Quadrupole Moment of the Lowest Rotational Band in Even Calcium Nuclei

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The static electric quadrupole moment of the 2_1^+ state in ^{42}Ca was measured to be $(-18.9 \pm 8.1)e$ fm^2. A $B(E2; 2_1^+ \to 0_1^+) = (81.5 \pm 3.0)e^2$ fm^4 was obtained. The $E2$ properties of the lowest seven levels were used to decompose the levels into $(fp)^2$ states plus a deformed band with $\beta_2 = +0.375 \pm 0.006$. The band structure, moments of inertia, and β_2 values in ^{42}Ca and ^{40}Ca are nearly identical, implying that the lowest $K = 0^+$ band in ^{40}Ca is also prolate.

The observation[1,2] of low-lying rotational bands coexisting with spherical states in the closed-shell nuclei ^{16}O and ^{40}Ca provoked considerable interest in the structure of these nuclei. The origin of these deformed levels is attributed to core excitation.[3,4] In particular, the lowest rotational band has been interpreted to be predominantly a 4p-4h (four-particle, four-hole) excitation at a large prolate deformation. It is necessary to measure a static electric quadrupole moment of a member of the band to determine the sign of deformation. Unfortunately, static moment measurements are impractical for ^{16}O and ^{40}Ca because of the high excitation energy and weak $B(E2)$ strengths to the ground state from the deformed band members. However, the sign of deformation of the lowest rotational band in ^{40}Ca can be obtained from a measurement of the static electric quadrupole moment of the lowest 2^+ state (Q_{2^+}) in ^{42}Ca. The low-lying energy-level spectrum and enhanced $B(E2)$ values in ^{42}Ca suggest[4] that the low-lying spectrum can be described as a mixture of a $(fp)^2$ spectrum and a low-lying deformed band similar in structure to that observed in ^{40}Ca. In particular, the lowest $(fp)^2$ and deformed 2^+ states are observed in the reaction ^{43}Ca$(d, t)^{42}$Ca to be strongly mixed.[6,7] The configurations $(f_{7/2})^2$ and $(fp)^2$ both predict a positive Q_{2^+} for the lowest 2^+ state. An admixture of a prolate deformed configuration in the lowest 2^+ state would result in a Q_{2^+} value appreciably more negative than the $(fp)^2$ prediction, making it possible to extract a moment for the deformed state.

The static electric quadrupole moment of the 1.524-MeV first excited state of ^{42}Ca was determined using the reorientation effect in Coulomb excitation. A 65-μg/cm^2 target of enriched ^{42}Ca was excited by a beam of 60-MeV ^{32}S ions. The inelastically scattered projectiles, as well as the recoiling target nuclei, were detected in surface barrier detectors at scattering angles of 25°, 30°, 35°, and 40°, in coincidence with de-excitation γ rays detected in four NaI(Tl) detectors. Three of the NaI(Tl) detectors were mounted in the scattering plane with the fourth detector mounted perpendicular to the scattering plane. The experiment was run for two different sets of angles of the three in-plane γ detectors to measure the attenuation of the γ-ray angular distribution due to the deorientation effect. The four particle detectors detected both the scattered projectile and the recoiling target nuclei, permitting simultaneous measurements of the inelastic cross section at eight widely different c.m. angles.[8] The combined time-of-flight resolution (1.3 nsec full width at half-maximum) and the particle energy resolution were sufficient to separate the ^{42}Ca nuclei clearly from the ^{32}S nuclei in every particle detector. Only the photopeak counts of the coincident 1.524-MeV γ ray were used in the analysis. Singles spectra were also recorded to normalize the cross sections to the Rutherford cross section.

Coulomb excitation calculations were done with the de Boer–Winther coupled-channels computer code.[9] Besides the ground state and first excited state at 1.524 MeV, states at 1.836 (0^+), 2.422 (2^+), and 2.750 MeV (4^+) were included. Matrix elements were calculated from known data.[10,11] Analysis of the experimental data followed the method of Ref. 8.

The static quadrupole moment extracted from these data is $Q_{2^+} = -(18.9 \pm 8.1)e$ fm^2. A value $B(E2;\ 2^+ \to 0^+) = (81.5 \pm 8.0)e^2$ fm^4 was also measured. The quoted error in the quadrupole moment is due only to statistical errors in the data. The sign of the interference term due to the 2_2^+ level contributed the only appreciable uncertainty in the effects due to the higher levels. The sign used is that given by the calculations discussed later. The other sign is in conflict with the model in addition to producing a very large moment of $-31e$ fm. Excitation of the target due to nuclear effects may have been non-negligible. At the beam energy used, the separation distance between the nuclear surfaces for a head-on collision is 5.4 fm. Recent measurements[8] show that the Q_{2^+} value obtained with this separation distance may be $2e$ to $3e$ fm^2 more negative than the result which would have been obtained with a minimum separation exceeding 6.0 fm. The higher energy was necessary, though, to obtain an acceptable counting rate, and the error introduced thereby is small compared to the other experimental errors. The quoted value for the Q_{2^+} has not been corrected for this effect.

The energy levels and $E2$ matrix elements in ^{42}Ca were calculated for the shell-model configurations $(f_{7/2})^2$ and $(fp)^2$ using the Kuo-Brown matrix elements. Calculations were also performed allowing 2p and 4p-2h configurations in the $(f_{7/2}\text{-}d_{3/2})$ vector space using an effective interaction due to Goode.[12] The latter configuration produced a low-lying level scheme in reasonable agreement with experiment, but none of the three configurations was able to predict the $E2$ matrix elements accurately and, in particular, Q_{2^+} was consistently predicted to be positive. In order to introduce the correlations required to produce a large negative Q_{2^+}, it is necessary to include the complete (sd) and (fp) shells. Flowers and Skouras[13] performed such a calculation allowing 2p and 4p-2h configurations in the (sd), (fp) vector space, but obtained level spectra and $B(E2)$ values which are little better than our (fd) calculation. Flowers and Skouras did not calculate the static quadrupole moments, which are the most sensitive test of the wavefunctions.

These shell-model calculations emphasize that the $E2$ properties require considerable admixtures of complicated core-excited configurations. Shell-model calculations in such a large space are neither feasible nor particularly useful. Instead, it is more useful to follow the Gerace and Green[4] approach and postulate that the low-lying spectrum can be described as a mixture of $(fp)^2$ states and some other complex states—presumably deformed. Gerace and Green calculated the interaction between deformed and spherical $(fp)^2$ states using Nilsson wave functions from which they predicted the wave functions and $E2$ matrix elements. However, we now have enough $E2$ information to reverse this procedure, that is, to calculate the wave functions from the experimental $E2$ matrix elements. The lowest 0^+, 2^+, 4^+, and 6^+ members of the $(fp)^2$ spectrum, calculated using the Kuo and Brown matrix elements, were assumed mixed with four states of presumably complex shell structure having spins 0^+, 2^+, 4^+, and 6^+. It was assumed that the $E2$ single-particle operator did not couple the $(fp)^2$ and complex states. There are nine experimental $E2$ matrix-element data which couple the pairs of 0^+, 2^+, and 4^+ levels. These data were used to determine the seven unknowns, that is, the neutron effective charge, the mixing parameters for the wave functions of the three pairs of levels, and the $E2$ matrix elements between the complex states C, i.e., $\langle C0^+\|E2\|C2^+\rangle$, $\langle C2^+\|E2\|C2^+\rangle$, and $\langle C2^+\|E2\|C4^+\rangle$. The resulting wave functions had amplitudes for the complex component of $+0.520\pm0.001$, -0.760 ± 0.012, and $+0.442\pm0.051$ for the lowest 0^+, 2^+, and 4^+, respectively, where the sign of the $(f_{7/2})^2$ component of the wave function has been chosen positive. The wave-function admixtures and experimental excitation energies were used to determine the unperturbed energies for the $(fp)^2$ and complex states. As shown in Fig. 1, the unperturbed excitation energies of the complex states have a rotational character in that they fit the relation $E(J)=[(1.345\pm0.009)+(0.0906\pm0.0008)\times J(J+1)]$ MeV shown as a solid line in Fig. 1. The lowest rotational band in ^{40}Ca, also shown in Fig. 1, obeys the relation $E(J)=[(3.345\pm0.009)+(0.0962\pm0.0009)J(J+1)]$ MeV, that is, the moment of inertia differs by 6% from the value for the uncoupled ^{42}Ca levels. Furthermore, in both nuclei a second $K=0^+$ band and a $K=0^-$ band appear to occur at the same excitation energies above the lowest $K=0^+$ band. An additional 2^+

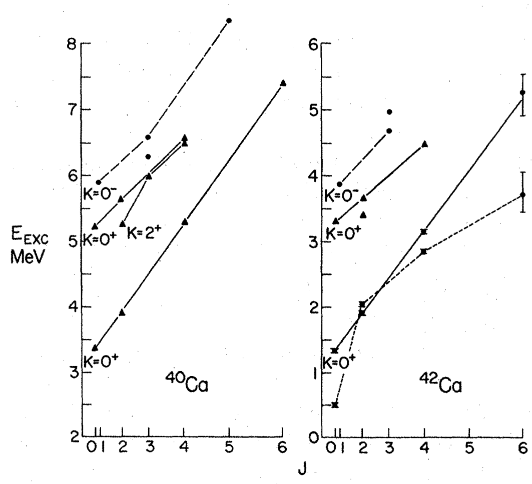

FIG. 1. Energy levels in ^{40}Ca and ^{42}Ca plotted versus $J(J+1)$ to illustrate the rotational bands in these nuclei. The lowest band in ^{42}Ca corresponds to the unperturbed energies of the complex states. The low-lying levels in ^{42}Ca connected by the dashed line are the unperturbed $(fp)^2$ spectrum.

level is known in ^{42}Ca at an energy corresponding to the first member of the $K=2^+$ band in ^{40}Ca. The observation of enhanced in-band $E2$ transitions is the only sure identification of a rotational band. Several $E2$ transitions are known[5] in ^{40}Ca which correspond to the intrinsic quadrupole moments listed in Table I. The $E2$ matrix elements for the complex states in ^{42}Ca determined from the mixed model, correspond to the intrinsic quadrupole moments also listed in Table I. Note that within the experimental errors, the moments in ^{42}Ca are constant and positive corresponding to a prolate rotor. The present extracted neutron effective charge $e_n = +(0.646 \pm 0.039)e$ and a constant intrinsic quadrupole moment yield $E2$ matrix elements in excellent agreement with the available experimental data. Only one 6^+ level and a single corresponding $E2$ matrix element are known. This $E2$ matrix element is close to the $(fp)^2$ value which allows two solutions for the 6^+ wave-function admixture, -0.028 ± 0.002 or $+0.450 \pm 0.129$, depending on the relative sign of the deformed and $(fp)^2$ parts of the wave function. Jamshidi and Alford[7] have observed two new levels in ^{42}Ca at 5.189 and 5.790 MeV which are populated by pure $L=3$ transitions in the reaction ^{43}Ca$(d,t)^{42}$Ca with spectroscopic factors of 0.13 ± 0.01 and 0.17 ± 0.02, respectively, suggesting possible spins of 0^+ and 6^+ for these levels. The rotational model predicts an unperturbed 6^+ level at (5.152 ± 0.030) MeV. If the mixing amplitude is assumed to be -0.028 ± 0.002, the energy of this state will be increased to 5.154 ± 0.030 MeV by the perturbation. Using the other solution increases the energy of the upper 6^+ state to 5.65 ± 0.30 MeV. Thus, unfortunately, both solutions result in a 6^+ level in agreement with a possible experimental 6^+ level. However, the solution with the 5.790-MeV level as 6^+ predicts a (d,t) spectroscopic factor in excellent agreement with Jamshidi and Alford, whereas the other solution predicts a spectroscopic factor to the 5.189-MeV level 200 times too small. Thus, we infer that the 5.790-MeV level has spin 6^+. The 5.189-MeV level is probably the second 0^+ level predicted at about this energy by the $(fp)^2$ model.

TABLE I. Intrinsic electric quadrupole moments for ^{40}Ca (Ref. 5) and for the complex states of ^{42}Ca, in units of e fm^2.

Band	^{40}Ca	^{42}Ca
Lowest $K=0^+$		
$0^+ \leftrightarrow 2^+$	$\lvert 112 \pm 8 \rvert$	$\lvert 127.8 \pm 1.8 \rvert$
$2^+ \leftrightarrow 2^+$	$\cdots$	$+112 \pm 33$
$2^+ \leftrightarrow 4^+$	$\lvert 138 \pm 14 \rvert$	$\lvert 110 \pm 73 \rvert$
Mean	$\lvert 118 \pm 7 \rvert$	$+127.7 \pm 1.8$
Second $K=0^+$		
$4^+ \leftrightarrow 2^+$	$\lvert 170^{+42}_{-32} \rvert$	$\cdots$
First $K=2$		
$3^+ \leftrightarrow 2^+$	$\lvert 144 \pm 28 \rvert$	$\cdots$
$4^+ \leftrightarrow 2^+$	$\lvert 130^{+42}_{-21} \rvert$	$\cdots$

The constant intrinsic quadrupole moments and energies of these uncoupled complex states in ^{42}Ca suggest that they are members of a rather good prolate spheroidal rotational band. The average intrinsic quadrupole moment in ^{42}Ca corresponds to $\beta_2 = +0.434 \pm 0.006$ for a uniform charge distribution with $R = 1.2A^{1/3}$ fm and to $\beta_2 = 0.375$ ± 0.006 for a Fermi charge distribution with a value of $\langle \rho r^2 \rangle$ taken from electron scattering data. The mean intrinsic quadrupole moment for the corresponding $K=0^+$ band in ^{40}Ca corresponds to $|\beta_2| = 0.355 \pm 0.024$ for a Fermi charge distribution. The essentially identical moments of inertia, relative spacing of bands, and intrinsic quadrupole moments in ^{40}Ca and ^{42}Ca are convincing evidence that the band structures in these nuclei are similar in nature. Thus, our measurement of a positive sign for the deformation of the rotational band in ^{42}Ca almost certainly implies that the lowest $K=0$ rotational band in ^{40}Ca has a prolate deformation.

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[1]E. B Carter, G. E. Mitchell, and R. H. Davies, Phys. Rev. 133B, 1421 (1964).

[2]R. W. Bauer, A. M. Bernstein, G. Heymann, E. P. Lippincott, and N. S. Wall, Phys. Lett. 14, 129 (1965).

[3]G. E. Brown and A. M. Green, Nucl. Phys. 75, 401 (1966).

[4]W. J. Gerace and A. M. Green, Nucl. Phys. A93, 110 (1967).

[5]J. R. MacDonald, D. H. Wilkinson, and D. E. Alburger, Phys. Rev. C 3, 219 (1971).

[6]J. H. Bjerregaard, H. R. Blieden, O. Hansen, G. Sidenius, and G. R. Satchler, Phys. Rev. 136B, 1348 (1964).

[7]A. Jamshidi and W. P. Alford, to be published.

[8]P. M. S. Lesser, D. Cline, P. Goode, and R. N. Horoshko, University of Rochester Nuclear Structure Research Laboratory Report No. 43 (to be published).

[9]A. Winther and J. de Boer, in *Coulomb Excitation,* edited by K. Alder and A. Winther (Academic, New York, 1966), p. 303.

[10]W. J. Kossler, J. Winkler, and C. D. Kavaloski,

Phys. Rev. 177, 1725 (1969).
[11]R. Hartmann and H. Grawe, Nucl. Phys. A164, 209 (1971).
[12]P. Goode, private communication.
[13]B. H. Flowers and L. D. Skouras, Nucl. Phys. A136, 353 (1969).

Nuclear Rotation in Heavy-Ion Scattering*

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The possibility is discussed that the observed energy dependence of the real part of the optical potential in ^{16}O-^{16}O elastic scattering arises from rotation of nuclei induced by their interaction. An estimate indicates that the effect may have the required magnitude.

An intriguing feature of recent observations of the elastic scattering of ^{16}O by ^{16}O and other nuclei of about the same mass is that the real part of the appropriate optical potential appears to increase with bombarding energy.[1] This increase in depth is contrary to the general experience with nucleon–heavy-nucleus scattering. It may, however, be due to rotational features of the encounter not ordinarily considered in scattering calculations. While quantitative proof of this suggestion is elusive, a simple plausibility argument makes it seem attractive. We consider here only the ^{16}O on ^{16}O data analyzed by Siemssen.[1]

As long as the internal structure of the ground states of the approaching nuclei remains undisturbed, their only contribution to the angular momentum is through the motion of their centers of mass as considered in the usual treatment of scattering. The interaction between the two nuclei can influence the internal structure in such a way as to give each of them an effective intrinsic moment of inertia $\mathscr{I}$. In the model considered here, the deformed ^{16}O nuclei then rotate with the same angular velocity as does the internuclear displacement $\vec{R}$. This rotation need not remove the nuclei from the elastic channel, for the modification of internal structure is nearly adiabatic, the velocity of a 60-MeV ^{16}O amounting to less than that of a 4-MeV nucleon, for example.

The Hamiltonian, in the center of mass, for such a system is

$$H = \frac{\hbar^2}{2\mu}\frac{1}{R^2}\frac{\partial}{\partial R}R^2\frac{\partial}{\partial R} + \frac{\hbar^2}{2}\frac{L^2}{2\mu R^2 + 2\mathscr{I}} + V(R),$$

where V is the usual optical-model potential written with $\vec{R}$ as the relative coordinate, μ is the reduced mass, $\vec{L}$ is the total angular momentum of the system, and $\mathscr{I}$ is the moment of inertia of an ^{16}O nucleus about its center.

The influence of the moment of inertia of the two deformed ^{16}O can be expressed in V_{eff} thus:

$$H = -\frac{\hbar^2}{2\mu}\frac{1}{R^2}\frac{\partial}{\partial R}R^2\frac{\partial}{\partial R} + \frac{\hbar^2 L^2}{2\mu R^2} + V_{eff}(R),$$

$$V_{eff} = V + \frac{\hbar^2 L^2}{2\mu R^2}\left(\frac{1}{1+\gamma} - 1\right)$$

$$= V - \frac{\hbar^2 L^2}{2\mu R^2}\frac{\gamma}{1+\gamma} \quad \text{with } \gamma = \frac{2\mathscr{I}}{\mu R^2}.$$

Since V is negative, the additional term increases the depth of the potential. The moments arise from the interaction of the nuclei and should increase as the nuclei approach each other (i.e., with decreasing R), and $[\gamma/(1+\gamma)]/R^2$ increases even more rapidly. However, Siemssen analyzes his scattering results in terms of a V_{eff} of the same shape as V but increased by a factor $(1+0.25E)$; and it is convenient in this estimate to consider the additional term to have roughly the same Woods-Saxon (WS) shape (in the outer part which affects the scattering).

To estimate $\mathscr{I}$, we use the picture of the states of ^{16}O given by Brown and Green,[2] who have shown that the low-lying, even-parity spectrum of ^{16}O can be explained by introducing primitive deformed states [a 2p-2h (two-particle, two-hole) state at 8.51 MeV and a 4p-4h state at 6.51 MeV] above a spherical unperturbed state at 2.31 MeV. These states then perturb each other through two-body interactions and can be made to reproduce the low-lying spectrum. The ground state consists mainly of the 2.31-MeV state pushed down by an admixture of the 2p-2h state. The 4p-4h state has the strongest $E2$ transition to the ground state and is expected to be strongly admixed by the internuclear perturbation. There is a rotational band built on it, and the separation of the