view of the low threshold ($v_D \ll a_e$) and relative insensitivity to T_e/T_i, this cross-field ion acoustic instability may be responsible for anomalous transport and ion heating in collisionless shocks,[7, 14] neutral-sheet phenomena (e.g., Earth's magnetic tail), and other situations where there is drift of electrons relative to ions across a magnetic field.

We are indebted to Mr. William Gould for assistance with the computations.

*Research partially supported by the U. S. Air Force Office of Scientific Research, the U. S. Atomic Energy Commission, the Office of Naval Research, and TRW Internal Research and Development program.

†Present address: Culham Laboratory, Abingdon, Berkshire, England.

‡Present address: Physics Department, Massachusetts Institute of Technology, Cambridge, Mass. 02139.

[1]V. I. Aref'ev, Zh. Tech. Phys. 39, 1973 (1969) [Sov. Phys. Tech. Phys. 14, 1487 (1970)].

[2]H. V. Wong, Phys. Fluids 13, 757 (1970).

[3]S. P. Gary and J. J. Sanderson, J. Plasma Phys. 4, 739 (1970); S. P. Gary, J. Plasma Phys. 4, 753 (1970).

[4]D. W. Forslund, R. L. Morse, and C. W. Nielson, Phys. Rev. Lett. 25, 1266 (1970).

[5]M. Lampe, W. Manheimer, J. B. McBride, J. H. Orens, R. Shanny, and R. N. Sudan, Phys. Rev. Lett. 26, 1221 (1971).

[6]D. E. T. F. Ashby and A. Paton, Plasma Phys. 9, 359 (1967).

[7]N. A. Krall and P. C. Liewer, Phys. Rev. B 4, 2094 (1971).

[8]B. D. Fried and S. Conte, *The Plasma Dispersion Function* (Academic, New York, 1961).

[9]Specifically, we note that ω/kc_s is close to 1. Then

$$\frac{\omega}{k_z a_e} \approx \frac{k}{k_z}\frac{v_D - c_s}{a_e} = \frac{k}{k_z}\left(\frac{m}{2M}\right)^{1/2}\left(\frac{1-v_D}{c_s}\right),$$

which is large only for propagation very nearly perpendicular to $\vec{B}$. For modest values of v_D/c_s and for most orientations of $\vec{k}$, this quantity will actually be small.

[10]R. J. Taylor and K. R. Mackenzie, in *Proceedings of the International Conference on Physics of Quiescent Plasmas* (Ecole Polytechnique, Paris, 1969), Pt. 3, p. 57; R. J. Taylor, D. R. Baker, and H. Ikezi, Phys. Rev. Lett. 24, 205 (1970).

[11]R. J. Taylor, Ph.D. thesis, University of California, Los Angeles, 1970 (unpublished).

[12]J. M. Sellen, W. Berstein, and R. F. Kemp, Rev. Sci. Instrum. 36, 316 (1965).

[13]Although accounting for the experimental observations by other low-frequency instabilities cannot be absolutely eliminated, we note that (1) the ion stream produced by the DP machine has negligible velocity shear, so that the Kelvin-Helmholtz instability is not a likely candidate, and (2) the instability is strongest in the central region of the plasma, where the density gradient is smallest, which argues against drift-wave instabilities. In addition, of course, the frequency, even in the ion rest frame, is well above the ion cyclotron frequency, i.e., outside the range of the fluid approximations generally used for such instabilities.

[14]The acoustic modes observed in the collision shock experiments of J. W. M. Paul, in Proceedings of the Fourth Conference on Plasma Physics and Controlled Nuclear Fusion Research, Madison, Wisconsin, 1971 (to be published), Paper J9, could well include the modes discussed here.

Saturation of the Decay Instability for Comparable Electronic and Ion Temperatures*

E. Valeo, C. Oberman, and F. W. Perkins
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08540
(Received 10 December 1971)

We present a model for the nonlinear evolution of the electron-plasma ion-acoustic decay instability when the ratio of electron and ion temperatures is of order 1. Ion nonlinear Landau damping is found to be the dominant saturation mechanism. We calculate the saturated spectral intensity and pump absorption coefficient. An estimate is given of the energy that the enhanced fluctuations absorb from an electromagnetic wave incident on an inhomogeneous plasma.

There is increasing evidence that the enhanced fluctuation levels arising from the excitation of parametric instabilities are responsible for the enhanced heating and anomalous absorption observed in a wide variety of laboratory,[1-3] numerical,[4] and ionospheric[5] experiments, where a plasma is irradiated by an intense monochromatic electromagnetic wave. We believe many of the essential features of these experiments are contained in the model presented here for computing the saturation level of these instabilities.

We treat the nonlinear evolution of the decay[6,7] instability in a homogeneous plasma in which the pump wave $\vec{E}(t)$ can be regarded as monochromatic with frequency ω_0 and can be treated in the dipole approximation, i.e.,

$$\vec{E}(t)=\vec{E}_0\sin(\omega_0 t). \tag{1}$$

We thus defer to future work discussion of the important effects associated with average plasma inhomogeneities and the self-consistent propagation of the pump. Those wave vectors $\vec{k}$ for which the decay conditions $\omega_0=\omega_{\vec{k}}+\omega_a$ are approximately satisfied will be unstable. Here $\omega_{\vec{k}}=\omega_p[1+\frac{3}{2}(k\lambda_D)^2]$ is the plasma-wave frequency and $\omega_a=k(T_e/m_i)^{1/2}(1+3T_i/T_e)^{1/2}$ is the acoustic-wave frequency, with $\lambda_D=(T_e/4\pi ne^2)^{1/2}$ the electron Debye length, and T_j the temperature of species j.

The ratio of electron and ion temperatures will be chosen to be approximately 1, a value relevant to ionospheric plasma parameters. In this limit, for the small wave-vector modes which we consider, the plasma-wave damping decrement $\nu_e \ll \nu_a$, the acoustic-wave damping decrement. The damping rate ν_e is taken as the sum of electron-ion collisional and electron Landau damping, and ν_a is principally due to ion Landau damping. The physical effects associated with this parameter regime are drastically different from those when $T_e \gg T_i$. (We do not consider this regime here!)

We choose the pump intensity so that the maximum parametric growth rate γ_M satisfies the inequalities

$$\nu_a \gg \gamma_M=\nu_e\left(\frac{\nu_e}{16\omega_k}\frac{\omega_a}{\nu_a}\frac{E_0^{\,2}}{4\pi nT_e}-1\right)\gtrsim \nu_e.$$

It is well known that under these conditions another instability, the oscillating two-stream instability,[7,8] may be excited with comparable growth rate. However, the active region of $\vec{k}$ space is down by a factor ν_e/ν_a, and hence we neglect it from consideration.

With this choice of $E_0^{\,2}$ and T_e/T_i, considerable analytic progress may be made in obtaining the spectral intensity. The principal simplification resulting from the relative ordering $\nu_e \ll \nu_a \sim \omega_a$ occurs because on the kinetic time scale ν_e^{-1} the ion waves do not represent additional degrees of freedom, but rather relax quickly to functions of the nonlinear low-frequency charge density. Thus, only the evolution of the plasma waves need be followed on the long time scale. It follows that nonlinear Landau damping is the dominant nonlinear mechanism.

The kinetic equation for plasma waves in the random-phase approximation is

$$\frac{1}{2}\frac{dy}{d\tau}(x,\mu)\equiv y(x,\mu)\left\{\frac{E^2\mu^2}{1+x^2}-1-\frac{\partial}{\partial x}\int_{-1}^{+1}d\mu'[\mu^2(3\mu'^2-1)+(1-\mu'^2)]\,y(x,\mu')\right\}+\delta\mu^2. \tag{2}$$

All variables are dimensionless and are defined as follows:

$$y(x,\mu)=\alpha\langle|\vec{E}_{\vec{k}}(t)|^2\rangle/8\pi VT_e,$$

$$\alpha=(12\pi)^{-1}\omega_k(k\lambda_D)^2\nu_e^{-1}(n\lambda_D^{\,3})^{-1}(m_e/m_i)^{1/2},$$

$$E^2=(\omega_k/16\nu_e)(E_0^{\,2}/4\pi nT_e)(\omega/_a\nu_a),$$

$$\tau=\nu_e t,\quad x=(\omega_0-\omega_k-\omega_a)/\nu_a,$$

$$\mu=\cos\theta,\quad \delta=(k\lambda_D/24\sqrt{2\pi})(\omega_k/\nu_e n\lambda_D^{\,3})E^2.$$

Here $\langle|\vec{E}_{\vec{k}}(t)|^2\rangle/8\pi V$ is the ensemble average[9] of the fluctuation electric field intensity, V is the system volume, and t is the time. Spherical coordinates are used with the polar axis directed along $\vec{E}_0$. In obtaining this form we have assumed solutions which have azimuthal symmetry and which are even functions of μ. The first term on the right-hand side of Eq. (2) is the parametric growth term. The second term is the ion nonlinear Landau damping term[9] which leads to saturation.

Estimates[10] of the relative strengths of ion as compared with electron nonlinear Landau damping show that the former is highly dominated in the small wave-vector expansion. A further approximation which allows us to obtain an analytic result for the spectral intensity is the approximation of the integral form of the nonlinearity by an integral differential form. This form for the ion nonlinear Landau damping is obtained[11] under the assumption that the scale for change of the spectral intensity $I_{\vec{k}}$ with $|\vec{k}|$ is greater than that for $Z\equiv(\omega_{\vec{k}}-\omega_{\vec{k}'})/(|\vec{k}-\vec{k}'|v_{T_i})$. We take the width of the linear instability region as an estimate of the scale in $|\vec{k}|$ over which $I_{\vec{k}}$ changes. If we define the spherical angle between $\vec{k}$ and $\vec{k}'$ as $\Delta\theta$, then the condition of applicability may be written $2\sin(\Delta\theta/2)<1$. We find some $\Delta\theta=\pi$ below, and thus our approximation is only marginally valid for such large angles. The predicted angular dependence of $I_{\vec{k}}$ may be in error, but we expect the total energy in the saturated state to be well estimated since the order of magnitude of the nonlinearity is insensitive to the shape of the spectral intensity. Studies of the kinetic equation using the more exact form of the nonlinearity will be reported at a later date.

The last term in Eq. (2) is the Cherenkov emission term,[12] much enhanced above the thermal level, and is determined by ion emission upshifted by the pump frequency for all but very small values of the pump intensity.

Although real frequency shifts of the modes due both to the pump and to the nonlinearity may not be neglected *a priori*, we have neglected these since it is to be expected that the principal effect of frequency shifts will be to change the region of space which contains the enhanced level of fluctuations so that the frequency matching condition is satisfied between the nonlinear mode frequencies and the pump.

We neglect the possibility of saturation by quasilinear modification of the distribution function for two reasons. First, the phase velocities of the enhanced plasma waves are large with the consequence that very few particles participate in quasilinear diffusion. Secondly, in a nonuniform plasma, such as the ionosphere, the spatial region containing enhanced fluctuations is small compared to the mean free path of electrons with velocities near the phase velocity of the plasma waves. As a result electrons which undergo quasilinear diffusion rapidly leave the unstable region.

We note that although in what follows we seek only the time-independent solution to Eq. (2), we can indeed use it to study the time evolution of the spectral intensity on the ν_e^{-1} time scale.

A formal solution to Eq. (2) is

$$y = \delta\mu^2/[a^2(x) - b^2(x)\mu^2], \tag{3}$$

whose integral over μ is immediately obtained as

$$\int_{-1}^{+1} d\mu\, y(x,\mu) = \frac{\delta}{2ab}\ln\left(\frac{a/b+1}{a/b-1}\right)^2. \tag{4}$$

Since $\delta \ll 1$, the source may be treated as a small perturbation for those modes which are appreciably excited if, as we assume, $E > 1+\delta$. We develop a solution which is asymptotic in the small parameter δ.

The solution for y, our primary result, is obtained to dominant order as

$$y(x,\mu) = \delta\mu^2[2\exp(-\bar{y}/\delta) + 1 - \mu^2]^{-1}, \tag{5}$$

where

$$\bar{y}(x) = \tfrac{1}{2}\{E^2[\arctan x + \arctan(E^2-1)^{1/2}] - x - (E^2-1)^{1/2}\}. \tag{6}$$

Of course, the solution is dominant only in the region of x for which $\bar{y} > 0$, and we disregard the small ("thermal") corrections to y outside this region. The spectral intensity is sharply peaked along the direction of the pump wave. In fact, its angular half-width $\theta_{1/2}$ is exponentially small, i.e.,

$$\theta_{1/2} \sim \exp\left\{-\frac{24(2\pi)^{1/2}}{k\lambda_D}\frac{\nu_e}{\omega_k}n\lambda_D^3\left[\arctan(E^2-1)^{1/2} - \frac{(E^2-1)^{1/2}}{E^2}\right]\right\}. \tag{7}$$

The total energy in the high-frequency component of the excited spectrum is also of interest. The dominant result for the integral of y over μ is, from Eq. (4),

$$\int d\mu\, y(x,\mu) = \bar{y}. \tag{8}$$

Integration of this result between the two zeros of $\bar{y}$ yields

$$\int_{-(E^2-1)^{1/2}}^{x_0} dx \int_{-1}^{+1} d\mu\, y(x,\mu) = \tfrac{1}{2}\{x_0[x_0 + (E^2-1)^{1/2}] - \tfrac{1}{2}x_0^2 - \tfrac{1}{2}E^2\ln[(x_0^2+1)/E^2]\}.$$

If $E^2 \gg 1$, then the second zero x_0 given by $x_0 \simeq \pi E^2$; and thus

$$\int dx \int d\mu\, y = \tfrac{1}{4}\pi^2 E^4$$

or, in dimensional units,

$$\frac{\text{wave energy}}{\text{particle energy}} = \frac{\pi}{2}\left(\frac{\nu_e}{\omega_k}\right)E^4. \tag{9}$$

We note that the parametric growth and the Cherenkov emission terms contain additional x dependence which has been neglected in Eq. (2). We have included these dependences in other calculations with the result that the wave energy at saturation, Eq. (9), is only slightly modified and that the angular half-width $\theta_{1/2}$ depends on x but is never greater than the result of Eq. (7).

We now compute the rate of work done by the pump, and we use this to define an effective energy absorption rate $\nu_{\rm eff}$ by $\nu_{\rm eff}E_0^2/8\pi \equiv (1/T)\int_T dt\, \vec{E}_0(t)\cdot\vec{j}_0(t)$.

Since ion nonlinear Landau damping conserves plasma-wave energy,[13] to $O(k^2\lambda_D^2)$, we may write

$$\nu_{\rm eff}\frac{E_0^2}{8\pi} = 2\int d^3k\,\nu_e \frac{\langle|E_{\vec{k}}(t)|^2\rangle}{4\pi V} = \frac{\pi\nu_e^2 n T_e}{\omega_k}\left(\frac{E_0}{E_T}\right)^4.$$

The energy flow is as follows: First, the pump field suffers ion nonlinear Landau damping thereby driving the plasma waves unstable. Second, these waves grow to a sufficiently large level that they in turn are stabilized by coupling via ion nonlinear Landau damping to stable plasma waves, causing an enhancement in their level. Finally, collisional and/or Landau damping of the plasma waves transfers the energy to the electrons.

Our result (9) allows an estimate of the total energy absorbed from a plane electromagnetic wave incident on a nonuniform plasma slab. The method is to obtain the local heating rate in the unstable region by multiplying (9) by $2\nu_e$, compute E_0 from WKB theory for electromagnetic waves in a lossless plasma, and integrate over the unstable region. The result for the nonlinear absorption coefficient Γ (valid when the absorption is weak, i.e., Γ less than 1) is

$$\Gamma = \frac{\pi}{2}\frac{H\nu_c}{c}\frac{\omega_a}{4\nu_a}\frac{E_V^2}{E_{TC}^2}\ln\left[\frac{1}{4}\left(\frac{H\omega}{c}\right)^{2/3}\right], \tag{10}$$

where E_{TC} is the threshold field determined by $\nu_e = \nu_c$, E_V is the incident electromagnetic-wave field strength in vacuum, and H is the electron density scale length. If the nonlinear absorption is to dominate classical absorption, E_V must exceed E_{TC}.

We acknowledge valuable discussions with W. L. Kruer, J. M. Dawson, V. M. Goldman, and D. F. DuBois. One of the authors (E.J.V.) was supported by a National Science Foundation Predoctoral Fellowship.

*Work supported in part by the U. S. Atomic Energy Commission under Contract No. AT(30-1)-1238, by the U. S. Air Force Office of Scientific Research under Contract No. F 44620-70-C-0033, and by the Advanced Research Projects Agency under Order No. F 30602-72-C-0053.

[1]H. P. Eubank, Phys. Fluids 14, 2551 (1971).

[2]T. K. Chu and H. W. Hendel, Bull. Amer. Phys. Soc. 16, 1297 (1971).

[3]H. Dreicer, J. C. Ingraham, and D. B. Henderson, Phys. Rev. Lett. 26, 1616 (1971).

[4]W. L. Kruer, P. K. Kaw, J. M. Dawson, and C. Oberman, Phys. Rev. Lett. 24, 987 (1970).

[5]W. F. Utlaut and R. Cohen, Science 174, 245 (1971).

[6]D. F. DuBois and M. V. Goldman, Phys. Rev. Lett. 14, 544 (1965).

[7]K. Nishikawa, J. Phys. Soc. Jap. 24, 916, 1152 (1968).

[8]J. R. Sanmartin, Phys. Fluids 13, 1533 (1970).

[9]A. Rogister and C. Oberman, J. Plasma Phys. 2, 33 (1968), and 3, 119 (1969).

[10]E. J. Valeo, Ph.D. thesis, Dept. of Astrophysical Sciences, Princeton University, 1971 (unpublished).

[11]A. A. Galeev, V. I. Karpman, and R. Z. Sagdeev, Dokl. Akad. Nauk SSSR 157, 1088 (1964) [Sov. Phys. Dokl. 9, 651 (1965)].

[12]D. F. DuBois and M. V. Goldman, Phys. Rev. 164, 207 (1967).

[13]B. B. Kadomtsev, *Plasma Turbulence* (Academic, New York, 1965), p. 66.