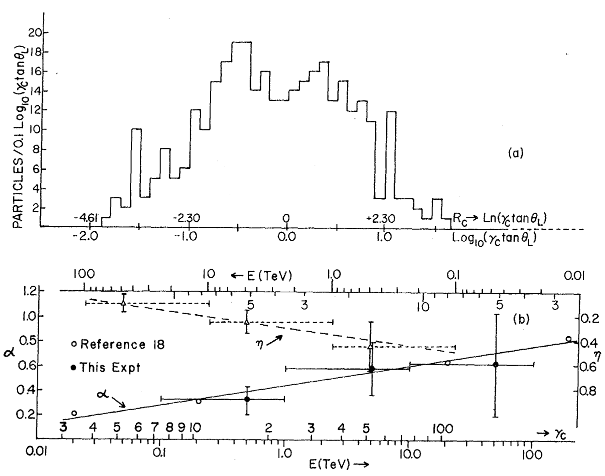

FIG. 1. (a) Angular distribution in $\log_{10}(\gamma_C\tan\theta_L)$ of secondary particles $R_c \to \ln(\gamma_C\tan\theta_L)$. (b) Correlation between (i) α and E^{beam}, (ii) η and E^{beam}.

There is a direct relation between the differential production cross section and the average multiplicity $\bar{n}_c$ for the particles produced in the interaction. In the present data, a single event contributes as many entries to the plot as it has charged prongs. Thus the total inclusive cross section of n_c charged particles (i.e., n-body cross section) is $\Sigma_c = \sum_n n_c \sigma_n$ or $\bar{n}_c = \Sigma_c/\sigma_{inel}$, where σ_{inel} is the total inelastic cross section for all collisions over which the average multiplicity $\bar{n}_c$ of charged tracks is defined.

If the distribution function $f(x, p_t)$ in Eq. (2) is already asymptotic and is constant in x for $x=0$, then

$$\begin{aligned}\bar{n}_c &= \Sigma_c/\sigma_{inel} \\ &= (1/\sigma_{inel})\int d^2\sigma/[dp_l\, d(p_t^2)]\, d(p_t^2)\, dp_l \\ &= \frac{\pi}{\sigma_{inel}}\int f(x,p_t)[x^2 + \tfrac{4}{s}(p_t^2+m^2)]^{-1/2} dx\, d(p_t^2) \\ &= \left[\frac{\pi}{\sigma_{inel}}\int f(0,p_t)\, d(p_t^2)\right]\ln s + \text{const} \\ &= C\ln\gamma_c + \text{const} \approx \frac{C}{2}\ln\left(\frac{E^{beam}}{M_p}\right) + \text{const}. \quad (3)\end{aligned}$$

Hence $\bar{n}_c$ increases logarithmically if $f(0, p_t) \ne 0$. The average multiplicity is linear in $\ln s$, and from the hypothesis of limiting fragmentation it follows that the central part of the spectrum is constant. The scaling behavior can be detected most easily if the variable p_l is replaced by the rapidity R and dR by dp_l/E in Eq. (1). Thus from the rapidity distribution we get $C = (1/\sigma_{tot})\, |d\sigma/dR|_{R_C=0}$. From Fig. 1(a) we have calculated the value of C, which is ~ 2.8.

Recently Akerlof *et al.*[15] and Abolins *et al.*[16] have observed a peak in the distribution $d^2\sigma/d\Omega\, dp$ at low p_t^2 values, $<0.2\ (\text{GeV}/c)^2$, in the proton momentum spectrum in pp interactions. For secondary kaons and pions the distributions are both characterized by a sharper peak $\exp(-15p_t^2)$ for small $p_t^2 < 0.2$ $(\mathrm{GeV}/c)^2$, which levels off to a more general dependence $\exp(-3p_t^2)$ for larger p_t^2, > 0.2 $(\mathrm{GeV}/c)^2$. The slope increases with the energy; for example, the slopes for p_t^2 distributions for pions are 9 and 15 from primary protons of momenta 6[16] and 12.5[15] GeV/c, respectively. This observation has also been made at higher energies, 100–1500 GeV, by Ratner *et al.*[17] They have found that pion and proton cross sections both drop roughly as $\exp(-3.5p_t^2)$ for events with $p_t^2 > 0.2$ $(\mathrm{GeV}/c)^2$, much the same as at lower energy. Unfortunately they did not have results for events with $p_t^2 < 0.2$ $(\mathrm{GeV}/c)^2$. Most of our events have $p_t^2 \leq 0.09$ $(\mathrm{GeV}/c)^2$. We are reporting here in Fig. 2 the p_t^2 distribution for the secondary particles for two values of p_l, i.e., $p_l \leq 6$ GeV/c (lab) and $p_l > 6$ GeV/c (lab). We find that both the curves have practically the same slope value, ~35. We also looked into the dependence of the production cross section on the p_l (c.m. system) distribution which is again Gaussian for small p_l. The slope of the p_l^2 distribution, however, is

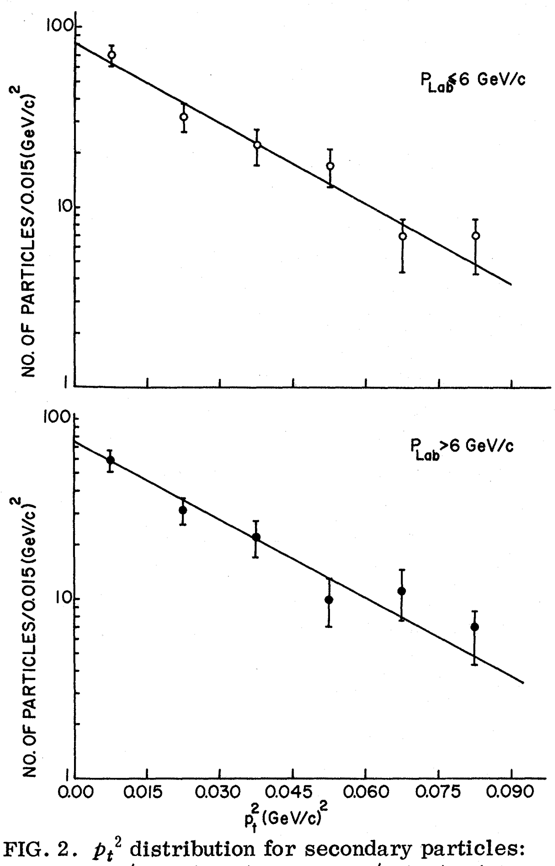

FIG. 2. p_t^2 distribution for secondary particles: (i) $p_l \leq 6$ GeV/c (lab); (ii) $p_l > 6$ GeV/c (lab). (This cut in p_l at 6 GeV/c lab, for secondary particles, gives approximately equal numbers of particles in both the distributions.)

clearly flatter than the p_t^2 distribution. The behavior in Fig. 2 shows that the production cross section is probably factorable into functions of p_l and p_t in the c.m. system, that is, $d^2\sigma/d\Omega dp = F(p_t)G(p_l)$, where F and G are some functions of their respective variables. Previously[15] the factorability has been tested at large p_t^2 values and now in the present experiment is examined in the small-p_t^2 region where the sharp forward peak appears with a value $\exp(-35p_t^2)$. The structure observed at small p_t values for hadrons produced in deep-inelastic hadron collisions is quite striking and needs further investigation.

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