Relationship between the Width for Decay from a Doorway State and the Ratio between the Numbers of the Correlating and Open Channels

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The width $\langle\Gamma_d\uparrow\rangle$ for the decay out of the doorway state is derived as a function of the average total level width $\langle\Gamma_\mu\rangle$, the channel correlation coefficient $C_{cc'}$, the ratio between the number n_d of the correlating channels and the number n of the open channels, and the level-overlapping ratio $\langle\Gamma_\mu\rangle/D$. Using the function, the relation between the ratio n_d/n and (1) the width $\langle\Gamma_d\uparrow\rangle$ and (2) the ratio $\langle\Gamma_\mu\rangle/D$ are discussed. The results indicate that the latter relation is in agreement with the experiment of Shotter, Fisher, and Scott.

Recent experimental results[1-3] indicate the existence of channel cross-correlations between different channels which make the doorway state one of the most interesting problems in the study of nuclear structure. As Lane[4] recently explained, the correlation effects in resonance capture γ radiation are due to the presence of the doorway state, as otherwise the channel correlation will not occur between two different channels. One can, therefore, assume that if there is a channel correlation between two different channels, these channels must decay out of the same doorway state in a compound nuclear reaction.

In this communication, I assume that the channel correlation is in fact a result of a doorway state, and derive the average width $\langle\Gamma_d\uparrow\rangle$ for the decay out of the doorway state as a function of the average total level width $\langle\Gamma_\mu\rangle$, the channel correlation coefficient $C_{cc'}$, the ratio between the number n_d of the correlating channels and the number n of the open channels, and the level-overlapping ratio $\langle\Gamma_\mu\rangle/D$. From this function, the average width $\langle\Gamma_d\uparrow\rangle$ may be estimated by measuring the values of $C_{cc'}$, $\langle\Gamma_\mu\rangle$, and D. I also compare this function with the relation $\Gamma_\mu = |A_{\mu d}|^2\Gamma_d\uparrow$ given by Lemmer,[5] and discuss the relation between the ratio n_d/n and (1) the width $\langle\Gamma_d\uparrow\rangle$ and (2) the ratio $\langle\Gamma_\mu\rangle/D$.

According to Moldauer,[6] if a compound nuclear reaction proceeds in a high-excitation region where many levels are overlapping, the transmission coefficient $T_c^{\text{c.n.}}$, the average resonance-absorption coefficient $\langle\theta_{\mu c}\rangle_\mu$, the quantity $\langle\gamma_{\mu c}\times\gamma_{\mu c'}\rangle_\mu$ for channel correlation, and the average level spacing D have the following relationship:

$$T_c^{\text{c.n.}} = \langle\theta_{\mu c}\rangle_\mu - \textstyle\sum_{c'} 2\pi^2 D^{-2}|\langle\gamma_{\mu c}\gamma_{\mu c'}\rangle_\mu|^2$$

$$\text{(neglecting the level correlation).} \tag{1}$$

Let us define $T_{cc'}{}^d$ as a partial transmission coefficient for decay out of the doorway state; then

$$T_c^{\text{c.n.}} = \langle\theta_{\mu c}\rangle_\mu - \textstyle\sum_{c'} T_{cc'}{}^d,$$

so that

$$T_{cc'}{}^d = 2\pi^2 D^{-2}|\langle\gamma_{\mu c}\gamma_{\mu c'}\rangle_\mu|^2 \tag{2}$$

$$= 2\pi\langle\Gamma_{dcc'}\uparrow\rangle_\mu/D, \tag{3}$$

where $\langle\Gamma_{dcc'}\uparrow\rangle_\mu$ is a partial width for the decay out of the doorway state, and the quantities $\gamma_{\mu c}$ and $\gamma_{\mu c'}$ are the single-channel reduced-width amplitudes of level μ. The partial level width $\Gamma_{\mu c}$, the reduced-width amplitude $\gamma_{\mu c}$, and the total level width Γ_μ have the following relations: $|\gamma_{\mu c}{}^2| = \Gamma_{\mu c}$ and $\Gamma_\mu = \sum_c \Gamma_{\mu c}$. Thus it follows that

$$\langle\Gamma_{dcc'}\uparrow\rangle_\mu = \pi D^{-1}|\langle\gamma_{\mu c}\gamma_{\mu c'}\rangle_\mu|^2. \tag{4}$$

By using the relationship[7]

$$|\langle\gamma_{\mu c}\gamma_{\mu c'}\rangle_\mu|^2 = C_{cc'}{}^2|\langle\gamma_{\mu c}{}^2\rangle_\mu{}^{1/2}\langle\gamma_{\mu c'}{}^2\rangle_\mu{}^{1/2}|^2, \tag{5}$$

where $C_{cc'}{}^2 = C_{cc'}(0)/[C_{cc}(0)C_{c'c'}(0)]^{1/2}$, and where $C_{cc'}(0)$, $C_{cc}(0)$, and $C_{c'c'}(0)$ are the channel cross-correlation, the auto-cross-correlation of channel c, and the auto-cross-correlation of channel c', respectively, then one obtains

$$\langle\Gamma_{dcc'}\uparrow\rangle_\mu = (\pi/D)C_{cc'}{}^2\langle\gamma_{\mu c}{}^2\rangle_\mu\langle\gamma_{\mu c'}{}^2\rangle_\mu, \tag{6}$$

and the width $\langle\Gamma_d\uparrow\rangle$ for the decay out of the doorway state can be obtained by summing Eq. (6) over all combinations of the correlating channels n_d, so that

$$\langle\Gamma_d\uparrow\rangle = \sum_{\substack{c=1\\ c\neq c'}}^{n_d}\sum_{c'=1}^{n_d}\langle\Gamma_{dcc'}\uparrow\rangle_\mu,$$

$$= \frac{\pi}{D}\sum_{\substack{c=1,\\ c\neq c'}}^{n_d}\sum_{c'=1}^{n_d} C_{cc'}{}^2\langle\Gamma_{\mu c}\rangle_\mu\langle\Gamma_{\mu c'}\rangle_\mu. \tag{7}$$

Because the value $C_{cc'}$ depends only on the number of doorway states which are common to channels c and c'[4] at the excitation energy E_d, so the value $C_{cc'}$ is independent of the quantities $\langle\gamma_{\mu c}{}^2\rangle_\mu$ and $\langle\gamma_{\mu c'}{}^2\rangle_\mu$, it follows that

$$\langle\Gamma_d\uparrow\rangle = \frac{\pi}{D}\langle C_{cc'}{}^2\rangle \sum_{\substack{c=1,\\ c\neq c'}}^{n} \sum_{c'=1}^{n} \langle\Gamma_{\mu c}\rangle_\mu \langle\Gamma_{\mu c'}\rangle_\mu, \tag{8}$$

where

$$\langle C_{cc'}{}^2\rangle = C_{cc'}{}^2 \sum_{\substack{c=1\\ c\neq c'}}^{n_d} \sum_{c'=1}^{n_d} \langle\Gamma_{\mu c}\rangle_\mu \langle\Gamma_{\mu c'}\rangle_\mu \times\Big[\sum_{\substack{c=1\\ c\neq c'}}^{n} \sum_{c'=1}^{n} \langle\Gamma_{\mu c}\rangle_\mu \langle\Gamma_{\mu c'}\rangle_\mu\Big]^{-1},$$

and n is the number of all possible decay channels out of the total level width Γ_μ, and of course $n \geq n_d$.

If this is a reaction of a large Q value, and the outgoing particles are not suppressed by the barrier penetration, Eq. (8) becomes

$$\langle\Gamma_d\uparrow\rangle \simeq \frac{n_d(n_d-1)C_{cc'}{}^2}{n^2}\frac{\pi\langle\Gamma_\mu\rangle}{2D}\langle\Gamma_\mu\rangle \tag{9}$$

$$\approx\left(\frac{n_d}{n}\right)^2 C_{cc'}{}^2\frac{\pi\langle\Gamma_\mu\rangle}{2D}\langle\Gamma_\mu\rangle \quad (\text{for } n\gg 1). \tag{10}$$

Therefore, for a given excitation energy E_d, where $\langle\Gamma_\mu\rangle$ and D are fixed, the width $\langle\Gamma_d\uparrow\rangle$ increases as the ratio n_d/n increases.

Unfortunately, it is experimentally not possible to measure the channel cross-correlations between all of the decay channels, and normally the data are obtained only for a limited number of low-lying levels. If we assume that the ratio n_d/n roughly equals the ratio $(n_d)_{\text{expt}}/(n)_{\text{expt}}$, Eq. (10) may be transformed into

$$\langle\Gamma_d\uparrow\rangle \approx \left(\frac{n_d}{n}\right)^2_{\text{expt}} C_{cc'}{}^2\frac{\pi\langle\Gamma_\mu\rangle}{2D}\langle\Gamma_\mu\rangle, \tag{11}$$

where $(n_d/n)_{\text{expt}} = (n_d)_{\text{expt}}/(n)_{\text{expt}}$ with $(n_d)_{\text{expt}}$ and $(n)_{\text{expt}}$ the experimental numbers of n_d and n, respectively. Therefore by measuring the values of $C_{cc'}$, $\langle\Gamma_\mu\rangle$, and D, we can calculate the width $\langle\Gamma_d\uparrow\rangle$ by using Eq. (11).

Now, let us compare Eq. (11) with the relationship $\Gamma_\mu = |A_{\mu d}|^2\Gamma_d\uparrow$ given by Lemmer,[5] and obtain

$$|A_{\mu d}|^2 = \frac{2D/\pi\langle\Gamma_\mu\rangle}{(n_d/n)_{\text{expt}}{}^2 C_{cc'}{}^2}, \tag{12}$$

where the quantity $|A_{\mu d}|^2$ is the probability that the doorway state is present in a compound state at excitation energy E_μ. Because $|A_{\mu d}|^2 \leq 1$, it follows that

$$\frac{2D}{\pi\langle\Gamma_\mu\rangle} \leq \left(\frac{n_d}{n}\right)^2_{\text{expt}} C_{cc'}{}^2. \tag{13}$$

For the doorway states which are common to channels c and c', $C_{cc'}\approx 1$ for a single doorway state and $C_{cc'}\approx K^{-1/2}$ for K overlapping doorway states.[4]

In experiment, as a matter of fact, the errors (statistical and finite-range data errors) are so large that only the values of $C_{cc'}$ for a single doorway are significant. Therefore, the number $(n_d)_{\text{expt}}$ is obtained only from the large values of $C_{cc'}$, which belong to a single doorway state. Thus, we put $C_{cc'} = 1$ in Eqs. (11) and (13) and obtain for a single doorway state

$$\langle\Gamma_d\uparrow\rangle \approx \left(\frac{n_d}{n}\right)^2_{\text{expt}} \frac{\pi\langle\Gamma_\mu\rangle}{2D}\langle\Gamma_\mu\rangle, \tag{14}$$

$$2D/\pi\langle\Gamma_\mu\rangle \leq (n_d/n)_{\text{expt}}{}^2. \tag{15}$$

Because $n_d \leq n$, it follows that

$$(2D/\pi\langle\Gamma_\mu\rangle)^{1/2} \leq (n_d/n)_{\text{expt}} \leq 1. \tag{16}$$

Equation (16) tells us that, if the experiments are performed in the region where

$$\langle\Gamma_\mu\rangle/D \approx 2/\pi, \tag{17}$$

for a single doorway state, the largest value of $(n_d/n)_{\text{expt}}$ is close to 1 when the value of $(2D/\pi\langle\Gamma_\mu\rangle)^{1/2}$ is close to 1.

Fortunately, there is experimental evidence to prove this conclusion. Shotter, Fisher, and Scott[1] recently found, in their experiment on $^{28}\text{Si}(p,p')^{28}\text{Si}^*$, 22 correlations out of the 36 possible different channel combinations, which corresponds to $(n_d)_{\text{expt}} \approx 7$ out of $(n)_{\text{expt}} = 9$. The reason why they got such a high value of $(n_d/n)_{\text{expt}}$ is because they did the experiment in the excitation region around $\langle\Gamma_\mu\rangle/D \approx 2$, which is reasonably close to $2/\pi$.

On the other hand, Hsu *et al.*[2] and Lee *et al.*[3] obtained $(n_d)_{\text{expt}} = 3$ out of $(n)_{\text{expt}} = 5$ for the excitation region around $\langle\Gamma_\mu\rangle/D \approx 15$ and $\langle\Gamma_\mu\rangle/D \approx 20$, respectively. It is clear that the ratio $(n_d/n)_{\text{expt}}$ of Shotter, Fisher, and Scott is larger than that of Hsu *et al.*[2] and Lee *et al.*[3] The results are listed in Table I. In the table, the width $\langle\Gamma_d\uparrow\rangle$ is calculated by using Eq. (14), and the theoretical values of n_d/n are also included. The agreement between the experimental and theoretical values of n_d/n are good, especially in the results of Ref. 1.

Finally, I make the following conclusions:

TABLE I. Quantities obtained from experiments: ^{28}Si(p,p')^{28}Si, Ref. 1; ^{28}Si(d,p)^{29}Si, Ref. 2; and ^{24}Mg(d,p)^{25}Mg, Ref. 3. The theoretical values of the ratio n_d/n and the width $\langle\Gamma_d\uparrow\rangle$ are calculated by using Eqs. (16) and (14), respectively.

	$\frac{\langle\Gamma_\mu\rangle}{D}$	$\langle\Gamma_\mu\rangle$ (keV)	$(n)_{\rm expt}$	$\left(\frac{n_d(n_d-1)}{2}\right)_{\rm expt}$	$(n_d)_{\rm expt}$	$\left(\frac{2D}{\pi\langle\Gamma_\mu\rangle}\right)^{\frac{1}{2}} \leq \left(\frac{n_d}{n}\right)_{\rm theor} \leq 1$	$\left(\frac{n_d}{n}\right)_{\rm expt}$	$\langle\Gamma_d\uparrow\rangle$ (keV)
^{28}Si(p,p')^{28}Si	2	110	9	22	7	$0.58 \leq \left(\frac{n_d}{n}\right)_{\rm theor} \leq 1$	0.8	200
^{28}Si(d,p)^{28}Si	15	30	5	3	3	$0.21 \leq \left(\frac{n_d}{n}\right)_{\rm theor} \leq 1$	0.6	243
^{24}Mg(d,p)^{25}Mg	20	40	5	3	3	$0.18 \leq \left(\frac{n_d}{n}\right)_{\rm theor} \leq 1$	0.6	432

(a) The width $\langle\Gamma_d\uparrow\rangle$ depends so strongly on the ratio n_d/n that the exact ratio n_d/n is needed to determine $\Gamma_d\uparrow$. In order to get $(n_d/n)_{\rm expt}$ as close to n_d/n as possible, an experiment with a large value of $(n)_{\rm expt}$ should be performed. (b) By counting the number $(n_d)_{\rm expt}$ only the large values of $C_{cc'}(0)$ can be included. Therefore, the width $\langle\Gamma_d\uparrow\rangle$ obtained in this way is the width for decay out of a *single* doorway state. (c) The number $(n_d)_{\rm expt}$ depends upon the calculation of $C_{cc'}(0)$, so that the calculations have to eliminate the influence from some gross structure resonance or modulation effect.[9]

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Reactions to Unbound Final States*

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It is pointed out that the strength of (^{3}He,d) reactions to unbound states is correlated with the penetrability for proton decay of these states. The anomalously weak transitions to s states fit into this pattern. Additional data from the reaction ^{116}Sn(^{3}He,d)^{117}Sb* are presented. Some questions regarding extrapolations to other reations are raised.

Considerable recent interest in stripping reactions to unbound analog states was initiated by a Letter reporting the (^{3}He,d) reaction on Zr and Mo isotopes, in which McGrath *et al.*[1] found anomalies in such transitions. In particular, $l=2$ transitions were seen strongly whereas $l=0$ transitions, expected in the same nuclei, were absent or very weak. The effect was contrary to the re-