it is not clear why He^3-He^4 mixtures differ in their tricritical behavior from these simple magnetic systems, although recent series expansion results[16] on the Blume-Emery-Griffiths[17] model suggest that the differences in the Hamiltonian may be responsible. Thus, we feel that there is a whole new range of thermodynamic phase diagram to be studied theoretically. In particular, we feel it would be most interesting to see the results of more careful experiments on real systems, and of series expansions studies for a variety of lattices.

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Transcendence of the Law of Baryon-Number Conservation in Black-Hole Physics*

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The following result is stated: A black hole in its final state can be endowed with no exterior scalar, vector, or spin-2 meson fields. We conclude that such a black hole is not subject to the strong interactions which are mediated by meson fields such as the π, ρ, and f, and that consequently a useful definition of baryon number cannot be given for such an object.

It is the present belief that, from the point of view of an exterior observer, the final state of the total gravitational collapse of a massive star is a stationary black hole.[1] It has been conjectured that mass, charge, and angular momentum are the *only* independent properties of such black holes. More precisely, this Israel-Carter conjecture states that the exterior of an isolated stationary black hole is *completely* described by a Kerr-Newman (charged Kerr) solution of the Einstein-Maxwell equations. Such a solution has only three parameters: the mass, charge, and angular momentum of the black hole.[1] This conjecture is supported by much evidence, but principally by theorems of Israel,[2] Carter,[3] Hawking,[4] and Wald.[5]

The above theorems do not explicitly allow for the effects of the weak and strong interactions of the stellar material out of which the black hole was formed. Hartle[6] and Teitelboim[7] have considered the possible effects of the weak interactions on the problem. Hartle concludes that a Kerr black hole cannot interact with the exterior world by means of the weak interactions which are mediated by neutrino fields. By a different method, Teitelboim obtains a similar result for the spherically symmetric case, and concludes that the black-hole's lepton number cannot be measured from its exterior by means of the weak interactions (which are characteristic of all leptons). This conclusion supports Wheeler's argument that the law of conservation of lepton number is transcended in black-hole physics.[8] Both Hartle's and Teitelboim's results support the Israel-Carter conjecture in the realm of the weak interactions.

Wheeler has also argued that the law of conservation of baryon number is transcended in black-hole physics.[8,9] An exterior observer, witnessing the collapse of a star into a black hole, loses the possibility of verifying the conservation of baryon number, because he loses all causal contact with the baryons of the star (red-shift effect, etc.). This is what transcendence of the law means. It is instructive to contrast baryon number and charge. Suppose the star carries a net charge. The law of charge conservation is *not* transcended in the collapse because the charge leaves "footprints," that is, it gives rise to an exterior electromagnetic field of the black hole. The exterior observer can then determine the charge of the black hole by means of the familiar Gaussian integral, and can verify that charge is conserved in the collapse.[1]

One knows that all baryons are strongly interacting. They are the sources of various meson fields, just as charge is the source of the electromagnetic field. Can the black hole in question be endowed with exterior meson fields (associated with the baryons) in the same way that it can be endowed with an exterior electromagnetic field (associated with the charge)? Such meson fields could serve as "footprints" for the baryons, and allow the exterior observer to verify the law of baryon-number conservation. True, the meson fields, being massive, would fall off exponentially at large distances from the black hole. This feature would preclude the determination of the baryon number of the black hole through a Gaussian integral (which requires an r^{-2} fall off). But there is no *a priori* reason why baryon number could not be one of the parameters of the solution (one more general than the Kerr-Newman) which would describe the black hole with exterior meson fields. The observer could still measure the baryon number in some way. We thus see that the possibility of exterior meson fields existing represents a potential difficulty in Wheeler's argument that the law of baryon-number conservation is transcended. It is hardly necessary to emphasize that this possibility is also a threat to the Israel-Carter conjecture.

It would not be correct to infer from the massiveness of mesons that the exterior meson fields would extend only a microscopic distance (10^{-13} cm) out from the black-hole's horizon. Such a conclusion based on flat-space reasoning cannot be correct for the region near the black hole, where the geometry varies appreciably over distances comparable to the gravitational radius. Under these conditions it is more appropriate to expect that the rate of spatial variation of the meson fields would be governed, not by the Compton wavelengths in question, but by the gravitational radius. The fields could well extend for a few gravitational radii out from the black hole before the (Yukawa) exponential falloff would set in. Therefore, the effects of the strong interactions cannot *a priori* be expected to be negligible on a macroscopic scale.

In view of all the complications that *could* have arisen as a result of the strong interactions, it is satisfying that the following simple result holds: *A black hole in its final (static or stationary) state cannot be endowed with any exterior massive scalar, vector, or spin-2 meson fields.* We shall give the complete proof of this result elsewhere,[10] but a simplified proof of a special case appears below. Our result rules out classical Klein-Gordon (scalar), Proca (vector), and Fierz-Pauli (spin-2) massive fields.[11] It is possible to modify the proofs to show that charged fields are also excluded, even from the exterior of a charged black hole.[10] With this extension, the result rules out all the known meson fields.

We expect our classical result to be relevant also when quantum effects are taken into account for the following reasons: Integral-spin fields, such as the ones we consider, can be split up into longitudinal and transverse components.[12] It is also possible to do this in curved space. (An example is the split for the electromagnetic field achieved by Arnowitt, Deser, and Misner.[13]) It is well known that only the transverse components (the truly dynamical ones) are quantized. The longitudinal ones are always described classically.[12] The statement that the black hole is in its final (or ground) state means that excitations of the meson fields are absent from the exterior (no real mesons). Thus, the transverse components must be zero, except for the inevitable *random* vacuum fluctuations. Our result then shows that the (classical) longitudinal components must vanish identically in the balck hole's exterior.

The strong interactions are known to be mediated by meson fields of the scalar (i.e., π), vector (i.e., ρ), and spin-2 (i.e., f) types. On the basis of the above discussion we conclude that a black hole in its final state cannot interact with the exterior world by means of the strong interactions. The reason is that the black hole cannot be endowed with the longitudinal meson fields that are required for such an interaction to take place. *Thus an exterior observer cannot* *use the strong interactions to probe a black hole in its final state and, in particular, to measure its baryon number.* (Using a different method, Teitelboim has reached a similar conclusion for the special case of measurements carried out, by means of the strong interactions mediated by a scalar meson field, on a Schwarzschild black hole.[14]) Thus we find that, in the collapse of a star into a black hole, the baryons of the star leave no "footprints," so that an exterior observer finds it impossible to verify operationally that baryon number is conserved in the collapse. Thus Wheeler's argument that the law of conservation of baryon-number is transcended has *no* loopholes.

We may also state our above conclusion by saying that, from an exterior observer's viewpoint, a black hole in its final state has no well-defined baryon number. Of course the observer can define the baryon number of the black hole to be the number of baryons that fell into it. But this procedure merely reduces the law of baryon-number conservation to a tautology. In any case, in black-hole physics the law is no longer an operationally well-defined physical statement. (All that has been said of baryon number also applies to strangeness.)

Our conclusion that all meson fields are absent from the exterior of a black hole in its final state means that the strong interactions make no contribution to the stress-energy tensor $T^{\mu\nu}$ in the exterior. The results of Hartle[6] and of Teitelboim[7] suggest that the same is true of the weak interactions. Thus $T^{\mu\nu}$ must be purely electromagnetic in nature. Under these conditions the uniqueness theorems mentioned earlier[2-5] show that (subject to some technical conditions) the black-hole exterior is completely described by a Kerr-Newman solution. Thus we find that the Israel-Carter conjecture is correct, even when the effects of the strong and (possibly) the weak interactions are taken into account. Not even the gravitational field of the black hole supplies any information about baryon number. We may mention here that all our conclusions (no exterior meson fields, nonmeasurability of baryon number, and the fact that the black-hole exterior has a Kerr-Newman character even in the presence of the strong interactions) hold as well in Brans-Dicke theory.[10]

We now illustrate the ideas behind the proof of our result by considering the case of a *massive* scalar meson field in the exterior of a rotating stationary black hole. By a theorem of Hawking[4] we know that the black-hole exterior must also be axisymmetric and the horizon must be topologically spherical. We may choose the following metric for the exterior[3] (signature +2):

$$ds^2 = W(d\rho^2 + dz^2) + A\,dt^2 + B\,d\varphi^2 + C\,dt\,d\varphi, \tag{1}$$

where W, A, B, and C are independent of the time t and the symmetry angle φ. The requirement that causality hold in the exterior (no closed timelike or null curves) leads to the conclusion that $W \geq 0$, and that W can vanish only on isolated points in the ρz plane.[10] (This condition is satisfied for Kerr-Newman black-hole exteriors.)

The horizon is (by definition) a null, nonsingular hypersurface. Thus its normal n_μ obeys $n_\mu n^\mu = 0$ (μ runs over all four coordinates). Because of the symmetries, $n_t = n_\varphi = 0$, so that

$$W^{-1}(n_\rho{}^2 + n_z{}^2) \to 0 \tag{2}$$

on the horizon. The scalar field ψ obeys the Klein-Gordon equation,

$$\psi_{,\mu;}{}^{\mu} - m^2 = 0, \tag{3}$$

which in view of the symmetries ($\psi_{,t} = \psi_{,\varphi} = 0$) reduces to

$$[W^{-1}(-g)^{1/2}\psi_{,\rho}] + [W^{-1}(-g)^{1/2}\psi_{,z}]_{,z} - m^2\psi(-g)^{1/2} = 0. \tag{4}$$

Multiplying (4) by ψ, integrating over the entire black-hole exterior (volume element $d^4x = d\rho\,dz \times dt\,d\varphi$), and integrating by parts once we get

$$\int W^{-1}\psi(\psi_{,\rho}n_\rho + \psi_{,z}n_z)\,d\sigma = \int [W^{-1}(\psi_{,\rho}{}^2 + \psi_{,z}{}^2) + m^2\psi^2](-g)^{1/2}d^4x, \tag{5}$$

where n_μ is now the normal to the boundary of the black-hole exterior (the horizon and spatial infinity), and $n_\mu d\sigma$ is the vector three-dimensional hypersurface element. The part of the boundary at spatial infinity contributes nothing because ψ falls off exponentially sufficiently far from the black hole.

The nonsingular character of the horizon will now be used in two ways to show that the boundary integral over the horizon in (5) vanishes. First, there must exist a (Kruskal-like) nonsingular coordinate system which extends past the horizon. We write out the definition of $n_\mu d\sigma$ in these coordinates, and notice that all the $n_\mu d\sigma$ are nonsingular, so that $d\sigma$ must be nonsingular on the horizon. But $d\sigma$ is an invariant, so it is also nonsingular in the (original) coordinates of (1). Second, the quantity $Q = \psi^2\psi_{,\mu}\psi_{,}{}^{\mu}$ can be expressed completely in terms of $T^{\mu}{}_{\mu}$ and $T^{\mu\nu}T_{\mu\nu}$, the scalars associated with the stress tensor for the field ψ. Thus Q is a physical scalar, and must be bounded on the (nonsingular) horizon:

$$\psi^2 W^{-1}(\psi_{,\rho}{}^2+\psi_{,z}{}^2) \text{ bounded on horizon.} \quad (6)$$

Now, Schwarz's inequality gives us for the boundary term of (5)

$$[\psi W^{-1}(\psi_{,\rho}n_\rho+\psi_{,z}n_z)]^2 \leq \psi^2 W^{-2}(\psi_{,\rho}{}^2+\psi_{,z}{}^2)(n_\rho{}^2+n_z{}^2). \quad (7)$$

If we now take into account (2), (6), and the fact that $d\sigma$ is nonsingular on the horizon, we see from (7) that the left-hand side of (5) vanishes.

The integrand of the right-hand side of (5) is positive definite (recall that W is non-negative) everywhere in the black-hole's exterior. It follows that the integral can vanish only if ψ vanishes identically in the black-hole's exterior. Thus a rotating black hole in its final state cannot be endowed with an exterior scalar meson field, a π meson field for example. The proofs for vector and spin-2 fields are more complicated than the above, but the basic idea is the same.[10] For nonrotating (static) black holes, axial symmetry is not guaranteed by Hawking's theorem, so one must assume the most general static metric; but this causes no particular problems.[10]

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Energy Straggling and Radiation Reaction for Magnetic Bremsstrahlung*

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When the average energy of the photon emitted by synchrotron radiation becomes appreciable compared to the energy of the particle, the particle will undergo straggling in its energy loss, which in turn broadens the radiation spectrum. The energy distributions of particles and the emitted photons are calculated using the method of quantum electrodynamics. The results are presented together with effects due to classical radiative reaction for experimental test. The significance of energy straggling in astrophysics is discussed briefly.

In ordinary circumstances the motion and radiation of a charged particle in a magnetic field can be adequately described by classical electrodynamics neglecting the radiation damping. The significance of the radiation effect is predicted by the parameter $R_c = \frac{2}{3}\gamma^2 H/H_c$, which represents the ratio of the radiative reaction force to the Lorentz force. Here $H_c = m^2c^4/e^3 = 6\times10^{15}$ G and $\gamma = E/mc^2$. On the other hand, the validity of classical radiation theory is limited by the condition $R_q = \frac{3}{2}\gamma H/H_q \ll 1$, where $H_q = m^2c^3/e\hbar = 4.4\times10^{13}$ G. The ratio of the mean energy of the emitted photons to the energy of the particle is $\sim R_q(1+R_q)^{-1}$. Therefore quantum recoils become significant at $R_q \gtrsim 1$, and for a consistent treatment one has to use the method of quantum